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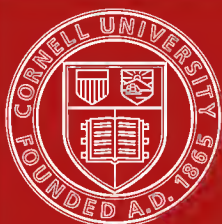
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Serial No. 9

DEPARTMENT OF COMMERCE

U. S. COAST AND GEODETIC SURVEY

E. LESTER JONES, SUPERINTENDENT

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GEODESY

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APPLICATION OF THE THEORY OF LEAST  
SQUARES TO THE ADJUSTMENT  
OF TRIANGULATION

BY

OSCAR S. ADAMS

COMPUTER

UNITED STATES COAST AND GEODETIC SURVEY

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# APPLICATION OF THE THEORY OF LEAST SQUARES TO THE ADJUSTMENT OF TRIANGULATION

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## GENERAL STATEMENT

In this publication the aim has not been to develop the theory of least squares, but to illustrate the application of the method to the problems arising in the adjustment of triangulation. The general idea has been to collect material in one volume that will serve as a working manual for the computer in the office and for such other members of the Survey as may desire to make these special applications. It has not been deemed necessary to insert the derivation of formulæ except in the case of a few special ones that are not usually found in the textbooks on least squares.

For the general theory reference should be made to such books as the following:

Crandall: *Geodesy and Least Squares*.

Helmert: *Die Ausgleichungsrechnung nach der Methode der kleinsten Quadrate*.

Jordan: *Handbuch der Vermessungskunde*, volume 1.

Merriman: *Textbook of Least Squares*.

Wright and Hayford: *Adjustment of Observations*.

Some of the simpler cases are treated first, such as the local adjustment at a station, the adjustment of a simple quadrilateral, etc. After these is given the development of the condition equations for latitude and longitude closures, followed by a sample adjustment including the condition equations for these closures, together with the equations for length and azimuth conditions.

A method of adjustment by the variation of geographic coordinates is then developed and applied first to a quadrilateral, then to a figure with a few new points connected with a number of fixed points. The same method is applied to the adjustment of a figure with latitude, longitude, length, and azimuth conditions. A sample adjustment of a vertical net is carried through and lastly there is given the development of the formulæ for the computation of vertical observations, together with examples of the method of computation.

## STATION ADJUSTMENT

The general rule followed by the observers of the Coast and Geodetic Survey is to measure the angles at each station in the order of azimuth, thus giving rise to no conditions except the horizon closure. Occasionally, however, sum angles are observed and, when this is done, other conditions are introduced in addition to the horizon closure making it necessary to adjust the angles at the station by the method of least squares. If all angles were observed in the same way, the weight of each would be unity and the adjustment would be made without regard to weights. In the adjustment given below the angles were measured by the usual Coast and Geodetic Survey repetition method; that is, six measures of the angles with the telescope direct and six with it reversed for each set. A station has been chosen at which there are angles measured with one, two, and three sets in order to illustrate the method of weighting.

*Observed angles, Gray Cliff*

Observed stations	Angle	Weight <i>p</i>	<i>v</i>	Adjusted final seconds*
	° ' " "			"
Boulder-Tower.....	65 06 26.6 30.9 30.4 } 29.3	3	$v_1$	29.9
Tower-Tyonek.....	19 46 27.7 25.0 28.0 } 26.9	3	$v_2$	27.5
Tyonek-Round Point.....	8 39 14.6 18.4 14.8 } 15.9	3	$v_3$	15.9
Round Point-Boulder.....	266 27 47.9	1	$v_4$	46.7
Round Point-Birch Hill.....	66 23 20.6 23.0 } 21.8	2	$v_5$	22.4
Birch Hill-Boulder.....	200 04 22.2 22.3 } 22.2	2	$v_6$	24.3
Boulder-Tyonek.....	84 52 56.2	1	$v_7$	57.4
Tyonek-Birch Hill.....	75 02 35.0	1	$v_8$	38.3
Round Point-Moose Point.....	64 32 12.9 09.8 } 11.4	2	$v_9$	11.3
Moose Point-Birch Hill.....	1 51 11.2	1	$v_{10}$	11.1

*List of directions, Gray Cliff*

Observed station	Direction	Adjusted final seconds*
	° ' " "	"
Boulder.....	0 00 00.0	00.0
Tower.....	65 06 29.3 + $v_1$	29.9
Tyonek.....	84 52 56.2 + $v_1 + v_2$	57.4
Round Point.....	93 32 12.1 + $v_1 + v_2 + v_3$	13.3
Moose Point.....	158 04 23.5 + $v_1 + v_2 + v_3 + v_9$	24.6
Birch Hill.....	159 55 34.7 + $v_1 + v_2 + v_3 + v_5 + v_{10}$	35.7

\* These values result from the computation on p. 13.

There have been formed a complete list of directions without using five of the angles, each of which, then, gives rise to a condition, there being five conditions in all. The equations expressing these conditions are formed in the following manner:

Angle Round Point-Boulder, observed,	266	27	$47.9 + v_4$
Angle Round Point-Boulder, from the list,	266	27	$47.9 - v_1 - v_2 - v_3$
Condition No. 1,			$0 = +0.0 + v_1 + v_2 + v_3 + v_4$
Angle Round Point-Birch Hill, from the list,	66	23	$22.6 + v_9 + v_{10}$
Angle Round Point-Birch Hill, observed,	66	23	$21.8 + v_5$
Condition No. 2,			$0 = +0.8 - v_5 + v_9 + v_{10}$

In the same way the other condition equations are formed. As a result there are finally:

*Condition equations*

1.  $0 = +0.0 + v_1 + v_2 + v_3 + v_4$
2.  $0 = +0.8 - v_5 + v_9 + v_{10}$
3.  $0 = -3.1 + v_1 + v_2 + v_3 + v_6 + v_9 + v_{10}$
4.  $0 = +0.0 + v_1 + v_2 - v_7$
5.  $0 = +3.5 + v_3 - v_8 + v_9 + v_{10}$

#### FORMATION OF NORMAL EQUATIONS BY DIFFERENTIATION

According to the theory of least squares, the most probable values will be determined by making the  $\Sigma p_n v_n^2$  a minimum, subject to the given conditions. By the method of Lagrangian multipliers the formation of the normal equations can be much simplified.

With the use of these the function  $u$  that is to be made a minimum is

$$u = 3v_1^2 + 3v_2^2 + 3v_3^2 + 1v_4^2 + 2v_5^2 + 2v_6^2 + 1v_7^2 + 1v_8^2 + 2v_9^2 + 1v_{10}^2 - 2C_1(+v_1 + v_2 + v_3 + v_4 + 0.0) - 2C_2(-v_5 + v_9 + v_{10} + 0.8) - 2C_3(+v_1 + v_2 + v_3 + v_6 + v_9 + v_{10} - 3.1) - 2C_4(+v_1 + v_2 - v_7 + 0.0) - 2C_5(+v_3 - v_8 + v_9 + v_{10} + 3.5).$$

The  $C$ 's are merely undetermined multipliers, the values of which will be determined by the solution. The factor 2 is included to obviate later on the use of the fraction  $\frac{1}{2}$ ; the minus sign is used for convenience. The function will be rendered a minimum if the partial differential coefficients with respect to  $v_1, v_2$ , etc., are equated to zero. By this means ten equations will be formed, giving the ten  $v$ 's expressed in terms of the  $C$ 's.

Differentiating with respect to  $v_1, v_2$ , etc., in succession and equating the results to zero, the following equations are obtained:

$$\begin{aligned} 3v_1 - C_1 - C_3 - C_4 &= 0 \\ 3v_2 - C_1 - C_3 - C_4 &= 0 \\ 3v_3 - C_1 - C_3 - C_5 &= 0 \\ v_4 - C_1 &= 0 \\ 2v_5 + C_2 &= 0 \\ 2v_6 - C_3 &= 0 \\ v_7 + C_4 &= 0 \\ v_8 + C_5 &= 0 \\ 2v_9 - C_2 - C_3 - C_5 &= 0 \\ v_{10} - C_2 - C_3 - C_5 &= 0 \end{aligned}$$

Therefore

$$\begin{aligned}
 v_1 &= +\frac{1}{3} C_1 + \frac{1}{3} C_3 + \frac{1}{3} C_4 \\
 v_2 &= +\frac{1}{3} C_1 + \frac{1}{3} C_3 + \frac{1}{3} C_4 \\
 v_3 &= +\frac{1}{3} C_1 + \frac{1}{3} C_3 + \frac{1}{3} C_5 \\
 v_4 &= +C_1 \\
 v_5 &= -\frac{1}{2} C_2 \\
 v_6 &= +\frac{1}{2} C_3 \\
 v_7 &= -C_4 \\
 v_8 &= -C_5 \\
 v_9 &= +\frac{1}{2} C_2 + \frac{1}{2} C_3 + \frac{1}{2} C_6 \quad \cdot \\
 v_{10} &= +C_2 + C_3 + C_5
 \end{aligned}$$

Thus all of the  $v$ 's are now expressed in terms of the  $C$ 's. These can now be substituted in the condition equations forming five normal equations containing five  $C$ 's and these equations may then be solved for the  $C$ 's. If the normals are formed from these values, fractions will occur in practically all of the coefficients. This can be avoided by replacing  $C_1$  by 6  $C_1'$ ,  $C_2$  by 6  $C_2'$ , etc. This is equivalent to using 12  $C_1'$ , 12  $C_2'$ , etc., in the original function instead of 2  $C_1$ , 2  $C_2$ , etc., which, of course, is perfectly valid.

The equations will then stand as follows:

$$\begin{aligned}
 v_1 &= +2 C_1' + 2 C_3' + 2 C_4' \\
 v_2 &= +2 C_1' + 2 C_3' + 2 C_4' \\
 v_3 &= +2 C_1' + 2 C_3' + 2 C_5' \\
 v_4 &= +6 C_1' \\
 v_5 &= -3 C_2' \\
 v_6 &= +3 C_3' \\
 v_7 &= -6 C_4' \\
 v_8 &= -6 C_5' \\
 v_9 &= +3 C_2' + 3 C_3' + 3 C_5' \\
 v_{10} &= +6 C_2' + 6 C_3' + 6 C_5'
 \end{aligned}$$

Dropping the prime and substituting these values in the first condition equation the following normal equation is obtained:

$$\begin{aligned}
 2 C_1 + 2 C_3 + 2 C_4 + 2 C_1 + 2 C_3 + 2 C_4 + 2 C_1 + 2 C_3 + 2 C_5 + 6 C_1 + 0.0 &= 0 \\
 + 12 C_1 &+ 6 C_3 + 4 C_4 + 2 C_5 + 0.0 = 0
 \end{aligned}$$

In a similar manner the other normal equations are formed, giving in all the following five equations:

$$\begin{aligned}
 + 12 C_1 &+ 6 C_3 + 4 C_4 + 2 C_5 + 0.0 &= 0 \\
 + 12 C_2 &+ 9 C_3 &+ 9 C_5 + 0.8 &= 0 \\
 + 6 C_1 &+ 9 C_2 + 18 C_3 + 4 C_4 + 11 C_5 - 3.1 &= 0 \\
 + 4 C_1 &+ 4 C_3 + 10 C_4 &+ 0.0 &= 0 \\
 + 2 C_1 &+ 9 C_2 + 11 C_3 &+ 17 C_5 + 3.5 &= 0
 \end{aligned}$$

This manner of forming the normal equations is called the method of correlates and is most conveniently carried out by means of a table of correlates formed as on page 11.

After the determination of the  $C$ 's by the solution of the normal equations, the  $v$ 's may be computed from the equations of the  $v$ 's

in terms of the  $C$ 's. In the tabulated form below the first column is multiplied by  $C_1$ , the second by  $C_2$ , etc. The sum of the first line multiplied by the  $\frac{6}{p}$  for that line gives  $v_1$ ; so also for the other  $v$ 's.

## Correlate equations

	$\frac{6}{p}$	1	2	3	4	5	$\Sigma$	$v$ 's*	Adopted $v$ 's
1	2	+1		+1	+1		+3	+0.618	+0.6
2	2	+1		+1	+1		+3	+0.618	+0.6
3	2	+1		+1		+1	+3	-0.050	-0.0
4	6	+1					+1	-1.182	-1.2
5	3		-1				-1	+0.585	+0.6
6	3			+1			+1	+2.133	+2.1
7	6				-1		-1	+1.230	+1.2
8	6					-1	-1	+3.234	+3.3
9	3		+1	+1		+1	+3	-0.069	-0.1
10	6		+1	+1		+1	+3	-0.138	-0.1

\* These values result from the computation on p. 13.

## FORMATION OF THE NORMAL EQUATIONS

After the condition equations are tabulated in correlates as above, the next step is the formation of the normal equations. In forming these the various products must be multiplied by  $\frac{1}{p}$  or by  $\frac{a}{p}$  in which  $p$  is the weight of the given  $v$  and  $a$  is some constant. (See the direct formation on p. 9.) It is most convenient to choose  $a$  so as to make most of the values integers, if this can be done without making the quantities too large. In this case 6 is the L. C. M. of the  $p$ 's, hence it is chosen for  $a$ . The normal equations are formed by taking the algebraic sums of  $\frac{6}{p}$  times the products of the various columns. Normal No. 1 is, in symbols—

$$\Sigma \frac{6}{p} \cdot 1 \cdot 1 + \Sigma \frac{6}{p} \cdot 1 \cdot 2 + \Sigma \frac{6}{p} \cdot 1 \cdot 3 + \Sigma \frac{6}{p} \cdot 1 \cdot 4 + \Sigma \frac{6}{p} \cdot 1 \cdot 5 + \eta + \left( \Sigma \frac{6}{p} \cdot 1 \cdot \Sigma + \eta^\dagger \right)$$

The algebraic sum of the sigma products in the formation checks or controls the formation of the normals. Each  $\Sigma$  line in the correlates is the algebraic sum of that line in the table. As is easily seen, the sum of the products of this column in the formation of the normals should check the algebraic sum of the coefficients of the normal. On the first normal  $+12 + 6 + 4 + 2 = +24$ , which is the same as the algebraic sum of the products in the correlates. The  $\Sigma$  column in the normals also includes the constant term. In the third normal  $+6 + 9 + 18 + 4 + 11 = +48$ . In the  $\Sigma$  columns of the normal  $+48 - 3.1 = +44.9$ .

†  $\eta$  is the constant term of the condition equation.

*Normal equations*

	1	2	3	4	5	$\eta$	$\Sigma$	$C's^*$
1	+12		+ 6	+ 4	+ 2	+0.0	+24	-0.19700
2		+12	+ 9	+ 4	+ 9	+0.8	+30.8	-0.19531
3			+18	+ 4	+11	-3.1	+44.9	+0.71069
4				+10		+0.0	+18	-0.20547
5					+17	+3.5	+42.5	-0.53917

\* These values result from the computation on p. 13.

## DISCUSSION OF METHOD OF SOLUTION OF NORMAL EQUATIONS

In the normal equations the coefficients in each equation occurring before the diagonal term are omitted, as the equations are symmetrical with regard to the diagonal line. The set just given when written in full is as follows:

	1	2	3	4	5	$\eta$	$\Sigma$
1	+12		+ 6	+ 4	+ 2	+0.0	+24
2		+12	+ 9		+ 9	+0.8	+30.8
3	+ 6	+ 9	+18	+ 4	+11	-3.1	+44.9
4	+ 4		+ 4	+10		+0.0	+18
5	+ 2	+ 9	+11		+17	+3.5	+42.5

It can be seen that the coefficients may be omitted to the left of the diagonal line and each equation may be read from the top down to the diagonal term and then across the page.

The Doolittle method of solution is used. Equation No. 1 is copied and then divided by the diagonal term (+12 in this case), the signs being changed. Since No. 2 does not occur on No. 1, this also is divided at once by the diagonal term with a change of sign. No. 3 has +6 on No. 1 and +9 on No. 2; accordingly, the divided coefficients of No. 1 are multiplied by +6 and those of No. 2 by +9 and these give the two products on No. 3. These are then added algebraically and divided by the diagonal term with change of sign to give  $C_3$  in terms of No. 4 and No. 5 plus a constant term. In a similar manner No. 4 and No. 5 are eliminated, the division on No. 5 giving the value of  $C_5$ . The back solution is then carried through  $C_4 = +0.17778C_5 - 0.10962$ . When the value of  $C_5$  is substituted,  $C_4 = -0.09585 - 0.10962 = -0.20547$ . So also for the remaining  $C's$ . For an explanation of the omission of the terms before the diagonal term, see page 22. For a full discussion of the Doolittle method of solution, see Wright & Hayford, Adjustment of Observations, page 114 et seq.

*Solution of normal equations*

1	2	3	4	5	$\eta$	$\Sigma$
+12 $C_1$		+ 6 - 0.5	+ 4 - 0.33333	+ 2 - 0.16667	+0.0 +0.0	+ 24 - 2
	+12 $C_2$	+ 9 - 0.75		+ 9 - 0.75	+0.8 -0.06667	+30.8 - 2.59667
	1 2	+18 - 3 - 6.75	+ 4 - 2	+11 - 1 - 6.75	-3.1 -0.6	+44.9 -12 -23.1
		+ 8.25 $C_3$	+ 2 - 0.24242	+ 3.25 - 0.39394	-3.7 +0.44848	+ 9.8 - 1.18788
		1 3	+10 - 1.3333 - 0.4848 + 8.1819 $C_4$	- 0.6667 - 0.7879 - 1.4546 + 0.17778 +17 - 0.3333 - 6.75 - 1.2803 - 0.2586 + 8.3778 $C_5$	+0.0 +0.8969 +0.8969 -0.10962 +3.5 -0.6 +1.4576 +0.1595 +4.5171 -0.53917	+18 - 8 - 2.3758 + 7.6242 - 0.93184 +42.5 - 4 -23.1 - 3.8606 + 1.3554 +12.8949 - 1.53917

*Back solution*

5	4	3	2	1
-0.53917	-0.10962	+0.44848	-0.06667	
	-0.09585	+0.21240	+0.40438	+0.08986
-0.53917		+0.04981	-0.53302	+0.06849
	-0.20547			-0.35535
		+0.71069	-0.19531	-0.19700

*Computation of corrections*

1	2	3	4	5
-0.197	-0.197	-0.197	-0.197	+0.195
+0.711	+0.711	+0.711	-0.197	+0.195
-0.205	-0.205	-0.539	6 -1.182	3 +0.585
+0.309	+0.309	-0.025		
2 +0.618	2 +0.618	2 -0.050		
6	7	8	9	10
+0.711	+0.205	+0.539	-0.195	-0.195
+0.711	+0.205	+0.539	+0.711	+0.711
3 +2.133	6 +1.230	6 +3.234	-0.539	-0.539
			-0.023	-0.023
			3 -0.069	6 -0.138

## ADJUSTMENT OF A QUADRILATERAL

## GENERAL STATEMENT

After the local conditions—that is, those arising from the relations of the angles to one another at each station—are satisfied there are general conditions arising from the geometrical relations necessary to form a closed figure which must be satisfied. To illustrate this, let the case of a quadrilateral be taken. The angles of each triangle should sum up to  $180^\circ$  plus the spherical excess of the triangle. Except in rare cases this does not happen with the observed angles; therefore condition equations are needed to bring it about. There are four triangles in a quadrilateral, but if three of them close the other will also close. There will then be three angle equations in the quadrilateral. A fourth equation must be included to insure that the lines at the pole will pass through the same point. When this condition is satisfied, and the triangles are closed, the same values will be obtained for the various sides when the computation is carried through different triangles.

In the adjustment of triangulation in the United States Coast and Geodetic Survey the method of directions is used; that is, an angle is considered as the difference of two directions.\* If  $v_1$  is the correction to the first direction in order of azimuth at a given station and  $v_2$  the correction to the second direction, the correction to the angle will be  $-v_1 + v_2$ , or the algebraic difference of the  $v$ 's applying to the directions. To avoid the use of so many  $v$ 's, the custom is to write (1) instead of  $v_1$ ; thus the angle given above will have the correction symbol  $-(1) + (2)$ , in which 1 and 2 are not quantities but the subscripts of the corresponding  $v$ 's.

An angle equation simply states that the sum of the corrections to the angles of a given triangle is to equal the failure in the closure of the triangle. In the triangle  $A_3A_2A_1$  (see fig. 1 on p. 16) the angle at  $A_2$  is to be corrected by  $-(1) + (2)$ , the angle at  $A_1$  by  $-(4) + (6)$ , and the angle at  $A_3$  by  $-(8) + (9)$ . The sum of the angles needs to be increased by  $2''.3$  to make up the sum of  $180^\circ$  plus the spherical excess. (See triangle on p. 23.) Therefore  $-(1) + (2) - (4) + (6) - (8) + (9) = +2.3$ , or, as it is usually written,  $0 = -2.3 - (1) + (2) - (4) + (6) - (8) + (9)$ .

Three angle equations in a quadrilateral will bring about the closure of the four triangles, but it is possible to have all of the triangles close and still the sides fail to check when computed through different triangles. To make the computation of lengths consistent a side equation must be added to the three angle equations. In figure 1 on page 16 the sides can be made consistent in the following manner:

---

\* See lists of directions on p. 16.

In the triangle  $A_1A_4A_2$

$$\frac{\text{side } A_2A_4}{\text{side } A_1A_4} = \frac{\text{sine angle } A_1}{\text{sine angle } A_2} = \frac{\text{sine } [-(5)+(6)]}{\text{sine } [-(1)+(3)]}$$

in the triangle  $A_1A_3A_4$

$$\frac{\text{side } A_1A_4}{\text{side } A_3A_4} = \frac{\text{sine angle } A_3}{\text{sine angle } A_1} = \frac{\text{sine } [-(7)+(9)]}{\text{sine } [-(4)+(5)]}$$

and in the triangle  $A_2A_3A_4$

$$\frac{\text{side } A_3A_4}{\text{side } A_2A_4} = \frac{\text{sine angle } A_2}{\text{sine angle } A_3} = \frac{\text{sine } [-(2)+(3)]}{\text{sine } [-(7)+(8)]}$$

If the sides are consistent, the product of these three equations gives

$$\frac{\text{sine } [-(5)+(6)] \text{ sine } [-(7)+(9)] \text{ sine } [-(2)+(3)]}{\text{sine } [-(1)+(3)] \text{ sine } [-(4)+(5)] \text{ sine } [-(7)+(8)]} = 1$$

In a spherical triangle the same equation is obtained by using the sine of the side in place of the side. In the end the equation given above results, since the sines of the sides cancel out as did the sides above.

Passing to logarithms, we have

$$\log \text{ sine } [-(5)+(6)] + \log \text{ sine } [-(7)+(9)] + \log \text{ sine } [-(2)+(3)] - \log \text{ sine } [-(1)+(3)] \\ - \log \text{ sine } [-(4)+(5)] - \log \text{ sine } [-(7)+(8)] = 0$$

As this will not be exactly true when the observed angles or angles adjusted only for closing errors of the triangles are used except in rare cases, a condition equation must be formed to accomplish this result. From the table of logarithms we find the amount of change of the log sine of the given angle for 1'' change in the angle, and this multiplied by the  $v$ 's applying to the angle will give the change in the log sine of that angle. It is customary to consider the log sines in six places of decimals, hence the change in the log sine for 1'' will be taken as units in the sixth place of decimals.

	°	'	''	
log sine [-(5)+(6)]	20	50	56.7	=9.5513374-5.53(5)+5.53(6)
log sine [-(7)+(9)]	61	47	35.0	=9.9450972-1.13(7)+1.13(9)
log sine [-(2)+(3)]	32	09	01.2	=9.7260280-3.35(2)+3.35(3)
Total.....				=9.2224626-3.35(2)+3.35(3)-5.53(5)+5.53(6) -1.13(7)+1.13(9)
	°	'	''	
log sine [-(1)+(3)]	133	53	46.3	=9.8576926+2.03(1)-2.03(3)
log sine [-(4)+(5)]	26	40	23.5	=9.6521506-4.19(4)+4.19(5)
log sine [-(7)+(8)]	31	03	42.5	=9.7126180-3.50(7)+3.50(8)
Total.....				=9.2224612+2.03(1)-2.03(3)-4.19(4)+4.19(5) -3.50(7)+3.50(8)
9.2224626		-3.35(2)+3.35(3)		-5.53(5)+5.53(6)-1.13(7) +1.13(9)
9.2224612+2.03(1)		-2.03(3)-4.19(4)+4.19(5)		-3.50(7)+3.50(8)
0=+1.4-2.03(1)-3.35(2)+5.53(3)+4.19(4)-9.72(5)+5.53(6)+2.37(7)-3.50(8)				+1.13(9)

(See tabulated form of this equation on p. 17.)

This condition requires the lines from  $A_2$ ,  $A_1$ , and  $A_3$  to pass through the same point at  $A_4$ . If  $v$ 's are found that satisfy this equation, at the same time satisfying the three angle equations given on page 17, they will render the quadrilateral consistent in all respects.

In a full quadrilateral (see figure 1) there are four conditions. These can be put in as three angle equations and one side equation, or two angle and two side, or one angle and three side equations. (See article by C. A. Schott, Appendix No. 17, United States Coast and Geodetic Survey Report of 1875, p. 280.) To illustrate the fact, a quadrilateral is adjusted using two angle equations and two side equations. (See p. 20.) In order to hold the closure of the triangles, the logarithms in the side equations must be found at least to seven places to hold the closure to tenths of seconds. Of course this method would never be used in practice, as the side equations require much more work, but the fact is interesting as an illustration of what can be done in the method of adjustment. Four side equations or four angle equations could not be used, for the fourth is functionally related to the other three, and hence they would not be independent conditions.

In a set of equations, if an identical one is included, the diagonal term of the reduced normal will become zero with the possible exception, of course, of a few units in the last place of solution due to accumulations. In any case, if the reduced diagonal term falls below unity, there may be danger of instability, since in this case any accumulations in the last place of the solution are increased when the normal is divided by this term.

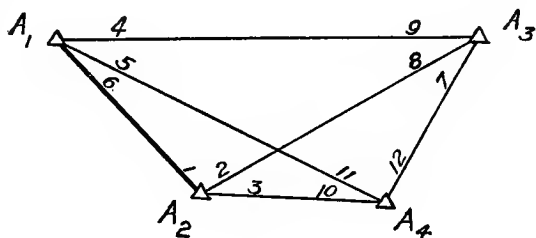


FIG. 1.

### Lists of directions

Stations observed	Directions after local adjustment	Final sec- onds*	Stations observed	Directions after local adjustment	Final sec- onds*
STATION $A_1$			STATION $A_3$		
$A_2$ .....	0 00 00.0	59.8	$A_4$ .....	0 00 00.0	00.7
$A_3$ .....	26 40 23.5	23.5	$A_1$ .....	31 03 42.5	42.0
$A_4$ .....	47 31 20.2	20.5	$A_2$ .....	61 47 35.0	34.8
STATION $A_2$			STATION $A_4$		
$A_1$ .....	0 00 00.0	59.5	$A_3$ .....	0 00 00.0	00.1
$A_3$ .....	101 44 45.1	46.1	$A_2$ .....	25 15 16.2	16.9
$A_4$ .....	133 53 46.3	45.8	$A_1$ .....	116 47 20.0	19.2

*Angle equations \**

$$0 = -2.3 - (1) + (2) - (4) + (6) - (8) + (9).$$

$$0 = +3.6 - (2) + (3) - (7) + (8) - (10) + (12).$$

$$0 = +2.2 - (4) + (5) - (7) + (9) - (11) + (12).$$

*Side equation*

Symbol	Angle	Logarithm	Tabular difference	Symbol	Angle	Logarithm	Tabular difference
	° ' "				° ' "		
-7+9	61 47 35.0	9.9450972	+1.13	-4+5	26 40 23.5	9.6521506	+4.19
-5+6	20 50 56.7	9.5513374	+5.53	-1+3	133 53 46.3	9.8576926	-2.03
-2+3	32 09 01.2	9.7260280	+3.35	-7+8	31 03 42.5	9.7126180	+3.50
		9.2224626				9.2224612	

$$0 = +1.4 - 2.03(1) - 3.35(2) + 5.38(3) + 4.19(4) - 9.72(5) + 5.53(6) + 2.37(7) - 3.50(8) + 1.13(9).$$

## FORMATION OF NORMAL EQUATIONS BY DIFFERENTIATION

The function  $u$  to be rendered a minimum is the sum of the squares of the  $v$ 's, subject to the four given conditions.

$$u = v_1^2 + v_2^2 + v_3^2 + v_4^2 + v_5^2 + v_6^2 + v_7^2 + v_8^2 + v_9^2 + v_{10}^2 + v_{11}^2 + v_{12}^2 - 2C_1(-2.3 - v_1 + v_2 - v_4 + v_6 - v_8 + v_9) - 2C_2(+3.6 - v_2 + v_3 - v_7 + v_8 - v_{10} + v_{12}) - 2C_3(+2.2 - v_4 + v_5 - v_7 + v_9 - v_{11} + v_{12}) - 2C_4(+1.4 - 2.03 v_1 - 3.35 v_2 + 5.38 v_3 + 4.19 v_4 - 9.72 v_5 + 5.53 v_6 + 2.37 v_7 - 3.50 v_8 + 1.13 v_9)$$

Differentiating with respect to the  $v$ 's in succession and equating to zero, there result after transposition the following equations:

$$v_1 = -C_1 - 2.03 C_4$$

$$v_2 = +C_1 - C_2 - 3.35 C_4$$

$$v_3 = +C_2 + 5.38 C_4$$

$$v_4 = -C_1 - C_3 + 4.19 C_4$$

$$v_5 = +C_3 - 9.72 C_4$$

$$v_6 = +C_1 + 5.53 C_4$$

$$v_7 = -C_2 - C_3 + 2.37 C_4$$

$$v_8 = -C_1 + C_2 - 3.50 C_4$$

$$v_9 = +C_1 + C_3 + 1.13 C_4$$

$$v_{10} = -C_2$$

$$v_{11} = -C_3$$

$$v_{12} = +C_2 + C_3$$

By the substitution of these values in the four condition equations the following normal equations result:

$$+6 C_1 - 2 C_2 + 2 C_3 + 4.65 C_4 - 2.3 = 0$$

$$-2 C_1 + 6 C_2 + 2 C_3 + 2.86 C_4 + 3.6 = 0$$

$$+2 C_1 + 2 C_2 + 6 C_3 - 15.15 C_4 + 2.2 = 0$$

$$+4.65 C_1 + 2.86 C_2 - 15.15 C_3 + 206.0470 C_4 + 1.4 = 0$$

These normal equations are formed most easily by means of the tabular form of the correlate equations given on page 18.

The sum of the squares of each column gives the diagonal term in that equation in the normals. All coefficients before the diagonal term are omitted; each equation is read by starting at the top of the tabular form below, reading down the column to the diagonal term, and then along the horizontal line. Compare the full normals given above with the tabular form below. After the diagonal terms are determined column No. 1 in the correlates is multiplied by column No. 2 and the algebraic sum of the products taken for the coefficient of normal No. 1 on No. 2; this is also the coefficient of No. 2 on No. 1. Column No. 1 times No. 3, with the algebraic sum of the products, gives the coefficient of No. 1 on No. 3 in the normals; also No. 3 on No. 1. Finally, the algebraic sum of the products of column No. 1 by column No. 4 gives the coefficient of normal No. 1 on No. 4. The algebraic sum of the products of column No. 1 by the  $\Sigma$  column should check the algebraic sum of the coefficients of normal No. 1. To this should be added algebraically the constant term of normal No. 1 and the sum placed in the  $\Sigma$  column of normal No. 1. (See the table of normals below.)

In the same way the sum of the products of column No. 2 times column No. 3 is determined for the second normal, and by continuing the process all of the normals are formed.

After the  $C$ 's are determined by the solution of the normals the  $v$ 's are most conveniently computed by multiplying column No. 1 in the correlates by  $C_1$ , column No. 2 by  $C_2$ , column No. 3 by  $C_3$ , and column No. 4 by  $C_4$ . Then the algebraic sum of line No. 1 gives  $v_1$ ; of No. 2,  $v_2$ , etc. (See the computation of the  $v$ 's on p. 19.)

Correlate equations

	1	2	3	4	$\Sigma$	$v$ 's *	Adopted $v$ 's	$v^2$
1	-1			-2.03	-3.03	-0.503	-0.5	0.25
2	+1	-1		-3.35	-3.35	+1.004	+1.0	1.00
3		+1		+5.38	+6.38	-0.501	-0.5	0.25
4	-1		-1	+4.19	+2.19	-0.227	-0.2	0.04
5			+1	-9.72	-8.72	-0.015	-0.0	0.00
6	+1			+5.53	+6.53	+0.242	+0.3	0.09
7		-1	-1	+2.37	+0.37	+0.663	+0.7	0.49
8	-1	+1		-3.50	-3.50	-0.493	-0.5	0.25
9	+1		+1	+1.13	+3.13	-0.170	-0.2	0.04
10		-1			-1	+0.099	+0.1	0.01
11			-1		-1	+0.740	+0.7	0.49
12		+1	+1		+2	-0.840	-0.8	0.64
							$\Sigma v^2$	3.55

Normal equations

1	2	3	4	$\eta$	$\Sigma$	$C$ 's *
+6	-2 +6	+2 +2 +6	+ 4.65 + 2.86 - 15.15	-2.3 +3.6 +2.2	+ 8.35 + 12.46 - 2.95	+0.6547 -0.0994 -0.7401



ADJUSTMENT OF A QUADRILATERAL BY THE USE OF TWO ANGLE  
AND TWO SIDE EQUATIONS \*

(See fig. 1 on p. 16.)

*Angle equations*

$$0 = -2.3 - (1) + (2) - (4) + (6) - (8) + (9)$$

$$0 = +3.6 - (2) + (3) - (7) + (8) - (10) + (12)$$

*Side equations*

Symbol	Angle	Logarithm	Tabular difference	Symbol	Angle	Logarithm	Tabular difference
-7+9	61 47 35.0	9.9450972	+1.13	-4+5	26 40 23.5	9.6521506	+4.19
-5+6	20 50 56.7	9.5513374	+5.53	-1+3	133 53 46.3	9.8576926	-2.03
-2+3	32 09 01.2	9.7260280	+3.35	-7+8	31 03 42.5	9.7126180	+3.50
		9.2224626				9.2224612	

$$0 = +1.4 - 2.03(1) - 3.35(2) + 5.38(3) + 4.19(4) - 9.72(5) + 5.53(6) + 2.37(7) - 3.50(8) + 1.13(9)$$

-2+3	32 09 01.2	9.7260280	+3.35	-7+8	31 03 42.5	9.7126180	+3.50
-11+12	91 32 03.8	9.9988442	-0.06	-4+5	26 40 23.5	9.6521506	+4.19
-8+9	30 43 52.5	9.7084309	+3.54	-1+2	101 44 45.1	9.9908094	-0.44
-5+6	20 50 56.7	9.5513374	+5.53	-10+11	25 15 16.2	9.6300613	+4.46
		8.9856405				8.9856393	

$$0 = +1.2 - 0.44(1) - 2.91(2) + 3.35(3) + 4.19(4) - 9.72(5) + 5.53(6) + 3.50(7) - 7.04(8) + 3.54(9) + 4.46(10) - 4.40(11) - 0.06(12)$$

*Correlate equations*

	1	2	3	4	$\Sigma$	$v's^\dagger$	Adopted $v's$
1	-1		-2.03	-0.44	-3.47	-0.495	-0.5
2	+1	-1	-3.35	-2.91	-6.26	+0.996	+1.0
3		+1	+5.38	+3.35	+9.73	-0.502	-0.5
4	-1		+4.19	+4.19	+7.38	-0.227	-0.2
5			-9.72	-9.72	-19.44	-0.013	-0.0
6	+1		+5.53	+5.53	+12.06	+0.240	+0.3
7		-1	+2.37	+3.50	+4.87	+0.659	+0.7
8	-1	+1	-3.50	-7.04	-10.54	-0.500	-0.5
9	+1		+1.13	+3.54	+5.67	-0.159	-0.2
10		-1		+4.46	+3.46	+0.113	+0.1
11				-4.40	-4.40	+0.717	+0.7
12		+1		-0.06	+0.94	-0.830	-0.8

*Normal equations*

	1	2	3	4	$\eta$	$\Sigma$	$C's^*$
1	+6	-2	+4.65	+9.45	-2.3	+15.80	+0.2328
2		+6	+2.86	-8.80	+3.6	+1.66	-0.8398
3			+206.0470	+208.2153	+1.4	+423.1723	+0.16435
4				+276.0980	+1.2	+486.1633	-0.16302

\* For triangles, see p. 23

† These values result from the computation on p. 21.

*Solution of normal equations*

1	2	3	4	$\eta$	$\Sigma$
+6 $C_1$	-2 +0.33333	+ 4.65 - 0.775	+ 9.45 - 1.575	-2.3 +0.38333	+ 15.80 - 2.63333
1	+6 -0.6667	+ 2.86 + 1.55	- 8.80 + 3.15	+3.6 -0.7667	+ 1.66 + 5.2667
	+5.3333 $C_2$	+ 4.41 - 0.82688	- 5.65 + 1.05938	+2.8333 -0.53125	+ 6.9266 - 1.29875
	1	+206.0470 - 3.6038	+208.2153 - 7.3238	+1.4 +1.7825	+423.1723 - 12.2450
	2	- 3.6465	+ 4.6719	-2.3428	- 5.7275
		+198.7967 $C_3$	+205.5634 - 1.034038	+0.8397 -0.004224	+405.1998 - 2.038262
		1	+276.0980 - 14.8838	+1.2 +3.6225	+486.1633 - 24.8850
		2	- 5.9855	+3.0015	+ 7.3379
		3	-212.5604	-0.8683	-418.9920
			+ 42.6683 $C_4$	+6.9557 -0.16302	+ 49.6240 - 1.16302

*Back solution*

4	3	2	1
-0.16302	-0.00422 +0.16857	-0.5312 -0.1727	+0.3833 +0.2568
-0.16302	+0.16435	-0.1359	-0.1274 -0.2799
		-0.8398	+0.2323

*Computation of corrections*

1	2	3	4	5	6
-0.2328 -0.3336 +0.0717	+0.2328 +0.8398 -0.5506 +0.4744	-0.8398 +0.8842 -0.5461	-0.2328 +0.6880 -0.6831	-1.5975 +1.5846	+0.2328 +0.9089 -0.9015
-0.4947 -0.5	+0.9964 +1.0	-0.5017 -0.5	-0.2273 -0.2	-0.0129 -0.0	+0.2403 +0.3
7	8	9	10	11	12
+0.8398 +0.3895 -0.5706	-0.2328 -0.8398 -0.5752 +1.1477	+0.2328 +0.1857 -0.5771	+0.8398 -0.7271	+0.7173	-0.8398 +0.0098
+0.6587 +0.7	+1.1477 -0.5001 -0.5	-0.1586 -0.2	+0.1127 +0.1	+0.7173 +0.7	-0.8300 -0.8

## SOLUTION OF A SET OF NORMALS INCLUDING TERMS USUALLY OMITTED

A set of four normal equations is solved below with inclusion of the terms omitted in the Doolittle method of solution.

*Solution of normals*

1	2	3	4	$\eta$	$\Sigma$
+6 -1 C <sub>1</sub>	-2 +0.33333	+ 2 - 0.33333	+ 4.65 - 0.775	-2.3 +0.38333	+ 8.35 - 1.39167
-2 +2 (1)	+6 -0.6667 +5.3333 -1 C <sub>2</sub>	+ 2 + 0.6667 + 2.6667 - 0.50001	+ 2.86 + 1.55 + 4.41 - 0.82688	+3.6 -0.7667 +2.8333 -0.53125	+ 12.46 + 2.7833 + 15.2433 - 2.85814
+2 -2	+2 +0.6667 (1) -2.6667 (2)	+ 6 - 0.6667 - 1.3333	- 15.15 - 1.55 - 2.205	+2.2 +0.7667 -1.4167	- 2.95 - 2.7833 - 7.6217
		+ 4 - 1 C <sub>3</sub>	- 18.905 + 4.72025	+1.55 -0.3875	- 13.355 + 3.33875
+4.65 -4.65	+2.86 +1.55 -4.41	-15.15 - 1.55 (1) - 2.205 (2) +18.905 (3)	+206.0470 - 3.6038 - 3.6465 - 89.3498 +109.4469 - 1 C <sub>4</sub>	+1.4 +1.7825 - 2.3428 +7.3257 +8.1654 -0.07461	+199.8070 - 6.4712 - 12.6044 - 63.1191 +117.6123 - 1.07461

## DISCUSSION OF THE SOLUTION

The quantities in heavy type are the ones omitted in the Doolittle method of solution of normal equations. They sum up to zero with the possible variation of a few units in the last place of the solution. This shows that the method is one of curtailed substitution. It can also be seen that the quantity in the  $\Sigma$  column is the direct sum of all the quantities in each horizontal line including those in heavy type. All of the quantities in heavy type occur in the regular solution. This is of value in the control of the solution. If an equation fails to check the  $\Sigma$  column after it is added up, the error can generally be located by adding back through noting that the coefficient is changed in sign because it is multiplied by  $-1$ . Note the product of equation No. 1 on No. 4;  $-1.55$  and  $+1.55$  are the products of No. 1 on No. 3 and No. 2, respectively;  $-4.65$  is the coefficient of No. 4 on No. 1 with sign changed. The method is the same in all cases. Care should be taken with such coefficients as No. 2 and No. 3 on No. 1. They have the same value with opposite sign. If a mistake should be made on them the  $\Sigma$  column control would not catch it. Care should be taken not to make a mistake in the  $\eta$  column and a compensating one in the  $\Sigma$  column. There is most danger of this in the addition. The control would not catch this and it would take much labor to correct it later.

After each equation is added, it should be added horizontally to check the  $\Sigma$  column. If the check fails an error has been made and it must be found before proceeding. A slight variation in the last place of the solution is of course unavoidable. After the division of each equation by the reduced diagonal term, a horizontal addition should be made (including, of course,  $-1$ ) to check the correctness of the division. No time is ever lost in using care in the solution of the equations. It takes so much time and labor to rectify a mistake later that every means should be employed to detect and correct it in the solution. The larger the set, the more important it is to be on guard against errors. It is possible to carry a set through with almost absolute assurance that the solution is correct.

If, in a given equation, the solution fails to check and the check of adding back through is satisfied, a mistake has been made somewhere in the solution columns and a compensating mistake in the  $\Sigma$  column. This can be caught by building up the omitted columns to the left of the given equation. They should each sum up to zero. If any one does not, the mistake in addition has been made in that equation in the column of the one being eliminated.

*Solution of triangles \**

Symbol	Station	Observed angle	Correction	Spherical angle	Spherical excess	Plane angle	Logarithm
		° ' "	"	"	"	"	
- 8+ 9	$A_2-A_1$	30 43 52.5	+0.3	52.8	0.0	52.8	3.772745
- 1+ 2	$A_3$	101 44 45.1	+1.5	46.6	0.1	46.5	0.291568
- 4+ -6	$A_1$	47 31 20.2	+0.5	20.7	0.0	20.7	9.990809
			+2.3		0.1		9.867787
	$A_3-A_1$						4.055122
	$A_3-A_2$						3.932100
	$A_2-A_1$						3.772745
-10+11	$A_4$	25 15 16.2	+0.6	16.8	0.0	16.8	0.369936
- 1+ 3	$A_2$	133 53 46.3	+0.0	46.3	0.1	46.2	9.857693
- 5+ 6	$A_1$	20 50 56.7	+0.3	57.0	0.0	57.0	9.551339
			+0.9		0.1		4.000374
	$A_4-A_1$						3.694020
	$A_4-A_2$						
	$A_2-A_3$						3.932100
-10+12	$A_4$	116 47 20.0	-0.9	19.1	0.1	19.0	0.049306
- 2+ 3	$A_2$	32 08 61.2	-1.5	59.7	0.0	59.7	9.726023
- 7+ 8	$A_3$	31 03 42.5	-1.2	41.3	0.0	41.3	9.712614
			-3.6		0.1		3.707429
	$A_4-A_3$						3.694020
	$A_4-A_2$						
	$A_1-A_3$						4.055122
-11+12	$A_4$	91 32 03.8	-1.5	02.3	0.1	02.2	0.000156
- 4+ 5	$A_1$	26 40 23.5	+0.2	23.7	0.0	23.7	9.652151
- 7+ 9	$A_3$	61 47 35.0	-0.9	34.1	0.0	34.1	9.945096
			-2.2		0.1		3.707429
	$A_4-A_3$						4.000374
	$A_4-A_1$						

\* For the method of solution of triangles see United States Coast and Geodetic Survey Special Publication No. 8, p. 6.

## Position computation,

STATION  $A_3$ 

$\alpha$ Second angle } $\alpha$ $\Delta\alpha$	$A_2$ to $A_1$			First angle of triangle	$\lambda$ $\Delta\lambda$	$^{\circ}$ 156	$'$ 20	$''$ 26.6
	$A_1$ and $A_3$					+101	44	46.6
	$A_2$ to $A_3$					258	05	13.2
$\alpha'$ $A_3$ to $A_2$					$\lambda'$	+	8	05.9
						180	00	00.0
						78	13	19.1
$\phi$ $\Delta\phi$	$^{\circ}$ 60	$'$ 56	$''$ 01.089	$A_2$		149	34	19.237
	+		56.720			-	9	15.880
	60	56	57.809			149	25	03.357
$\phi'$				$A_3$				
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 60	$'$ 56	$''$ 29	$s$ $\cos \alpha$	3.932100	$s^2$ $\sin^2 \alpha$	7.86420	$h^2$ $D$
				B	9.314765	C	9.98109	3.5122
					8.509299		1.65750	2.3224
1st term 2d and 3d terms }	$''$ h			1.756164		9.50279		5.8346
	-57.0380					0.3183		
	+ 0.3184					0.0001		
- $\Delta\phi$	-56.7196							
	$s$ $\sin \alpha$	3.932100		$\sin \frac{1}{2}(\phi+\phi')$	2.744981			
	$A'$	9.990544			9.941572			
$\sec \phi'$	8.508600		2.686553					
		0.313737						
	2.744981							
	$''$							
	$\Delta\lambda$	-555.8800		- $\Delta\alpha$	-485.89			

STATION  $A_4$ 

$\alpha$ Second angle $\alpha$ $\Delta\alpha$	$A_2$ to $A_3$			First angle of triangle	$\lambda$	$^{\circ}$ 258	$'$ 05	$''$ 13.2
	$A_3$ and $A_4$					+ 32	08	59.7
	$A_2$ to $A_4$					290	14	12.9
$\alpha'$ $A_4$ to $A_2$					$\lambda'$	+	4	29.0
						180	00	00.0
						110	18	41.9
$\phi$ $\Delta\phi$	$^{\circ}$ 60	$'$ 56	$''$ 01.089	$A_2$		149	34	19.237
	-		55.340			-	5	07.795
	60	55	05.749			149	29	11.442
$\phi'$				$A_4$				
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 60	$'$ 55	$''$ 33	$s$ $\cos \alpha$	3.694020	$s^2$ $\sin^2 \alpha$	7.38804	$h^2$ $D$
				B	9.538954	C	9.94466	3.484
					8.509299		1.65750	2.322
1st term 2d and 3d terms } - $\Delta\phi$	$''$ h			1.742273		8.99020		5.806
	+55.2425					0.0978		
	+ 0.0979					1		
	+55.3404							
	$s$ $\sin \alpha$	3.694020		$\sin \frac{1}{2}(\phi+\phi')$	2.488262			
	$A'$	9.972328			9.941507			
$\sec \phi'$	8.508601		2.429769					
	0.313313							
	2.488262							
	$''$							
$\Delta\lambda$	-307.7953			- $\Delta\alpha$	-269.01			

\* For an explanation of the forms for computing differences of latitude, longitude, and azimuth see United States Coast and Geodetic Survey Special Publication No. 8, pp. 6-11.



*List of geographic positions, Turnagain Arm, Alaska. Valdez datum*

Station	Latitude and longitude	Seconds in meters	Azimuth	Back azimuth	To station	Distance	Logarithm
	° ' "		° ' "	° ' "		Meters	
$A_3$	60 56 57.809	1789.4	78 13 19.1	258 05 13.2	$A_2$	8552.6	3.932100
	149 25 03.357	50.5	108 57 11.9	288 46 47.7	$A_1$	11353.3	4.055122
$A_4$	60 55 05.749	177.9	110 18 41.9	290 14 12.9	$A_2$	4943.3	3.694020
	149 29 11.442	172.4	135 33 58.7	315 27 11.4	$A_1$	10008.6	4.000374
			227 06 01.0	47 09 37.8	$A_3$	5098.3	3.707429

## DEVELOPMENT OF CONDITION EQUATIONS FOR LATITUDE AND LONGITUDE CLOSURES

After the conditions arising from the closure of triangles and from the equality of sides or lengths computed by different routes have been satisfied, cases frequently arise where azimuth, latitude, and longitude conditions must be satisfied. There is given now a development of a form of condition equations that will bring about a closure in geographic position.

Discrepancies in latitude and longitude arise whenever a chain of triangulation or a traverse closes on itself. The discrepancies may be distributed throughout the whole loop or in a selected portion of it, depending upon the circumstances. Of course the most rigid adjustment would require the discrepancies to be distributed throughout the whole chain. At times, however, this would require more labor than the importance of the work would justify. Also some parts of the loop may be much better determined than other parts, in which case the more poorly determined part should be required to make up the discrepancies.

The discussion of the form of equations to be employed to effect the closure without discrepancies will be based upon the position computation formulæ employed by the United States Coast and Geodetic Survey. (See United States Coast and Geodetic Survey Special Publication No. 8, p. 8.)

The amount to be distributed being, of course, small compared with the total change in latitude and longitude, the only term of the latitude computation formula that need be considered is the first one. No appreciable changes due to the adjustment will take place in the other terms.

The formation of the equations must always start from a line fixed in length and azimuth. If a scheme of triangulation should start from a fixed line and run to two points which are fixed in position but are not the ends of a single line, then the formation of the equations for each of the two points must start from the fixed line.

There are, of course, two elements that enter into the determination of the position of any point as computed from a known point; these

are the distance from the known point and the azimuth of the line from the known to the unknown point.

In the triangle 1 2 3, let 1 and 2 be fixed in position, and let us consider what change in the position of 3 will be produced by small changes in angles  $A$ ,  $B$ , and  $C$ . The length to be carried forward is 1 to 3. Starting with the length 1 to 2, we have  $\log 1 \text{ to } 3 = \log 1 \text{ to } 2 - \log \sin B + \log \sin A$ . The change in length, then, depends upon the changes in angles  $A$  and  $B$  and the change in azimuth of the line 1 to 3 depends upon the change in angle  $C$ . The angles  $A$  and  $B$ , therefore, are called the length angles and angle  $C$  the azimuth angle.

If we can derive a linear expression for the effect of each of these separately, the total effect will be the sum of the two.

Let  $\delta_A$  and  $\delta_B$  represent the change of the log sin for a change of one second in the angles  $A$  and  $B$ ;  $v_A$  and  $v_B$  the number of seconds change in angles  $A$  and  $B$ , respectively. Then the change in log sin  $A$  will equal  $\delta_A(v_A)$ , and the change in log sin  $B$  will equal  $\delta_B(v_B)$ ; therefore, the change in log 1 to 3 is  $+\delta_A(v_A) - \delta_B(v_B)$ . The change in the logarithm of the first term of the latitude due to the change in length is equal to  $+\delta_A(v_A) - \delta_B(v_B)$ . This is the change in the logarithm, but for convenience of computation it is better to determine what change in the antilogarithm will be produced by this change; or, in other words, to determine what this logarithmic change will amount to in seconds of arc. From the nature of logarithms, if we know the number to which a given logarithm corresponds, the change in the number due to any *small* change in the logarithm can be found by multiplying the logarithm change by the number and dividing by  $M$  (the modulus of the common system of logarithms). This can also be shown by differentiation.

Let  $y = \log_{10} x$

$$dy = M \frac{dx}{x}$$

Therefore  $dx = \frac{x}{M} dy$ ,  $dy$  being the small change in the log and  $dx$  the corresponding small change in the number.

$+\delta_A(v_A) - \delta_B(v_B)$  must then be multiplied by  $(\phi_B - \phi_C)$ , (in which  $\phi_B$  is the computed latitude of 3 and  $\phi_C$  is the latitude of 1), and the product divided by  $M$ ; this will give the change in seconds in the latitude of 3 due to the change in length of 1 to 3.

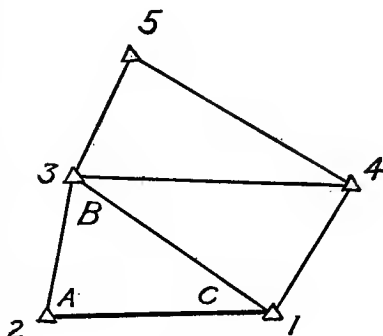


FIG. 2.

Next must be considered the change in latitude due to the change in the azimuth angle  $C$ . If  $s$  is the length in meters of 1 to 3, the length of the small arc through which 3 turns is equal to  $s(v_c)$  arc  $1''$  (as  $ds = r d\theta$  for a circle about the origin in polar coordinates),  $v_c$

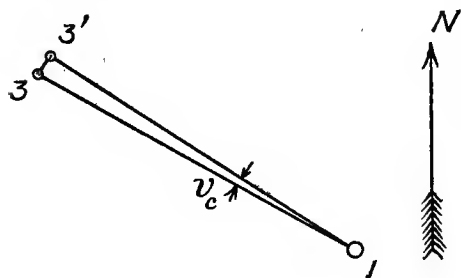


FIG. 3.

equals the number of seconds change in angle  $C$ , and arc  $1''$  is included to reduce this angle to circular measure.

Let 3 be the original position of 3 and 3' the position due to a small rotation of 1 to 3 about 1.

$$3 \text{ to } 3' = s v_c \text{ arc } 1''$$

The azimuth of 3 to 3' is  $90^\circ + \alpha$ . The change in latitude due to  $s(v_c)$  arc  $1''$  is equal to  $-s(v_c)$  arc  $1'' \cos (90^\circ + \alpha)$  times the  $B$  factor in the position computation,

$$= +s(v_c) \text{ arc } 1'' \sin \alpha B$$

$$= + (v_c) \text{ arc } 1'' B s \sin \alpha$$

But  $\lambda_B - \lambda_C = s \sin \alpha A' \sec \phi'$

$$\text{Therefore } s \sin \alpha = \frac{\lambda_B - \lambda_C}{A' \sec \phi'}$$

$$\text{Therefore the change in latitude} = \frac{B \text{ arc } 1''}{A' \sec \phi'} (\lambda_B - \lambda_C) (v_c).$$

In a similar way the change in longitude due to a change in length is,

$$\frac{\lambda_B - \lambda_C}{M} \left[ +\delta_A(v_A) - \delta_B(v_B) \right]$$

and the change in longitude due to the change in azimuth is,

$$s(v_c) \text{ arc } 1'' \sin (90^\circ + \alpha) A' \sec \phi' = + (v_c) \text{ arc } 1'' s \cos \alpha A' \sec \phi'$$

$$-s \cos \alpha B = \phi_B - \phi_C \text{ (neglecting the small terms)}$$

$$s \cos \alpha = -\frac{\phi_B - \phi_C}{B}$$

$$\text{Therefore change in longitude} = -\frac{A' \sec \phi' \text{ arc } 1''}{B} (\phi_B - \phi_C) (v_c).$$

The usage is to point off the log change for one second of arc as an integer in the sixth place of logarithms; therefore as a number the

$$\text{tabular difference} = \frac{\delta_A}{10^6} \text{ and } \frac{\delta_B}{10^6}.$$

The total change in seconds of latitude in the triangle is

$$d\phi = \frac{(\phi_B - \phi_C)}{10^6 M} \left[ +\delta_A(v_A) - \delta_B(v_B) \right] \pm \frac{B \text{ arc } 1''}{A' \sec \phi'} (\lambda_B - \lambda_C) (v_c).*$$

The total change in longitude is,

$$\frac{(\lambda_B - \lambda_C)}{10^6 M} \left[ +\delta_A(v_A) - \delta_B(v_B) \right] \mp \frac{A' \sec \phi' \text{ arc } 1''}{B} (\phi_B - \phi_C) (v_c).*$$

\* The upper sign being used for a right azimuth angle, the lower sign for a left.

In this way the change could be determined for each triangle in the chain and the sum placed equal to the discrepancy, but this would require a very great amount of work.

If any change takes place in the first triangle while the remaining triangles are for the moment supposed to remain fixed, this length change and azimuth change will affect not only this triangle, but will persist in each succeeding triangle. As a consequence the change of length and azimuth in the first triangle will be felt in the computation of every point after it in the chain. Let  $\phi_n$  and  $\lambda_n$  be the computed  $\phi$  and  $\lambda$  of the end point. The change in the first triangle will apply not merely to  $\phi_B - \phi_C$ , etc., but to  $\phi_n - \phi_C$ , etc.

Therefore the change in the final position due to the changes in the first triangle is, for latitude,

$$\frac{(\phi_n - \phi_C)}{10^6 M} \left[ + \delta_A(v_A) - \delta_B(v_B) \right] \pm \frac{B_C \text{ arc } 1''}{A_n \sec \phi_n} (\lambda_n - \lambda_C) (v_C),^*$$

and for longitude,

$$\frac{(\lambda_n - \lambda_C)}{10^6 M} \left[ + \delta_A(v_A) - \delta_B(v_B) \right] \mp \frac{A_n \sec \phi_n \text{ arc } 1''}{B_C} (\phi_n - \phi_C) (v_C).^*$$

In the same way the change in the final position due to changes in the second triangle can be determined, and so on through the whole chain. Each triangle will have an  $A$ ,  $B$ , and  $C$  angle,  $A$  being the length angle next to the known side;  $B$ , the one opposite the known side; and  $C$ , the azimuth angle.

The equations will finally stand

$$0 = +(\phi_n - \phi_{n'}) + \Sigma \left[ \frac{(\phi_n - \phi_C)}{10^6 M} \delta_A(v_A) - \frac{(\phi_n - \phi_C)}{10^6 M} \delta_B(v_B) \right] \\ + \Sigma \pm \frac{B_C \text{ arc } 1''}{A_n \sec \phi_n} (\lambda_n - \lambda_C) (v_C).^*$$

$$0 = +(\lambda_n - \lambda_{n'}) + \Sigma \left[ \frac{\lambda_n - \lambda_C}{10^6 M} \delta_A(v_A) - \frac{\lambda_n - \lambda_C}{10^6 M} \delta_B(v_B) \right] \\ + \Sigma \mp \frac{A_n \sec \phi_n \text{ arc } 1''}{B_C} (\phi_n - \phi_C) (v_C).^*$$

$\phi_n$  is the computed latitude of the final point and  $\phi_{n'}$ , the fixed latitude; so also for  $\lambda_n$  and  $\lambda_{n'}$ .

It is exact enough to take  $\phi_n - \phi_C$  and  $\lambda_n - \lambda_C$  to minutes and tenths of a minute, so that it is advisable to divide the equations by 60 since, as they stand,  $\phi_n - \phi_C$ , etc., are in seconds. Also it is best to multiply through by  $10^6 M$  to remove this factor from the denominator of the first summation.

\* Upper sign for right azimuth angle, lower for left.

Then we have

$$0 = +\frac{M}{60} 10^6 (\phi_n - \phi_n')'' + \Sigma [(\phi_n - \phi_c)' \delta_A(v_A) - (\phi_n - \phi_c)' \delta_B(v_B)] \\ + \Sigma \pm 10^6 M \frac{B_c \text{ arc } 1''}{A_n \sec \phi_n} (\lambda_n - \lambda_c)' (v_c). *$$

$$0 = +\frac{M}{60} 10^6 (\lambda_n - \lambda_n')'' + \Sigma [(\lambda_n - \lambda_c)' \delta_A(v_A) - (\lambda_n - \lambda_c)' \delta_B(v_B)] \\ + \Sigma \mp 10^6 M \frac{A_n \sec \phi_n \text{ arc } 1''}{B_c} (\phi_n - \phi_c)' (v_c). *$$

The  $A$  and  $B$  factors change so slowly that for any chain they can be taken for the mean  $\phi$  and also  $\sec \phi_n$  can be used in the same way. A table can then be prepared for functions designated as  $a_1$  and  $a_2$  and defined as follows:

$$a_1 = +10^6 M \frac{B \text{ arc } 1''}{A \sec \phi}$$

$$a_2 = -10^6 M \frac{A \sec \phi \text{ arc } 1''}{B},$$

the  $A$  and  $B$  factors and the  $\phi$  being used at a convenient interval. A table has been computed for latitudes starting at  $24^\circ$  for intervals of  $4^\circ$  up to  $56^\circ$ . The minus sign is used with  $a_2$  in order that the same sign can be used on the directions of the azimuth angle for both latitude and longitude equations. If the discrepancy to be made up by the adjustment is large, or if the chain extends over a great distance of latitude, it would be best to compute the values of  $a_1$  and  $a_2$  using  $A_n$  and  $\phi_n$  and the  $B$  for the mean  $\phi$ .

If the chain to be adjusted extends principally east and west, in place of  $\phi_n - \phi_c$  a summation of the first terms ( $h$ ) in the position computations should be used.  $\Sigma_n^c h$  would then replace  $\phi_n - \phi_c$ , the sign being used that would conform to  $\phi_n - \phi_c$ . These quantities should then be used throughout in forming the equations.

If the latitude and longitude equations are to be included in the main adjustment and the equations all solved simultaneously, the computation of the positions through the chain must be made with one length carried through the figures by means of the observed plane angles; that is, the angles as observed each diminished by  $\frac{1}{3}$  of the spherical excess of the triangle. This could be done by carrying the length through a selected chain of triangles and then computing each of the various positions over a single line. Both lines of the triangle could not be used because the observed plane angles must be used in carrying the length and, under ordinary circumstances, the triangle would not be closed. To obviate this difficulty, it is best to use only the observed  $A$  and  $B$  angles and to conclude the  $C$  angle, using, of course, the concluded correction symbols on this

\* Upper sign for right azimuth angle, lower for left,

angle. This method gives a much more reliable determination of the discrepancy, as it furnishes a check on each position, and thus prevents a mistake being left in the computation. If the figure adjustment is carried out first, there is no need to follow this method as the triangles would then be closed. In this case it is the general custom of the United States Coast and Geodetic Survey to choose the best chain of triangles and to form the equations through them, using the angle method in place of the direction method. Equations with absolute terms equal to zero must be included for the various triangles in order to hold them closed; also, if a length equation is included in the figure adjustment, it must be retained with zero discrepancy to hold the length. If the figure ends on a fixed line and a length equation is not put in the figure adjustment, the discrepancy must be put on the length equation used with the latitude and longitude equations. After adjustment is made for these final discrepancies the cross lines are computed by two sides and the included angle.

The best results are probably obtained by the solution of all the equations at once, but this entails so much work that the angle method is often used in chains of minor importance.

We have finally:

$$\frac{M}{60} 10^6 = 7238.24$$

$$0 = +7238.24(\phi_n - \phi_{n'})'' + \Sigma[(\phi_n - \phi_c)' \delta_A(v_A) - (\phi_n - \phi_c)' \delta_B(v_B)] + \Sigma \pm a_1(\lambda_n - \lambda_c)'(v_c).$$

$$0 = +7238.24(\lambda_n - \lambda_{n'})'' + \Sigma[(\lambda_n - \lambda_c)' \delta_A(v_A) - (\lambda_n - \lambda_c)' \delta_B(v_B)] + \Sigma \pm a_2(\phi_n - \phi_c)'(v_c).$$

In the equations  $v_A$ ,  $v_B$ , and  $v_c$  would be replaced by their correction symbols, care being taken to use  $v_c = -v_A - v_B$ , if the azimuth angle has been concluded in carrying the position computation through the chain.

If an azimuth equation occurs, the constant term must be corrected by  $+(\lambda_n - \lambda_{n'}) \times$  sine of the mean  $\phi$ , this being the amount that the azimuth will change from the changes in the back azimuths due to the changes in longitude.

It should be noted that whenever a discrepancy of position is adjusted into a section of a loop, an external condition is placed upon the chain, as at best only part of this discrepancy is due to errors in the chain, the rest being due to the remainder of the loop. It is necessary to hold some parts of the triangulation fixed; otherwise when a loop closure is put in it would frequently be necessary to readjust nearly all of the triangulation of the country. The result is, however, that some chains of triangulation, excellent in themselves, get some rather large corrections due to the position closure.

## EQUATIONS IN A NET

In the adjustment of a quadrilateral, use is made of the two kinds of condition equations that are necessary for the adjustment of any figure that does not contain external conditions such as length, azimuth, or loop closure. In fact most figures can be broken up into successive quadrilaterals. In forming the length equation, use is made of the two length angles in the various triangles passed through. In fig. 5, on page 37, the length angles are lettered *A* and *B*. The angle omitted is the azimuth angle of the given triangle. The log sin of the *A* angle is added to the first length and the log sin of the *B* angle to the final length. So with all of the triangles through which the length is carried. The discrepancy is found and the equation formed in the same way as in the case of an ordinary side equation. See the formation of the length equation on page 37. If the spherical angles are used a correction for arc to sine must be applied to each length. (See the table of these corrections in Special Publication No. 8, p. 17.)

An azimuth equation is formed by adding algebraically to the first azimuth the various azimuth angles up to the second line fixed in azimuth. When passing from one end of a line to another, the azimuth difference due to convergence of the meridians, must be applied as determined in the computation of positions. The algebraic sum of the *v*'s upon these angles must make up the discrepancy between the computed and fixed azimuths. See the computation on page 38.

The determination of the exact number of side and angle equations in a net and the manner in which they come in, is one of the difficulties encountered by a beginner in the adjustment of triangulation. This is especially true if the net is somewhat complicated. The best method for this determination is to plot the figure point by point. By plotting the triangle Tower, Turn, and Dundas, in the figure on page 34 one angle equation is determined. Add Lazaro by the lines Lazaro to Turn and Lazaro to Tower. This gives another angle equation, making two. Another angle equation and a side equation are obtained by putting in the line Lazaro to Dundas. This makes a total of three angle equations and one side equation for the quadrilateral, just as it should be. Next plot Nichols by the lines Nichols to Lazaro and Nichols to Tower; this is a closed triangle and gives a fourth angle equation. Put in Tow Hill by the lines Tow Hill to Nichols and Tow Hill to Lazaro; this does not give an angle equation as it is not a closed triangle. Draw the line Tow Hill to Tower; this gives a second side equation. In this way one can continue through the whole figure. If a full line Nichols to Turn were in the figure, it would give another angle and another side

equation. The angle equation added would have to include the directions on this line as would also the side equation. This method shows at once where the equations come in and what new  $v$ 's must appear in the equations.

Lines sighted over in only one direction have no effect on the number of angle equations. If the closed part of the figure is plotted, omitting all of the extra lines—that is, putting in each station with only two lines from those already plotted, a closed framework of the figure will be formed. The first triangle requires three lines, those after the first require two lines. The number of angle equations in the framework of the figure is thus equal to the number of lines in the figure minus the number of stations plus one. Every full line added to this framework gives another angle equation. Therefore, the whole number of angle equations in a net is equal to the whole number of full lines minus the number of occupied stations plus one.

The lines sighted over in one direction have the same effect on the number of side equations that the full lines have. If the full framework of the figure is plotted with two lines to each station from those already determined, no side equation will as yet appear in the figure. Every extra line put in gives a side equation. The first triangle fixes three stations; the stations after these require two lines to be used in plotting them. Thus the number of lines needed to plot the framework is equal to twice the number of stations minus three. The full number of side equations will then be equal to the number of all the lines minus twice the number of all the stations plus three.

Let  $n$  = total number of lines.

$n'$  = number of lines sighted over in both directions.

$S$  = total number of stations.

$S'$  = number of occupied stations.

Then

The number of angle equations in a net =  $n' - S' + 1$ .

The number of side equations in a net =  $n - 2S + 3$ .

These formulas should be used to check the number determined by directly plotting the figure.

In figure 4 on page 34,

$$n = 41$$

$$n' = 38$$

$$S = 18$$

$$S' = 17$$

Therefore number of angle equations =  $38 - 17 + 1 = 22$ .

number of side equations =  $41 - 36 + 3 = 8$ .

For convenience in solution it is best to use triangles with the larger angles for the angle equations, reserving the small angles to be used in the side equations. This will keep the large coefficients

in the side equations from appearing on the same directions as are used in the angle equations and will aid in the solution of the normals. The small angles need to appear in the side equations, as their tabular differences are proportionally much less affected by the dropping of decimal places than are those of the larger angles.

ADJUSTMENT OF A FIGURE WITH LATITUDE, LONGITUDE, AZIMUTH,  
AND LENGTH CLOSURE CONDITIONS

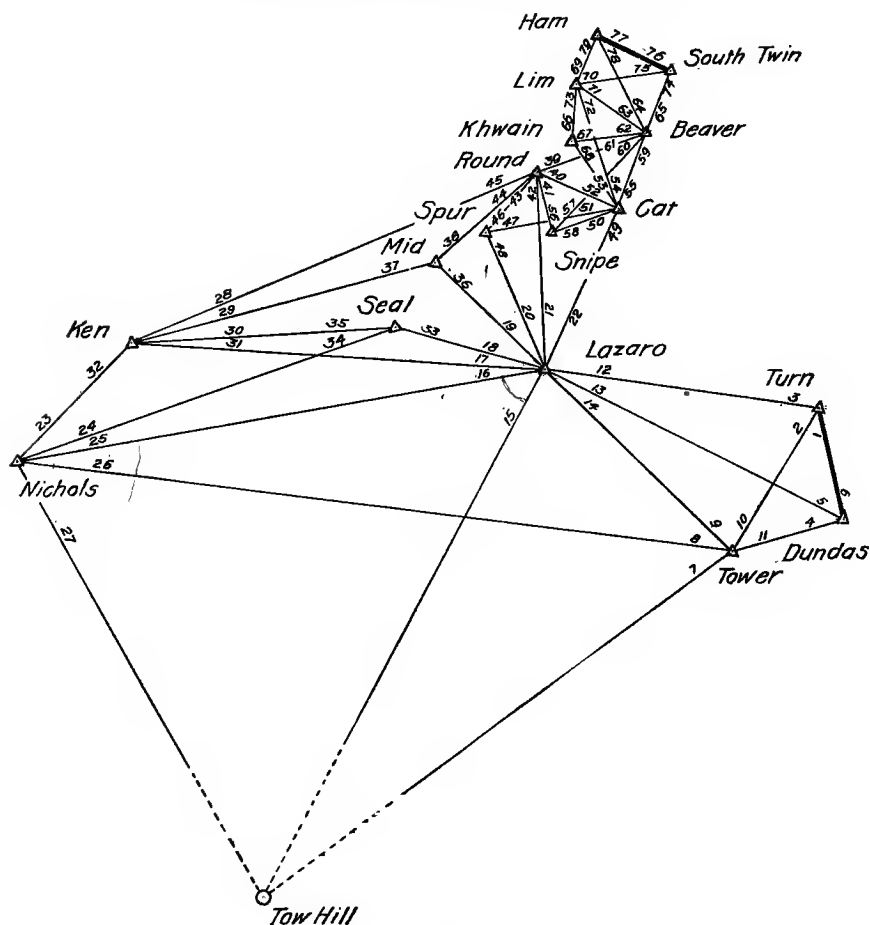


FIG. 4.

In this figure, in addition to the angle, side, and length conditions, there are included conditions for azimuth, latitude, and longitude.

*Angle equations*

$$0 = -5.5 - (1) + (2) - (4) + (6) - (10) + (11)$$

$$0 = +4.0 - (1) + (3) - (5) + (6) - (12) + (13)$$

$$0 = +12.0 - (2) + (3) - (9) + (10) - (12) + (14)$$

$$0 = -8.0 - (8) + (9) - (14) + (16) - (25) + (26)$$

$$0 = -2.9 - (16) + (17) - (23) + (25) - (31) + (32)$$

$$0 = -0.0 - (16) + (18) - (24) + (25) - (33) + (34)$$

$$0 = -3.2 - (23) + (24) - (30) + (32) - (34) + (35)$$

$$0 = -2.3 - (17) + (19) - (29) + (31) - (36) + (37)$$

$$0 = -3.2 - (17) + (21) - (28) + (31) - (42) + (45)$$

$$0 = -3.2 - (19) + (21) + (36) - (38) - (42) + (44)$$

$$0 = +5.0 - (21) + (22) - (40) + (42) - (49) + (52)$$

$$0 = -1.0 - (20) + (22) - (47) + (48) - (49) + (51)$$

$$0 = +3.7 - (40) + (43) - (46) + (47) - (51) + (52)$$

$$0 = +5.2 - (40) + (41) - (50) + (52) - (56) + (58)$$

$$0 = -2.5 - (39) + (40) - (52) + (55) - (59) + (61)$$

$$0 = +2.8 - (50) + (55) - (57) + (58) - (59) + (60)$$

$$0 = -5.1 - (53) + (55) - (59) + (62) - (67) + (68)$$

$$0 = -1.7 - (54) + (55) - (59) + (63) - (71) + (72)$$

$$0 = +2.1 - (62) + (63) - (66) + (67) - (71) + (73)$$

$$0 = +2.3 - (63) + (65) - (70) + (71) - (74) + (75)$$

$$0 = +6.1 - (64) + (65) - (74) + (76) - (77) + (78)$$

$$0 = +0.1 - (69) + (70) - (75) + (76) - (77) + (79)$$

*Azimuth equation*

$$0 = (-7.1^* - 0.2) - (1) + (2) + (9) - (10) - (14) + (21) + (40) - (42) - (52) + (55) - (59) + (63) + (70) - (71) - (75) + (76)$$

Computation of correction to azimuth constant:

$$\log 0.277 = 9.442$$

$$\log \sin. \text{ mean } \phi = 9.912$$

$$\log \text{ correction} = 9.354$$

$$\text{correction} = -0.2$$

*Side equations*

Symbol	Angle	Logarithm	Tabular difference	Symbol	Angle	Logarithm	Tabular difference
	° ' "				° ' "		
- 9+11	93 11 39.1	9.9993248	-0.12	-13+14	16 37 43.4	9.4566222	+7.05
-12+13	25 52 38.1	9.6399291	+4.34	- 1+ 3	110 36 08.7	9.9712965	-0.79
- 1+ 2	24 17 25.4	9.6142236	+4.67	-10+11	42 00 30.0	9.8255810	+2.34
		9.2534775				9.2534997	

$$0 = -22.2 - 5.46(1) + 4.67(2) + 0.79(3) + 0.12(9) + 2.34(10) - 2.46(11) - 4.34(12) + 11.39(13) - 7.05(14)$$

- 7+ 9	111 09 20.5	9.9696969	-0.81	{ + 7- 9 +14-15 }	21 40 38.8	9.5674745	+5.30
+15-16	36 08 04.3	9.7706188	+2.88	-25+27	89 23 18.6	9.9999753	+0.02
+25-27	30 04 51.8	9.7600325	+3.64	- 8+ 9	48 16 10.2	9.8729041	+1.88
-25+26		9.4403482				9.4403539	

$$0 = -5.7 - 4.49(7) + 1.88(8) + 2.61(9) - 5.30(14) + 8.18(15) - 2.88(16) - 0.74(25) + 3.64(26) - 2.90(27)$$

\* See computation on p. 38.

## Side equations—Continued

Symbol	Angle	Logarithm	Tabular difference	Symbol	Angle	Logarithm	Tabular difference
-16+18	38 29 18.8	9.7940405	+ 2.65	-24+25	12 35 33.3	9.3384902	+9.43
-23+24	27 51 39.2	9.6696203	+ 3.98	-30+32	126 31 06.3	9.9050754	-1.56
-30+31	9 31 16.8	9.2185744	+12.55	-17+18	15 56 21.5	9.4387305	+7.37
		8.6822352				8.6822961	

$$0 = -60.9 - 2.65(16) + 7.37(17) - 4.72(18) - 3.98(23) + 13.41(24) - 9.43(25) - 14.11(30) + 12.55(31) + 1.56(32)$$

-17+19	35 17 43.3	9.7617712	+ 2.98	-29+31	16 21 23.9	9.4496564	+7.17
-28+29	9 58 42.9	9.2387486	+11.97	-44+45	23 37 23.0	9.6028386	+4.81
-42+44	51 05 11.2	9.8910323	+ 1.70	-19+21	43 39 34.4	9.8390831	+2.21
		8.8915521				8.8915781	

$$0 = -26.0 - 2.98(17) + 5.19(19) - 2.21(21) - 11.97(28) + 19.14(29) - 7.17(31) - 1.70(42) + 6.51(44) - 4.81(45)$$

-49+52	89 47 52.8	9.9999973	+0.01	-21+22	26 22 55.0	9.6477280	+4.25
-20+21	26 37 18.2	9.6513730	+4.20	-46+48	105 41 48.0	9.9834943	-0.59
-46+47	33 20 40.5	9.7401044	+3.20	-51+52	35 09 14.0	9.7602524	+2.99
		9.3914747				9.3914747	

$$0 = +0.0 - 4.20(20) + 8.45(21) - 4.25(22) - 3.79(46) + 3.20(47) + 0.59(48) - 0.01(49) + 2.99(51) - 2.98(52)$$

-39+41	105 37 20.7	9.9836521	-0.59	-60+61	22 08 21.3	9.5761788	+5.17
-59+60	27 49 50.7	9.6691879	+3.99	-50+55	125 14 18.4	9.9120934	-1.49
-50+52	37 39 24.5	9.7859918	+2.73	-40+41	63 10 28.9	9.9505530	+1.07
		9.4388318				9.4388252	

$$0 = +6.6 + 0.59(39) + 1.07(40) - 1.66(41) - 4.22(50) + 2.73(52) + 1.49(55) - 3.99(59) + 9.16(60) - 5.17(61)$$

-71+73	59 25 24.7	9.9349784	+1.24	-62+63	43 56 28.3	9.8413093	+2.19
-59+62	58 43 17.2	9.9317900	+1.28	-53+55	59 03 08.4	9.9333037	+1.26
-53+54	18 17 51.0	9.4968619	+6.37	-72+73	22 50 29.2	9.5890357	+5.00
		9.3636303				9.3636487	

$$0 = -18.4 - 5.11(53) + 6.37(54) - 1.26(55) - 1.28(59) + 3.47(62) - 2.19(63) - 1.24(71) + 5.00(72) - 3.76(73)$$

-74+76	94 10 29.2	9.9988462	-0.15	-64+65	50 11 06.3	9.8854273	+1.76
-63+64	28 13 32.1	9.6748099	+3.92	-69+71	102 20 31.6	9.9898451	-0.46
-69+70	61 25 13.8	9.9435708	+1.15	-75+76	33 30 23.0	9.7419627	+3.18
		9.6172269				9.6172351	

$$0 = -8.2 - 3.92(63) + 5.68(64) - 1.76(65) - 1.61(69) + 1.15(70) + 0.46(71) + 0.15(74) + 3.18(75) - 3.33(76)$$

## Length equation

Symbol	Angle	Logarithm	Tabular difference	Symbol	Angle	Logarithm	Tabular difference
Turn-Dundas		-6 4.266771		Ham-South Twin		-1 3.898371	
- 4+ 6	113 41 59.6	9.9617359	-0.92	-10+11	40 00 30.0	9.8255810	+2.34
- 2+ 3	86 18 43.3	9.9990997	+0.14	-12+14	42 30 21.5	9.8297327	+2.30
- 7+ 9	111 09 20.5	9.9696969	-0.81	{ + 7- 9 +14-15 }	21 40 38.8	9.5674745	+5.30
+15-16 +25-27	36 08 04.3	9.7706188	+2.88	-25+27	89 23 18.6	9.9999753	+0.02
-23+25	40 27 12.5	9.8121311	+2.47	-31+32	116 59 49.5	9.9498922	-1.07
-28+31	26 20 06.8	9.6470132	+4.25	-42+45	74 42 34.2	9.9843478	+0.58
-21+22	26 22 55.0	9.6477280	+4.25	-49+52	89 47 52.8	9.9999973	+0.01
-39+40	42 26 51.8	9.8292505	+2.30	-59+61	49 58 12.0	9.8840631	+1.77
-54+55	40 45 17.4	9.8147959	+2.44	-71+72	36 34 55.5	9.7752272	+2.84
-63+65	78 24 38.4	9.9910544	+0.43	-74+75	60 40 06.2	9.9404164	+1.18
-69+70	61 25 13.8	9.9435708	+1.15	-77+79	85 04 23.4	9.9953924	+0.18
		2.6534656				2.6534708	

$$\begin{aligned}
0 = & -5.2 - 0.14(2) + 0.14(3) + 0.92(4) - 0.92(6) - 4.49(7) + 4.49(9) + 2.34(10) - 2.34(11) \\
& + 2.30(12) - 7.60(14) + 8.18(15) - 2.88(16) - 4.25(21) + 4.25(22) - 2.47(23) + 5.37(25) \\
& - 2.90(27) - 4.25(28) + 3.18(31) + 1.07(32) - 2.30(39) + 2.30(40) + 0.58(42) - 0.58(45) \\
& + 0.01(49) - 0.01(52) - 2.44(54) + 2.44(55) + 1.77(59) - 1.77(61) - 0.43(63) + 0.43(65) \\
& - 1.15(69) + 1.15(70) + 2.84(71) - 2.84(72) + 1.18(74) - 1.18(75) + 0.18(77) - 0.18(79)
\end{aligned}$$

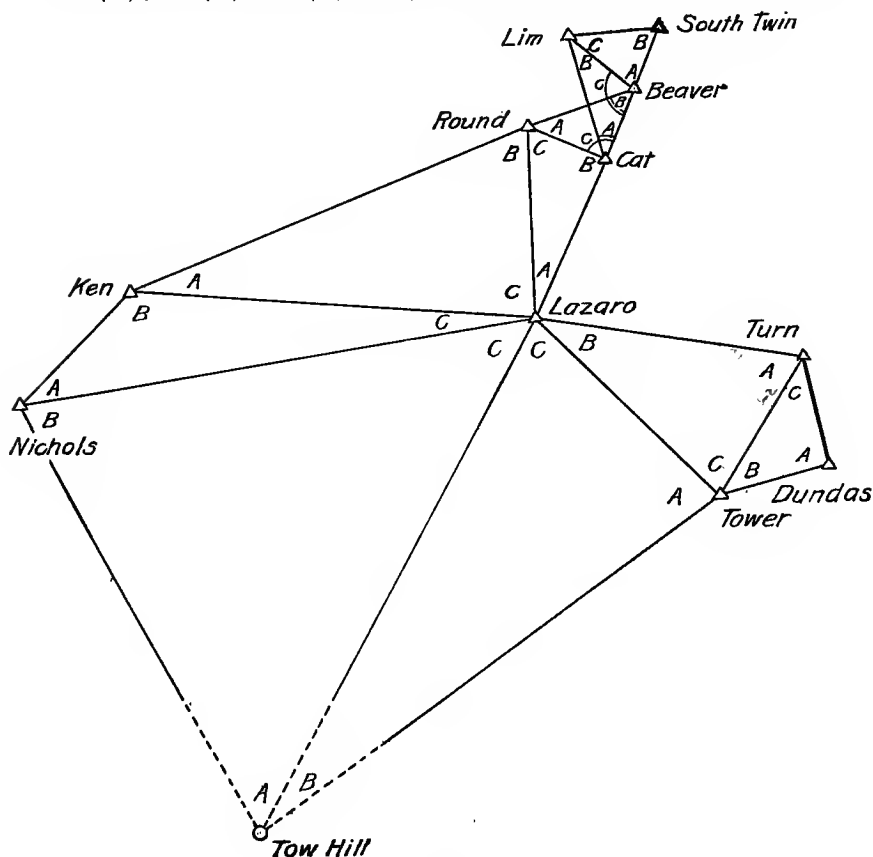


FIG. 5.

*Formation of azimuth equation*

	°	'	"		°	'	"
Turn-Dundas.....	357	33	07.7	Cat-Round.....	109	42	29.5
-1+2.....	24	17	25.4	-52+55.....	87	34	53.9
Turn-Tower.....	21	50	33.1	Cat-Beaver.....	197	17	23.4
Azimuth difference*....	-180	07	07.6	Azimuth difference.....	+180	01	40.2
Tower-Turn.....	201	43	25.5	Beaver-Cat.....	17	19	03.6
+9-10.....	51	11	09.1	-59+63.....	102	39	45.5
Tower-Lazaro.....	150	32	16.4	Beaver-Lim.....	119	58	49.1
Azimuth difference.....	-180	14	01.2	Azimuth difference.....	-180	05	20.3
Lazaro-Tower.....	330	18	15.2	Lim-Beaver.....	299	53	28.8
-14+21.....	101	38	55.1	+70-71.....	40	55	17.8
	22	32	57.3	Lim-South Twin.....	258	58	11.0
	78	57	17.7	Azimuth difference.....	+180	06	48.1
Lazaro-Round.....	173	27	25.3	South Twin-Lim.....	79	04	59.1
Azimuth difference.....	-180	01	37.8	-75+76.....	33	30	23.0
Round-Lazaro.....	353	25	47.5	South Twin-Ham.....	112	35	22.1
+40-42.....	63	49	17.6	Fixed azimuth.....	112	35	29.2
Round-Cat.....	289	36	29.9				
Azimuth difference.....	+180	05	59.6	Discrepancy.....			-7.1
Cat-Round.....	109	42	29.5				

*Preliminary computation of triangles*

Designation of angle	Symbol	Station	Observed angle	Cor-rection	Spher-ical angle	Spher-ical excess	Plane angle	Logarithm
			° ' "			"	° ' "	
B	-10+11 - 4+ 6	Turn-Dundas	42 00 30.0			0.2	24 17	4.266771
C		Tower	30.9			0.2		0.1744195
A		Dundas	113 41 59.6			0.1		9.6142483
		Tower-Dundas				0.5		9.9617359
		Tower-Turn						4.0554388
								4.4029264
B	-12+14 - 2+ 3	Turn-Tower	42 30 21.5			0.6	51 10	4.4029264
A		Lazaro	86 18 43.3			0.7		0.1702686
C		Tower	57.1			0.6		9.9990996
		Lazaro-Tower				1.9		9.8916183
		Lazaro-Turn						4.5722946
								4.4648133
B	-14+15 - 7+ 9	Lazaro-Tower	47 10 38.8			2.2	21 40	4.5722946
C		Tow Hill	07.2			2.2		0.4325371
A		Lazaro	111 09 20.5			2.1		9.8653119
		Tower						9.9696986
		Tow Hill-Tower				6.5		4.8701436
		Tow Hill-Lazaro						4.9745303
B	-25+27 -15+16	Lazaro-Tow Hill	89 23 18.6			3.6	36 08	4.9745303
C		Nichols	54 28 47.9			3.6		0.000248
A		Lazaro	04.3			3.6		9.9105723
		Tow Hill						9.7706083
		Nichols-Tow Hill				10.8		4.8851274
		Nichols-Lazaro						4.7451634

\* See position computation, p. 40.

*Preliminary computation of triangles—Continued*

Designation of angle	Symbol	Station	Observed angle	Cor-rection	Spher-ical angle	Sphe-ical excess	Plane angle	Logarithm
			° ' "			"	° ' "	
B	-31+32	Lazaro-Nichols						4.7451634
C		Ken	116 59 49.5			0.8		0.0501070
A	-23+25	Lazaro	60.2			0.7	22 32 59.5	9.5837509
		Nichols	40 27 12.5			0.7	11.8	9.8121294
		Ken-Nichols				2.2		4.3790213
		Ken-Lazaro						4.6073998
B	-42+45	Lazaro-Ken				0.6		4.6073998
C		Round	74 42 34.2			0.7	78 57 33.6	0.0156525
A	-28+31	Lazaro	20.9			0.6	20.2	9.9918811
		Ken	26 20 06.8				06.2	9.6470108
		Round-Ken				1.9		4.6149334
		Round-Lazaro						4.2700631
B	-49+52	Lazaro-Round				0.2		4.2700631
A		Cat	89 47 52.8			0.1	52.6	0.0000027
C	-21+22	Lazaro	26 22 55.0			0.1	54.9	9.6477276
		Round	12.6			0.1	63 49 12.5	9.9529926
		Cat-Round				0.4		3.9177934
		Cat-Lazaro						4.2230584
B	-59+61	Cat-Round				0.1		3.9177934
C		Beaver	49 58 12.0			0.0	11.9	0.1159371
A	-39+40	Cat	56.4			0.1	87 34 56.4	9.9996132
		Round	42 26 51.8			0.1	51.7	9.8292503
		Beaver-Round				0.2		4.0333437
		Beaver-Cat						3.8629808
B	-71+72	Beaver-Cat				0.0		3.8629808
C		Lim	36 34 55.5			0.1	55.5	0.2247728
A	-54+55	Beaver	47.2			0.0	102 39 47.1	9.9893057
		Cat	40 45 17.4			0.0	17.4	9.8147959
		Lim-Cat				0.1		4.0770593
		Lim-Beaver						3.9025495
B	-74+75	Beaver-Lim				0.0		3.9025495
A		South Twin	60 40 06.2			0.1	06.2	0.0595836
C	-63+65	Beaver	78 24 38.4			0.0	38.3	9.9910543
		Lim	15.5			0.0	40 55 15.5	9.8162528
		South Twin-Lim				0.1		3.9531874
		South Twin-Beaver						3.7783859

*Preliminary position computation,*

## STATION TOWER

$\alpha$ Second angle	Turn to Dundas Dundas and Tower						$^{\circ}$ 357 + 24	$'$ 33 17	$''$ 07.7 30.9
$\alpha$ $\Delta\alpha$	Turn to Tower						21 —	50 7	38.6 07.6
$\alpha'$	Tower to Turn				First angle of triangle		180 201 42	43 00	31.0 30.0
$\phi$ $\Delta\phi$	$^{\circ}$ 54 —	$'$ 48 12	$''$ 06.742 39.419	Turn		$\lambda$ $\Delta\lambda$	130 +	56 8	04.052 43.993
$\phi'$	54	35	27.323	Tower		$\lambda'$	131	04	48.045
$s$ $\cos \alpha$ B	4.4029264 9.9676416 8.5097251	$s^2$ $\sin^2 \alpha$ C	8.80585 9.14128 1.55459	$(\delta\phi)^2$ D	5.7609 2.3672	$-\frac{h}{s^2 \sin^2 \alpha}$ E	2.8803 7.9471 6.4574		
h	2.8802931		9.50172		8.1281			7.2848	
1st term	+759.0896	3d term	+0.0134						
2d term	+ 0.3175	4th term	-0.0019						
3d and 4th terms	+759.4071 + 0.0115		—6 4.4029264 9.5706383 8.5087480 0.2370138	Arg. $s$ $\Delta\lambda$	-11 + 5	$\sin \frac{\Delta\lambda}{2}(\phi + \phi')$ $\sec \frac{1}{2}(\Delta\phi)$	2.7193259 9.9117440 7		
$-\Delta\phi$	+759.4186	$s$ $\sin \alpha$ $A'$ $\sec \phi'$		Corr.	- 6		2.6310706 "		
$\frac{1}{2}(\phi + \phi')$	$^{\circ}$ 54 $'$ 41 $''$ 47		2.7193259 "				+427.63		
		$\Delta\lambda$	+523.9935						

## STATION LAZARO

$\alpha$ Second angle	Turn to Tower Tower and Lazaro						$^{\circ}$ 21 + 86	$'$ 50 18	$''$ 38.6 43.3
$\alpha$ $\Delta\alpha$	Turn to Lazaro						108 —	09 21	21.9 10.7
$\alpha'$	Lazaro to Turn				First angle of triangle		180 287 42	48 30	11.2 21.5
$\phi$ $\Delta\phi$	$^{\circ}$ 54 +	$'$ 48 4	$''$ 06.742 51.101	Turn		$\lambda$ $\Delta\lambda$	130 +	56 25	04.052 54.244
$\phi'$	54	52	57.843	Lazaro		$\lambda'$	131	21	58.296 —1
$s$ $\cos \alpha$ B	4.4648133 9.4936068 8.5097251	$s^2$ $\sin^2 \alpha$ C	8.92963 9.95564 1.55459	$(\delta\phi)^2$ D	4.9281 2.3672	$-\frac{h}{s^2 \sin^2 \alpha}$ E	2.4681 8.8853 6.4574		
h	2.4681452		0.43986		7.2953			7.8108	
1st term	-293.8632	3d term	+0.0020						
2d term	+ 2.7534	4th term	+0.0065						
3d and 4th terms	-291.1098 + 0.0085		+27 4.4648133 9.9778202 8.5087409 0.2401420	Arg. $s$ $\Delta\lambda$	-15 +42	$\sin \frac{\Delta\lambda}{2}(\phi + \phi')$ $\sec \frac{1}{2}(\Delta\phi)$	3.1915191 9.9125251		
$-\Delta\phi$	-291.1013	$s$ $\sin \alpha$ $A'$ $\sec \phi'$		Corr.	+27		3.1040442 "		
$\frac{1}{2}(\phi + \phi')$	$^{\circ}$ 54 $'$ 50 $''$ 32.3		3.1915191 "				+1270.70		
		$\Delta\lambda$	+1554.2441						

## primary triangulation

## STATION TOWER

$\alpha$ Third angle	Dundas to Turn Tower and Turn					$^{\circ}$ 177 -113	$'$ 33 41	$''$ 43.6 59.6
$\alpha$ $\Delta\alpha$	Dundas to Tower					63 —	51 7	44.0 43.1
$\alpha'$	Tower to Dundas					180 243	44	00.9 +1
$\phi$ $\Delta\phi$	$^{\circ}$ 54 —	$'$ 38 2	$''$ 09.559 42.236	Dundas	$\lambda$ $\Delta\lambda$	130 +	55 9	20.042 28.003
$\phi'$	54	35	27.323	Tower	$\lambda'$	131	04	48.045
$s$ $\cos \alpha$ B	4.0554388 9.6439767 8.5097372	$s^2$ $\sin^2 \alpha$ C	8.11087 9.90630 1.55194	$(\partial\phi)^2$ D	4.4203 2.3681	$-\frac{h}{s^2 \sin^2 \alpha}$ E	2.2091 8.0172 6.4528	
h	2.2091527 "		9.56911 "		6.7884		6.6791	
1st term	+161.8649	3d term	+0.0006					
2d term	+ 0.3708	4th term	-0.0005					
3d and 4th terms }	+162.2357							
	+0.0001							
$-\Delta\phi$	+162.2358	$s$ $\sin \alpha$ A'	$+\frac{3}{4}$ 4.0554388 9.9531493 8.5087480 0.2370138	Arg. $s$ $\Delta\lambda$	-2 +5	$\sin \frac{\Delta\lambda}{2}(\phi+\phi')$ $\sec \frac{1}{2}(\Delta\phi)$	2.7543502 9.9112981	
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 54 36 48.4	$\sec \phi'$	2.7543502 "	Corr.	+3		2.6656483 "	
		$\Delta\lambda$	+568.0025			$-\Delta\alpha$	+463.07	

## STATION LAZARO

$\alpha$ Third angle	Tower to Turn Lazaro and Turn				$^{\circ}$ 201 — 51	$'$ 43 10	$''$ 31.0 57.1
$\alpha$ $\Delta\alpha$	Tower to Lazaro				150 —	32 14	33.9 01.2
$\alpha'$	Lazaro to Tower				180 330	18	32.7
$\phi$ $\Delta\phi$	$^{\circ}$ 54 +	$'$ 35 17	$''$ 27.323 30.520	Tower	$\lambda$ $\Delta\lambda$	131 +	04 17 48.045 10.250
$\phi'$	54	52	57.843	Lazaro	$\lambda'$	131	21 58.295
$s$ $\cos \alpha$ B	4.5722946 9.9398800 8.5097404	$s^2$ $\sin^2 \alpha$ C	9.14459 9.38353 1.55122	$(\partial\phi)^2$ D	6.0428 2.3683	$-\frac{h}{s^2 \sin^2 \alpha}$ E	3.0219 8.5281 6.4516
h	3.0219150 "		0.07934 "		8.4111		8.0016
1st term	-1051.7559	3d term	+0.0258				
2d term	+ 1.2004	4th term	+0.0100				
3d and 4th terms }	-1050.5555						
	+ 0.0358						
	-1050.5197	$s$ $\sin \alpha$ A'	$-\frac{7}{4}$ 4.5722946 9.6917656 8.5087409 0.2401420	Arg. $s$ $\Delta\lambda$	-25 +18	$\sin \frac{\Delta\lambda}{2}(\phi+\phi')$ $\sec \frac{1}{2}(\Delta\phi)$	3.0129424 9.9119609
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 54 $'$ 44 $''$ 12.6	$\sec \phi'$	3.0129424 "	Corr.	- 7		2.9249033 " +841.20
		$\Delta\lambda$	+1030.2498			$-\Delta\alpha$	

*Preliminary position computation,*

## STATION TOW HILL

$\alpha$ Second angle	Lazaro to Tower Tower and Tow Hill				First angle of triangle	$^{\circ}$ 330 + 47	$'$ 18 10	$''$ 32.7 07.2
	Lazaro to Tow Hill					17 —	28 21	39.9 07.7
	Tow Hill to Lazaro					180 197 21	07 40	32.2 38.8
$\phi$ $\Delta\phi$ $\phi'$	$^{\circ}$ 54 —	$'$ 52 48	$''$ 57.843 31.870	Lazaro	$\lambda$ $\Delta\lambda$	131 +	21 25	58.295 57.539
	54	04	25.973 —1	Tow Hill	$\lambda'$	131	47	55.834
$s$ $\cos \alpha$ B	4.9745303 9.9794727 8.5097191	$s^2$ $\sin^2 \alpha$ C	9.94906 8.95521 1.55589	$(\delta\phi)^2$ D	6.9283 2.3667	$-\frac{h}{s^2 \sin^2 \alpha}$ E	3.4637 8.9043 6.4597	
h	3.4637221 " "		0.46016 " "		9.2950		8.8277	
1st term	+2908.8550	3d term	+0.1972					
2d term	+ 2.8851	4th term	—0.0673					
3d and 4th terms	+2911.7401		—117 4.9745303 9.4776065 8.5087606 0.2315532	Arg. $s$ $\Delta\lambda$	—159 + 42	$\sin \frac{\Delta\lambda}{2}(\phi + \phi')$ $\sec \frac{\Delta\lambda}{2}(\Delta\phi)$	3.1924389 9.9105687 108	
	+ 0.1299	$s$ $\sin \alpha$ $\Lambda'$						
	+2911.8700	$\sec \phi'$						
$\frac{1}{2}(\phi + \phi')$	$^{\circ}$ 54 28 41.9		3.1924389 " +1557.5388	Corr.	—117		3.1030184 " +1267.70	
		$\Delta\lambda$				$-\Delta\alpha$		

## STATION NICHOLS

$\alpha$ Second angle	Lazaro to Tow Hill Tow Hill and Nichols					$^{\circ}$ 17 + 54	$'$ 28 28	$''$ 39.9 47.9
$\alpha$ $\Delta\alpha$	Lazaro to Nichols					71 —	57 40	27.8 14.3
$\alpha'$	Nichols to Lazaro					180 251 89	17 23	13.5 18.6
	First angle of triangle							
$\phi$ $\Delta\phi$ $\phi'$	$^{\circ}$ 54 —	$'$ 52 9	$''$ 57.843 27.012	Lazaro	$\lambda$ $\Delta\lambda$	131 +	21 49	58.295 14.397
	54	43	30.831	Nichols	$\lambda'$	132	11	12.692 +1
$s$ $\cos \alpha$ B	4.7451634 9.4909674 8.5097191	$s^2$ $\sin^2 \alpha$ C	9.49033 9.95620 1.55589	$(\delta\phi)^2$ D	5.5072 2.3667	$-\frac{h}{s^2 \sin^2 \alpha}$ E	2.7458 9.4465 6.4597	
h	2.7458499 " +556.9933 + 10.0559		1.00242 " +0.0075 -0.0449		7.8739		8.6520	
1st term 2d term		3d term 4th term						
3d and 4th terms	+567.0492 — 0.0374		+92 4.7451634 9.9781021 8.5087447 0.2384494	Arg. $s$ $\Delta\lambda$	— 56 +148	$\sin \frac{\Delta\lambda}{2}(\phi + \phi')$ $\sec \frac{\Delta\lambda}{2}(\Delta\phi)$	3.4704688 9.9123203	
$-\Delta\phi$	+567.0118	$\frac{s}{\Lambda'}$ $\sec \phi'$						
$\frac{1}{2}(\phi + \phi')$	$^{\circ}$ 54 48 14.3		3.4704688 " +2954.3966	Corr.	+ 92		3.3827891 " +2414.3	
		$\Delta\lambda$				$-\Delta\alpha$		

## primary triangulation—Continued

## STATION TOW HILL

$\alpha$ Third angle	Tower to Lazaro Tow Hill and Lazaro					$^{\circ}$ 150 —111	$'$ 32 09	$''$ 33.9 20.5
$\alpha$ $d\alpha$	Tower to Tow Hill					39 —	23 35	13.4 02.4
$\alpha'$	Tow Hill to Tower					180 218	48	11.0
$\phi$ $d\phi$	$^{\circ}$ 54 —	$'$ 35 31	$''$ 27.323 01.351	Tower	$\lambda$ $d\lambda$	131 +	04 43	48.045 07.789
$\phi'$	54	04	25.972	Tow Hill	$\lambda'$	131	47	55.834
$s$ $\cos \alpha$ B	4.8701436 9.8881103 8.5097404	$s^2$ $\sin^2 \alpha$ C	9.74029 9.60494 1.55122	$(\partial\phi)^2$ D	6.5397 2.3683	$-h$ $s^2 \sin^2 \alpha$ E		3.2680 9.3452 6.4516
h	3.2679943		0.89645		8.9080			9.0648
1st term	+1853.5073	3d term	+0.0809			$(d\lambda)^3$ F		0.239
2d term	+7.8786	4th term	—0.1160					7.733
3d and 4th terms	+1861.3859 —0.0351		+16 4.8701436 9.8024699 8.5087606 0.2315532	Arg. $s$ $d\lambda$	—98 +114	$\sin \frac{1}{2}(\phi+\phi')$ $\sec \frac{1}{2}(d\phi)$		3.4129289 9.9097770 44
$-\Delta\phi$	+1861.3508	$s$ $\sin \alpha$ A' sec $\phi'$		Corr.	+16			3.3227103
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 54 19 56.6	$d\lambda$	+2587.7893			$-\Delta\alpha$		+2102.38

## STATION NICHOLS

$\alpha$ Third angle	Tow Hill to Lazaro Nichols and Lazaro					$^{\circ}$ 197 —36	$'$ 07 08	$''$ 32.2 04.3
$\alpha$ $d\alpha$	Tow Hill to Nichols					160 —	59 18	27.9 55.8
$\alpha'$	Nichols to Tow Hill					180 340	40	32.1
$\phi$ $d\phi$	$^{\circ}$ 54 +	$'$ 04 39	$''$ 25.972 04.860	Tow Hill	$\lambda$ $d\lambda$	131 +	47 23	55.834 16.859
$\phi'$	54	43	30.832 —1	Nichols	$\lambda'$	132	11	12.693
$s$ $\cos \alpha$ B	4.8851274 9.9756468 8.5097780	$s^2$ $\sin^2 \alpha$ C	9.77025 9.02568 1.54301	$(\partial\phi)^2$ D	6.7403 2.3709	$-h$ $s^2 \sin^2 \alpha$ E		3.3706 8.7959 6.4373
h	3.3705522		0.33894		9.1112			8.6038
1st term	—2347.2114	3d term	+0.1292					
2d term	+2.1824	4th term	+0.0402					
3d and 4th terms	—2345.0290 +0.1694		—70 4.8851274 9.5128381 8.5087447 0.2384494	Arg. $s$ $d\lambda$	—104 +34	$\sin \frac{1}{2}(\phi+\phi')$ $\sec \frac{1}{2}(d\phi)$		3.1451536 9.9101427 70
$-\Delta\phi$	—2344.8596	$s$ $\sin \alpha$ A' sec $\phi'$		Corr.	—70			3.0553033
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 54 23 58.9	$d\lambda$	+1396.8592			$-\Delta\alpha$		+1135.80

*Preliminary position computation,*

## STATION KEN

$\alpha$	Lazaro to Nichols						°	'	''	
Second angle	Nichols and Ken						+ 71	57	27.8	
							+ 22	33	00.2	
$\alpha$	Lazaro to Ken						94	30	28.0	
							—	30	53.7	
$\Delta\alpha$							180			
$\alpha'$	Ken to Lazaro						273	59	34.3	
						First angle of triangle	116	59	49.5	
$\phi$	°	'	''		Lazaro	$\lambda$				
$\Delta\phi$	54	52	57.843			$\Delta\lambda$	131	21	58.295	
	+	1	37.056				+	37	45.861	
$\phi'$	54	54	34.899		Ken	$\lambda'$	131	59	44.156	
$s$	4.6073998	$s^2$	9.21480		$(\partial\phi)^2$ D	4.0250 2.3667	—h	2.0125		
$\cos \alpha$	8.8953917	$\sin^2 \alpha$	9.99731				$s^2 \sin^2 \alpha$	E	9.2121	
B	8.5097191	C	1.55589						6.4597	
h	2.0125106		0.76800			6.3917		7.6843		
	''		''							
1st term	— 102.9226	3d term	+0.0002							
2d term	+ 5.8614	4th term	+0.0048							
	— 97.0612									
3d and 4th terms	+ 0.0050	$s$	+58		Arg. $s$ $\Delta\lambda$	—29 +87	$\sin \frac{\Delta\lambda}{2}(\phi+\phi')$ $\sec \frac{1}{2}(\Delta\phi)$	3.355234		
		$\sin \alpha$	4.6073998					9.912798		
$-\Delta\phi$	— 97.0562	$A'$	9.9986545							
		$\sec \phi'$	8.5087403							
	° ' ''		0.2404328							
$\frac{1}{2}(\phi+\phi')$	54 53 46.4		3.3552332		Corr.	+58		3.268032		
			''					''		
		$\Delta\lambda$	+2265.8607				— $\Delta\alpha$	+1853.67		

## STATION ROUND

$\alpha$ Second angle $\alpha$ $\Delta\alpha$ $\alpha'$ $\phi$ $\Delta\phi$ $\phi'$	Lazaro to Ken Ken and Round				First angle of triangle	$^{\circ}$ 94 + 78	$'$ 30 57	$''$ 28.0 20.9
	Lazaro to Round					173 —	27 1	48.9 37.8
	Round to Lazaro					180 353 74	26 42	11.1 34.2
						131 +	21 1	58.295 59.402
	$^{\circ}$ 54 +	$'$ 52 9	$''$ 57.843 58.318	Lazaro	$\lambda$ $\Delta\lambda$	131 +	21 1	58.295 59.402
	55	02	56.161	Round	$\lambda'$	131	23	57.697
$s$ $\cos \alpha$ B	4.2700631 9.9971678 8.5097191	$s^2$ $\sin^2 \alpha$ C	8.54013 8.11255 1.55589	$(\partial\phi)^2$ D	5.5538 2.3667	$-\frac{h}{s^2}$ $\sin^2 \alpha$ E	2.7769 6.6527 6.4597	
h	2.7769500 "		8.20857 "		7.9205		5.8893	
1st term	—598.3427	3d term	+0.0083					
2d term	+ 0.0161	4th term	+0.0001					
3d and 4th terms } $-\Delta\phi$ $\frac{1}{2}(\phi+\phi')$	—598.3266 + 0.0084	$s$ $\sin \alpha$ A'	—6 4.2700631 9.0562746 8.5087369 0.2419389	Arg. $s$ $\Delta\lambda$	—6 0	$\sin \frac{\Delta\lambda}{2}(\phi+\phi')$ $\sec \frac{1}{2}(\Delta\phi)$	2.077013 9.913183	
	$^{\circ}$ $'$ $''$ 54 57 57	$\sec \phi'$	2.0770129 "	Corr.	—6		1.990196 "	
			$\Delta\lambda$	+119.4024			$-\Delta\alpha$	+97.77

## primary triangulation—Continued

## STATION KEN

Third angle	Nichols to Lazaro Ken and Lazaro					° 251 — 40	' 17 27	'' 13.5 12.5
$\alpha$	Nichols to Ken					210	50	01.0
$\Delta\alpha$						+	9	22.8
$\alpha'$	Ken to Nichols					180 30	59	23.8
$\phi$	° 54	' 43	'' 30.831	Nichols	$\lambda$ $\Delta\lambda$	132	11	12.693
$\Delta\phi$	+	11	04.068			—	11	28.537
$\phi'$	54	54	34.899	Ken	$\lambda'$	131	59	44.156
$s$	4.3790213	$s^2$	8.75804			—h		2.8226
$\cos \alpha$	9.9338209	$\sin^2 \alpha$	9.41947	$(\partial\phi)^2$ D	5.6450 2.3676	$s^2 \sin^2 \alpha$		8.1775
B	8.5097307	C	1.55337			E		6.4553
h	2.8225729		9.73088		8.0126			7.4554
1st term	—664.6192	3d term	+0.0103					
2d term	+ 0.5381	4th term	+0.0029					
3d and 4th terms }	—664.0811		—2					
	+ 0.0132	$s$	4.3790213	Arg. $s$ $\Delta\lambda$	—10 + 8	$\sin \frac{1}{2}(\phi+\phi')$ $\sec \frac{1}{2}(\Delta\phi)$		2.837928
	—664.0679	$\sin \alpha$	9.7097334					9.912392
— $\Delta\phi$	° ' '' 54 49 02.9	$A'$ $\sec \phi'$	8.5087403 0.2404328					
$\frac{1}{2}(\phi+\phi')$			2.8379276	Corr.	— 2			2.750320
		$\Delta\lambda$	—688.5375			— $\Delta\alpha$		—562.76

## STATION ROUND

Third angle	Ken to Lazaro Round and Lazaro					° 273 — 26	' 59 20	'' 34.3 06.8
$\alpha$	Ken to Round					247	39	27.5
$\Delta\alpha$						+	29	17.8
$\alpha'$	Round to Ken					180 68	08	45.3
$\phi$	° 54	' 54	'' 34.899	Ken	$\lambda$ $\Delta\lambda$	131	59	44.156
$\Delta\phi$	+	8	21.261			—	35	46.458
$\phi'$	55	02	56.160 +1	Round	$\lambda'$	131	23	57.698 —1
$s$	4.6149334	$s^2$	9.22987			—h		2.7046
$\cos \alpha$	9.5799436	$\sin^2 \alpha$	9.93222	$(\partial\phi)^2$ D	5.4091 2.3666	$s^2 \sin^2 \alpha$		9.1621
B	8.5097172	C	1.55631			E		6.4604
h	2.7045942		0.71840		7.7757			8.3271
1st term	—506.5172	3d term	+0.0058					
2d term	+ 5.2288	4th term	+0.0212					
3d and 4th terms }	—501.2884		+49					
	+ 0.0270	$s$	4.6149334	Arg. $s$ $\Delta\lambda$	—30 +79	$\sin \frac{1}{2}(\phi+\phi')$ $\sec \frac{1}{2}(\Delta\phi)$		3.331722
	—501.2614	$\sin \alpha$	9.9661083					9.913254
— $\Delta\phi$	° ' '' 54 58 46.0	$A'$ $\sec \phi'$	8.5087369 0.2419389					
$\frac{1}{2}(\phi+\phi')$			3.3317224	Corr.	+49			3.244976
		$\Delta\lambda$	—2146.458			— $\Delta\alpha$		—1757.83

*Preliminary position computation,*

## STATION CAT

$\alpha$	Lazaro to Round					$^{\circ}$	$'$	$''$
Second angle	Round and Cat					173	27	48.9
$\alpha$						+ 26	22	55.0
$\Delta\alpha$	Lazaro to Cat					199	50	43.9
						+	4	21.5
$\alpha'$	Cat to Lazaro					180		
						19	55	05.4
						89	47	52.8
	First angle of triangle							
$\phi$	$^{\circ}$	$'$	$''$	Lazaro	$\lambda$			
$\Delta\phi$	54	52	57.843		$\Delta\lambda$	131	21	58.295
	+	8	28.257			-	5	19.363
$\phi'$	55	01	26.100	Cat	$\lambda'$	131	16	38.932
$s$	4.2230584	$s^2$	8.44612			-h		2.7062
$\cos \alpha$	9.9734102	$\sin^2 \alpha$	9.06164	$(\Delta\phi)^2$	5.4124	$s^2 \sin^2 \alpha$		7.5078
B	8.5097191	C	1.55589	D	2.3667	E		6.4597
h	2.7061877		9.06365		7.7791			6.6737
	"		"					
1st term	-508.3791	3d term	+0.0060					
2d term	+ 0.1158	4th term	+0.0005					
	-508.2633							
3d and 4th terms	+ 0.0065		-3					
$-\Delta\phi$	-508.2568	$s$	4.2230584	Arg.		$\sin \frac{\Delta\lambda}{2}(\phi+\phi')$		2.504284
		$A'$	9.5308211	$s$	-5	$\sec \frac{\Delta\phi}{2}$		9.913117
		$\sec \phi'$	8.5087375	$\Delta\lambda$	+2			
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$		2.5042844	Corr.	-3			2.417401
	54 57 42.0		"					"
		$\Delta\lambda$	-319.3629			$-\Delta\alpha$		-261.46

## STATION BEAVER

$\alpha$	Cat to Round					$^{\circ}$	$'$	$''$
Second angle	Round and Beaver					109	42	58.1
$\alpha$						+ 87	34	56.4
$\Delta\alpha$	Cat to Beaver					197	17	54.5
						+	1	40.2
$\alpha'$	Beaver to Cat					180		
						17	19	34.7
						49	58	12.0
	First angle of triangle							
$\phi$	$^{\circ}$	$'$	$''$	Cat	$\lambda$			
$\Delta\phi$	55	01	26.100		$\Delta\lambda$	131	16	38.932
	+	3	45.192			-	2	02.273
$\phi'$	55	05	11.292	Beaver	$\lambda'$	131	14	36.659
$s$	3.8629808	$s^2$	7.7260			-h		2.353
$\cos \alpha$	9.9798982	$\sin^2 \alpha$	8.9465	$(\Delta\phi)^2$	4.705	$s^2 \sin^2 \alpha$		6.672
B	8.5097090	C	1.5584	D	2.366	E		6.464
h	2.3525880		8.2309		7.071			5.489
	"		"					
1st term	-225.2102	3d term	+0.0012					
2d term	+ 0.0170	4th term						
	-225.1932							
3d and 4th terms	+0.0012		-1					
$-\Delta\phi$	-225.1920	$s$	3.8629808	Arg.		$\sin \frac{\Delta\lambda}{2}(\phi+\phi')$		2.087330
		$A'$	9.4732671	$s$	-1	$\sec \frac{\Delta\phi}{2}$		9.913657
		$\sec \phi'$	8.5087360	$\Delta\lambda$	0			
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$		2.0873301	Corr.	-1			2.000987
	55 03 18.7		"			$-\Delta\alpha$		-100.23
		$\Delta\lambda$	-122.2728					

## primary triangulation—Continued

## STATION CAT

$\alpha$ Third angle	Round to Lazaro Cat and Lazaro						$^{\circ}$ 353 — 63	$'$ 26 49	$''$ 11.1 12.6
$\alpha$ $\Delta\alpha$	Round to Cat						289 +	36 5	58.5 59.6
$\alpha'$	Cat to Round						180 109	42	58.1
$\phi$ $\Delta\phi$	$^{\circ}$ 55 —	$'$ 02 1	$''$ 56.161 30.061	Round	$\lambda$ $\Delta\lambda$		131 —	23 7	57.697 18.765
$\phi'$	55	01	28.100	Cat	$\lambda'$		131	16	38.932
$s$ $\cos \alpha$ B	3.9177934 9.5259756 8.5097071	$s^2$ $\sin^2 \alpha$ C	7.8356 9.9481 1.5586	$(\partial\phi)^2$ D	3.9069 2.3658	$-\hbar$ $s^2 \sin^2 \alpha$ E			1.9535 7.7837 6.4643
h	1.9534761 "		9.3423 "		6.2727				6.2015
1st term	+89.8413	3d term	+0.0002						
2d term	+ 0.2199	4th term	-0.0002						
3d and 4th terms	+90.0612								
$-\Delta\phi$	+90.0612	$s$ $\sin \alpha$ A'	3.9177934 9.9740335 8.5087375	Arg. $s$ $\Delta\lambda$	-1 +3	$\Delta\lambda$ $\sin \frac{1}{2}(\phi+\phi')$ $\sec \frac{1}{2}(\Delta\phi)$			2.642232 9.913558
$\frac{1}{2}(\phi+\phi')$	55 02 11.1	$\sec \phi'$	0.2416677	Corr.	+2				2.555790
		$\Delta\lambda$	-438.7654			$-\Delta\alpha$			-359.58

## STATION BEAVER

$\alpha$ Third angle	Round to Cat Beaver and Cat						$^{\circ}$ 289 — 42	$'$ 36 26	$''$ 58.5 51.8
$\alpha$ $\Delta\alpha$	Round to Beaver						247 +	10 7	06.7 40.0
$\alpha'$	Beaver to Round						180 67	17	46.7
$\phi$ $\Delta\phi$	$^{\circ}$ 55 +	$'$ 02 2	$''$ 56.161 15.131	Round	$\lambda$ $\Delta\lambda$		131 —	23 9	57.697 21.038
$\phi'$	55	05	11.292	Beaver	$\lambda'$		131	14	36.659
$s$ $\cos \alpha$ B	4.0333437 9.588562 8.5097071	$s^2$ $\sin^2 \alpha$ C	8.0667 9.9291 1.5586	$(\partial\phi)^2$ D	4.264 2.366	$-\hbar$ $s^2 \sin^2 \alpha$ E			2.132 7.996 6.464
h	2.1319070 "		9.5544 "		6.630				6.592
1st term	-135.4899	3d term	+0.0004						
2d term	+ 0.3584	4th term	+0.0004						
3d and 4th terms	-135.1315								
$-\Delta\phi$	+ 0.0008	$s$ $\sin \alpha$ A'	4.0333437 9.9645661 8.5087360	Arg. $s$ $\Delta\lambda$	-2 +5	$\Delta\lambda$ $\sin \frac{1}{2}(\phi+\phi')$ $\sec \frac{1}{2}(\Delta\phi)$			2.748992 9.913723
$\frac{1}{2}(\phi+\phi')$	55 04 03.7	$\sec \phi'$	0.2423463	Corr.	+3				2.662715
		$\Delta\lambda$	-561.0382			$-\Delta\alpha$			-459.95

*Preliminary position computation,*

## STATION LIM

$\alpha$	Beaver to Cat						$^{\circ}$	$'$	$''$
Second angle	Cat and Lim						17	19	34.7
							+102	39	47.2
$\alpha$	Beaver to Linn						119	59	21.9
$\Delta\alpha$							—	5	20.3
$\alpha'$	Lim to Beaver						180	54	01.6
	First angle of triangle						299	34	55.5
							36		
$\phi$	$^{\circ}$	$'$	$''$	Beaver		$\lambda$			
$\Delta\phi$	55	05	11.292			$\Delta\lambda$	131	14	36.659
	+	2	08.973				+	6	30.479
$\phi'$	55	07	20.265	Lim		$\lambda'$	131	21	07.138
$s$	3.9025495	$s^2$	7.8051						
$\cos \alpha$	9.6988310	$\sin^2 \alpha$	9.8751	$(\Delta\phi)^2$		4.222	$-\frac{h}{s^2} \sin^2 \alpha$		
B	8.5097044	C	1.5591	D		2.366	E		
h	2.1110849		9.2393			6.588			
1st term	"		"						
2d term	-129.1471	3d term	+0.0004						
	+ 0.1735	4th term	+0.0002						
	-128.9736		+2						
3d and 4th terms	+ 0.0006		3.9025495						
$-\Delta\phi$	-128.9730	$s$	9.9375769	Arg.					
	"	$\sin \alpha$	8.5087351	$s$		-1	$\sin \frac{\Delta\lambda}{2}(\phi+\phi')$		
	"	$\sec \phi'$	0.2427356	$\Delta\lambda$		+3	$\sec \frac{1}{2}(\Delta\phi)$		
$\frac{1}{2}(\phi+\phi')$	55 06 15.8		2.5915973	Corr.		+2			
		$\Delta\lambda$	+390.4787				$-\Delta\alpha$		

## STATION SOUTH TWIN

$\alpha$	Beaver to Lim						$^{\circ}$	$'$	$''$
Second angle	Lim and South Twin						119	59	21.9
							+ 78	24	38.4
$\alpha$	Beaver to South Twin						198	24	00.3
$\Delta\alpha$							+	1	27.7
$\alpha'$	South Twin to Beaver						180	25	28.0
	First angle of triangle						18	40	06.2
							60		
$\phi$	$^{\circ}$	$'$	$''$	Beaver		$\lambda$			
$\Delta\phi$	55	05	11.292			$\Delta\lambda$	131	14	36.659
	+	3	04.190				—	1	46.962
$\phi'$	55	08	15.482	South Twin		$\lambda'$	131	12	49.697
	Fixed latitude, 15.517						Fixed longitude, 49.974		
$s$	3.7783859	$s^2$	7.5568				$-\frac{h}{s^2} \sin^2 \alpha$		
$\cos \alpha$	9.9772093	$\sin^2 \alpha$	8.9984	$(\Delta\phi)^2$		4.530	2.265		
B	8.5097044	C	1.5591	D		2.366	E		
h	2.2652996		8.1143			6.896			
	"		"				5.285		
1st term	-184.2042	3d term	+0.0008						
2d term	+ 0.0130	4th term							
	-184.1912		-1						
3d and 4th terms	+ 0.0008	$s$	3.7783859	Arg.			2.029229		
$-\Delta\phi$	-184.1904	$\sin \alpha$	9.4992064	$s$		-1	$\sin \frac{\Delta\lambda}{2}(\phi+\phi')$		
	"	$\sec \phi'$	8.5087347	$\Delta\lambda$			$\sec \frac{1}{2}(\Delta\phi)$		
	"		0.2429024						
$\frac{1}{2}(\phi+\phi')$	55 06 43.4		2.0292293	Corr.		-1	1.943187		
		$\Delta\lambda$	-106.9619				"		
							$-\Delta\alpha$		
							-87.74		

Discrepancy in latitude:

$$\begin{aligned}
 & -0.035 \\
 & \times 7238.24 \div 100 = \\
 & -2.5334
 \end{aligned}$$

Discrepancy in longitude:

$$\begin{aligned}
 & -0.277 \\
 & \times 7238.24 \div 100 = \\
 & -20.0499
 \end{aligned}$$

primary triangulation—Continued

## STATION LIM

$\alpha$ Third angle	Cat to Beaver Lim and Beaver					$^{\circ}$ 197 — 40	$'$ 17 45	$''$ 54.5 17.4
$\alpha$ $\Delta\alpha$	Cat to Lim					156 —	32 3	37.1 39.9
$\alpha'$	Lim to Cat					180 336	28	57.2 — .1
$\phi$ $\Delta\phi$	$^{\circ}$ 55 +	$'$ 01 5	$''$ 26.100 54.165	Cat	$\lambda$ $\Delta\lambda$	131 +	16 4	38.932 28.206
$\phi'$	55	07	20.265	Lim	$\lambda'$	131	21	07.138
$s$ $\cos \alpha$ B	4.0770593 9.9625415 8.5097090	$s^2$ $\sin^2 \alpha$ C	8.1541 9.1999 1.5584	$(\partial\phi)^2$ D	5.099 2.366	$-\frac{h}{s^2 \sin^2 \alpha}$ E	2.549 7.354 6.464	
h	2.5493098		8.9124		7.465			6.367
1st term	—354.2499	3d term	+0.0029					
2d term	+ 0.0817	4th term	+0.0002					
	—354.1682							
3d and 4th terms	+ 0.0031	$s$ $\sin \alpha$ A'	4.0770593 9.999381 8.5087351	Arg. $s$ $\Delta\lambda$	—2 +2	$\Delta\lambda$ $\sin \frac{1}{2}(\phi+\phi')$ $\sec \frac{1}{2}(\Delta\phi)$	2.428468 9.913752	
$\frac{1}{2}(\phi+\phi')$	55 04 23.2	$\sec \phi'$	0.2427356	Corr.	0			2.342220
		$\Delta\lambda$	+268.2057			$-\Delta\alpha$		+219.90

## STATION SOUTH TWIN

$\alpha$ Third angle	Lim to Beaver South Twin and Beaver					$^{\circ}$ 299 — 40	$'$ 54 55	$''$ 01.6 15.5
$\alpha$ $\Delta\alpha$	Lim to South Twin					258 +	58 6	46.1 48.1
$\alpha'$	South Twin to Lim					180 79	05	34.2
$\phi$ $\Delta\phi$	$^{\circ}$ 55 +	$'$ 07	$''$ 20.265 55.217	Lim	$\lambda$ $\Delta\lambda$	131 —	21 8	07.138 17.440
$\phi'$	55	08	15.482	South Twin	$\lambda'$	131	12	49.698 — .1
$s$ $\cos \alpha$ B	3.9531874 9.2813974 8.5097018	$s^2$ $\sin^2 \alpha$ C	7.9064 9.9838 1.5597	$(\partial\phi)^2$ D	3.488 2.365	$-\frac{h}{s^2 \sin^2 \alpha}$ E	1.744 7.890 6.467	
h	1.7442866		9.4499		5.853			6.101
1st term	—55.4992	3d term	+0.0001					
2d term	+ 0.2818	4th term	+0.0001					
	—55.2174							
3d and 4th terms	+ 0.0002	$s$ $\sin \alpha$ A'	3.9531874 9.9919164 8.5087347	Arg. $s$ $\Delta\lambda$	—2 +4	$\Delta\lambda$ $\sin \frac{1}{2}(\phi+\phi')$ $\sec \frac{1}{2}(\Delta\phi)$	2.696741 9.914053	
$\frac{1}{2}(\phi+\phi')$	55 07 47.9	$\sec \phi'$	0.2429024	Corr.	+2			2.610794
		$\Delta\lambda$	—497.4404			$-\Delta\alpha$		—408.12



*Latitude equation*

$$\begin{aligned}
0 = & -2.5334 - 0.14(2) + 0.14(3) + 0.39(4) - 0.39(6) - 0.69(7) + 0.69(9) + 0.67(10) - 0.67(11) \\
& - 0.66(12) - 1.36(14) + 1.25(15) - 0.55(16) - 0.09(21) + 0.09(22) - 0.49(23) + 0.93(25) \\
& - 0.44(27) - 0.76(28) + 0.49(31) + 0.27(32) - 0.20(39) + 0.20(40) - 0.02(42) + 0.02(45) \\
& + 0.14(49) - 0.14(52) - 0.10(54) + 0.10(55) + 0.08(59) - 0.08(61) + 0.10(63) - 0.10(65) \\
& + 0.07(71) - 0.07(72) + 0.11(74) - 0.11(75)
\end{aligned}$$

*Longitude equation*

$$\begin{aligned}
0 = & -20.0499 + 1.20(2) - 1.20(3) - 0.59(4) + 0.59(6) + 0.41(7) - 0.41(9) - 0.35(10) + 0.35(11) \\
& + 1.39(12) - 0.34(14) - 0.75(15) - 0.30(16) + 0.67(21) - 0.67(22) - 0.34(23) + 0.07(25) \\
& + 0.27(27) - 0.17(28) - 0.29(31) + 0.46(32) - 0.16(39) + 0.16(40) - 0.62(42) + 0.62(45) \\
& + 0.20(49) - 0.20(52) - 0.07(54) + 0.07(55) - 0.32(59) + 0.32(61) + 0.07(63) - 0.07(65) \\
& - 0.16(71) + 0.16(72) - 0.06(74) + 0.06(75)
\end{aligned}$$

*Correlate*

[illegible]

equations

23	24	25	26	27	28	29	30	$x_{31}$	$l_{32}$	$\phi_{33}$	$\lambda_{34}$	$\Sigma$	
-0.55								-1				-3.55	1
+0.47								+1	-0.14	-0.14	+1.20	+2.39	2
+0.08									+0.14	+0.14	-1.20	+1.16	3
									+0.92	+0.39	-0.59	+0.28	4
												-1.00	5
									-0.92	-0.39	+0.59	+1.28	6
	-4.49								-4.49	-0.69	+0.41	-9.26	7
+0.01	+1.88											+0.88	8
+0.24	+2.61							+1	+4.49	+0.69	-0.41	+8.39	9
								-1	+2.34	+0.67	-0.35	+1.90	10
-0.25									-2.34	-0.67	+0.35	-1.91	11
-0.43									+2.30	+0.66	+1.39	+1.92	12
+1.14												+2.14	13
-0.71	-5.30							-1	-7.60	-1.36	-0.34	-16.31	14
	+8.18								+8.18	+1.25	-0.75	+16.86	15
	-2.88	-0.27							-2.88	-0.55	-0.30	-7.88	16
		+0.74	-0.30									-0.56	17
		-0.47										+0.53	18
			+0.52									+0.52	19
				-4.20								-5.20	20
			-0.22	+8.45				+1	-4.25	-0.09	+0.67	+6.56	21
				-4.25					+4.25	+0.09	-0.67	+1.42	22
		-0.40							-2.47	-0.49	-0.34	-5.70	23
		+1.34										+1.34	24
-0.74		-0.94							+5.37	+0.93	+0.07	+5.69	25
	+3.64											+4.64	26
	-2.90								-2.90	-0.44	+0.27	-5.97	27
			-1.20						-4.25	-0.76	-0.17	-7.38	28
			+1.92									+0.92	29
		-1.41										-2.41	30
		+1.25	-0.72						+3.18	+0.49	-0.29	+4.91	31
		+0.16							+1.07	+0.27	+0.46	+3.96	32
												-1.00	33
												0.00	34
												+1.00	35
												0.00	36
												+1.00	37
												-1.00	38
					+0.59				-2.30	-0.20	-0.16	-3.07	39
					+1.07			+1	+2.30	+0.20	+0.16	+2.73	40
					-1.66			-1	+0.58	-0.02	-0.62	-0.66	41
												-2.23	42
			-0.17									+1.00	43
												+1.65	44
		+0.65							-0.58	+0.02	+0.62	+0.58	45
		-0.48											
												-4.79	46
					-3.79							+3.20	47
					+3.20							+1.59	48
					+0.59							-1.66	49
					-0.01				+0.01	+0.14	+0.20	-6.22	50
												+2.99	51
					+2.99				-0.01	-0.14	-0.20	+0.40	52
					-2.98			-1				-6.11	53
						-5.11			-2.44	-0.10	-0.07	+2.76	54
						+6.37		+1	+2.44	+0.10	+0.07	+7.84	55
						-1.26							
						+1.49							
												-1.00	56
												-1.00	57
												+2.00	58
												-8.74	59
					-3.99	-1.28		-1	+1.77	+0.08	-0.32	+10.16	60
					+9.16								
												-5.70	61
									-1.77	-0.08	+0.32	+3.47	62
												+4.37	63
								+1	-0.43	+0.10	+0.07	+4.68	64
												+0.50	65
									+0.43	-0.10	-0.07		

*Correlate*

[illegible]

equations—Continued

23	24	25	26	27	28	29	30	$\alpha$ 31	$\epsilon$ 32	$\phi$ 33	$\lambda$ 34	$\Sigma$	
												- 1.00	66
												0.00	67
												+ 1.00	68
							-1.61		-1.15			- 3.76	69
							+1.15	+1	+1.15			+ 3.30	70
						-1.24	+0.46	-1	+2.84	+0.07	-0.16	- 0.03	71
						+5.00			-2.84	-0.07	+0.16	+ 3.25	72
						-3.76						- 2.76	73
							+0.15		+1.18	+0.11	-0.06	- 0.62	74
							+3.18	-1	-1.18	-0.11	+0.06	+ 0.95	75
							-3.33	+1				- 0.33	76
									+0.18			- 1.82	77
												+ 1.00	78
									-0.18			+ 0.82	79

## List of corrections

	$v$ 's.*	Adopted $v$ 's.	$v^2$ .		$v$ 's.*	Adopted $v$ 's.	$v^2$ .
1	+0.699	+0.7	0.49	41	-0.789	-0.8	0.64
2	+2.448	+2.4	5.76	42	-1.997	-2.0	4.00
3	-3.146	-3.2	10.24	43	+0.028	+0.0	0.00
4	-1.369	-1.4	1.96	44	+1.554	+1.6	2.56
5	-0.207	-0.2	0.04	45	+0.581	+0.6	0.36
6	+1.576	+1.6	2.56	46	+0.697	+0.7	0.49
7	+0.806	+0.7	0.49	47	-1.478	-1.5	2.25
8	-1.876	-1.9	3.61	48	+0.781	+0.8	0.64
9	+0.498	+0.5	0.25	49	+0.735	+0.8	0.64
10	-0.117	-0.1	0.01	50	+1.145	+1.2	1.44
11	+0.688	+0.7	0.49	51	+0.294	+0.3	0.09
12	+3.097	+3.1	9.61	52	-0.317	-0.3	0.09
13	+1.159	+1.2	1.44	53	-1.522	-1.5	2.25
14	-2.691	-2.7	7.29	54	-0.138	-0.1	0.01
15	-1.472	-1.4	1.96	55	-0.197	-0.2	0.04
16	-0.728	-0.7	0.49	56	+0.741	+0.7	0.49
17	+0.755	+0.8	0.64	57	+0.525	+0.5	0.25
18	-0.168	-0.1	0.01	58	-1.266	-1.3	1.69
19	+0.945	+1.0	1.00	59	-0.592	-0.6	0.36
20	-0.090	-0.1	0.01	60	-0.262	-0.2	0.04
21	+0.102	+0.1	0.01	61	+0.524	+0.6	0.36
22	-0.910	-0.9	0.81	62	+1.193	+1.2	1.44
23	-1.665	-1.6	2.56	63	+0.065	+0.1	0.01
24	+0.614	+0.7	0.49	64	+0.364	+0.4	0.16
25	-1.570	-1.5	2.25	65	-1.294	-1.3	1.69
26	+2.090	+2.1	4.41	66	-0.190	-0.2	0.04
27	+0.530	+0.5	0.25	67	-0.898	-0.9	0.81
28	-0.966	-1.0	1.00	68	+1.088	+1.1	1.21
29	+0.183	+0.2	0.04	69	-0.748	-0.8	0.64
30	-1.164	-1.2	1.44	70	-0.490	-0.5	0.25
31	+0.311	+0.3	0.09	71	+0.134	+0.1	0.01
32	+1.636	+1.6	2.56	72	+1.234	+1.2	1.44
33	-0.457	-0.4	0.16	73	-0.130	-0.2	0.04
34	+1.167	+1.2	1.44	74	+1.360	+1.3	1.69
35	-0.710	-0.7	0.49	75	-0.203	-0.2	0.04
36	-0.494	-0.5	0.25	76	-1.158	-1.2	1.44
37	+1.484	+1.5	2.25	77	+0.445	+0.4	0.16
38	-0.990	-1.0	1.00	78	-1.482	-1.5	2.25
39	-0.319	-0.3	0.09	79	+1.037	+1.0	1.00
40	+0.941	+0.9	0.81				
				Total.....			103.76

\* These values result from the computation on p. 69.

## Normal equations

	1	2	3	4	5	6	7	8	9	10	11	23	24	25	26	27	31	32	33	34	$\eta$	$\Sigma$	$C's^*$
1.....	+6	+2	-2									+0.53					+3	6.66	-2.26	+3.08	-5.5	-1.81	+0.0070
2.....		+6	+2									+2.20					+1	-3.08	-0.91	-2.00	+4.0	+11.21	+0.2070
3.....			+6	-2								-0.44	-7.91				-4	-11.77	-1.76	-0.07	+8.0	-13.95	-0.7922
4.....				+6	-2							+0.72	+2.14	+0.67			+2	+7.53	+0.57	-0.44	-8.0	+6.89	+1.9492
5.....					+6	+2	+2	-2	-2					-0.62	+0.42			+8.61	+1.75	+1.46	-2.9	+14.86	+1.1901
6.....						+6	-2						+2.14	-2.48				+8.25	+1.48	+0.37	-0.0	+13.76	+0.4571
7.....							+6	+6	+2	-2				+3.31				+3.54	+0.76	+0.80	-3.2	+11.21	+0.7099
8.....									+6	+2	-2			+0.51	-1.82			+3.18	+0.49	-0.29	-2.3	+3.77	+1.4844
9.....										+6	+2	-2		+0.51	+0.25	+8.45	+2	+2.02	+1.20	+1.79	-3.2	+19.02	+0.3254
10.....											-2	-2			+0.08	+8.45	+2	-4.83	-0.07	+1.29	-3.2	+7.72	+0.9897

	11	12	13	14	15	16	17	18	19	20	21	22	26	27	28	29	30	31	32	33	34	$\eta$	$\Sigma$	$C's$
11.....	+6	+2	+2										+0.05	-15.67	+1.66			-4	+6.76	-0.32	-2.52	+5.0	-3.04	-1.2334
12.....		+6	+2	-2										+0.34	+1.66			-2	+4.24	-0.05	-0.87	-1.0	+8.66	+0.8642
13.....			+6	-2	-2									+1.02	+4.22			-2	-2.31	-0.34	-0.36	+3.7	+7.37	+0.0285
14.....				+6	+6	+2	+2	+2						-2.98	-1.94	+0.02		+4	-2.31	-0.34	-0.36	+5.2	+11.43	-0.7413
15.....						+6	+2	+2	+2					+2.98					+3.51	+0.48	+1.23	-2.5	+13.78	-0.0027
16.....							+6	+2							+13.86	+0.02		+2	+0.67	+0.02	+0.39	+2.8	+38.76	-0.5247
17.....								-2	-2						+8.60	+8.60		+2	-0.67	+0.02	+0.39	-5.1	+22.06	+1.0884
18.....								+6	+6	-2					+5.48	-2.30	-4.38	+4	-3.00	+0.08	+0.85	-1.7	+11.03	+0.5038
19.....								+6	-2	-2						-8.18	-4.38	+2	+3.27	+0.03	+0.23	+2.1	-7.47	+0.1899
20.....									+6	+6	+2	-2				+0.95	+4.50	-4	+0.19	-0.35	-0.18	+2.3	+5.41	-0.0433

	21	22	23	24	25	26	27	28	29	30	31	32	33	34	$\eta$	$\Sigma$	$C$ 's
21	+6	+2								-10.92	+1	-	-0.93	-	+6.1	+5.03	-1.4817
22		+6								-3.75	+3	+	3.12	-	+0.1	+8.52	+1.0414
23											+1.50	+	5.5439	-	+2.22	+15.1601	+0.83524
24											+7.91	+	+151.8020	-	-5.7	+336.4739	+0.03872
25												+	0.8640	-	-6.09	+4.2703	+1.32912
26											-0.05	+	3.9252	+	2.60	+4.6556	+0.86320
27											+11.43	-	53.9453	+	0.0	+108.3339	-0.19135
28											+3.82	+	6.8009	-	+6.6	+205.2346	+0.02875
29											-0.93	+	37.6627	-	-18.4	+79.9881	+0.08500
30											-9.74	+	1.8338	-	-8.2	+48.4945	-0.19677
31											+16	+	6.82	+	7.3	+46.82	-1.37166
32												+	49.3161	-	-5.2	+478.9301	+0.02103
33												+	8.7558	-	-2.5334	+79.9111	-0.27282
34												+	351.4744	+	-20.0499	+18.6051	+2.16065

\* These values result from the computation on p. 68.



*normals*

25	31	32	33	34	$\eta$	$\Sigma$
	+3 -0.5	- 6.66 + 1.11	- 2.26 + 0.37667	+3.08 -0.51333	- 5.5 + 0.91667	- 1.81 + 0.30167
	+1 -1	- 3.08 + 2.22  - 0.86 + 0.16125	- 0.91 + 0.7533  - 0.1567 + 0.02938	-2.00 -1.0267  -3.0267 +0.56751	+ 4.0 + 1.8333  + 5.8333 - 1.09375	+ 11.21 + 0.6033  + 11.8132 - 2.21499
	-4 +1  -3 +0.75	- 11.77 + 2.22 + 0.43  - 13.56 + 3.39	- 1.76 - 0.7533 + 0.0783  - 2.435 + 0.60875	-4.07 +1.0267 +1.5133  -1.53 +0.3825	+12.0 - 1.8333 - 2.9167  + 7.25 - 1.8125	- 13.95 - 0.6033 - 5.9067  - 20.4600 + 5.115
	+1.50 -0.2650  -0.9562  +0.2788 -0.19664	+ 5.5439 + 0.5883 + 0.3263 - 4.3222  + 2.1363 - 1.50677	+ 0.9624 + 0.1996 + 0.0594 - 0.7762  + 0.4452 - 0.31401	-0.0639 -0.2721 +1.1482 -0.4877  +0.3245 -0.22888	- 2.22 + 0.4858 - 2.2130 + 2.3109  - 1.6363 + 1.15411	+ 15.1601 + 0.1599 - 4.4816 - 6.5216  + 4.3166 - 3.04458
+0.67   +0.67 -0.13413	+2 -1.5 -0.0162  +0.4838 -0.09685	+ 3.84 - 6.78 - 0.1243  - 3.0643 + 0.61345	+ 0.57 - 1.2175 - 0.0259  - 0.6734 + 0.13481	-0.44 -0.7650 -0.0189  -1.2239 +0.24502	- 8.0 + 3.6250 + 0.0952  - 4.2798 + 0.85678	+ 6.89 - 10.23 - 0.2512  - 3.5912 + 0.71893
-2.48 +0.2683  -2.2117 +0.42539	+0.1937  +0.1937 -0.03726	+ 8.25 - 1.2269  + 7.0231 - 1.35080	+ 1.48 - 0.2697  + 1.2103 - 0.23279	+0.37 -0.4900  -0.12 +0.02308	- 0.0 - 1.7135  - 1.7135 + 0.32957	+ 13.76 - 1.4378  + 12.3221 - 2.37000
+1.4732  -0.4606 +1.5067  +2.5103 -0.0186822	+7.91 -5.9325 -0.2493 -0.3391 -0.1320  +1.2571 -0.0093556	+151.8020 - 26.8149 - 1.9103 + 2.1478 - 4.7843  +120.4403 - 0.8963453	+24.5038 - 4.8152 - 0.3981 + 0.4720 - 0.8245  +18.9380 - 0.1409411	-7.2148 -3.0256 -0.2902 +0.8578 +0.0817  -9.5911 +0.0713792	- 5.7 +14.3369 + 1.4632 + 2.9998 + 1.1673  +14.2672 - 0.1061799	+336.4739 - 40.4597 - 3.8599 + 2.5171 - 8.3941  +286.2773 - 2.1305435
+3.31 -0.8508 -0.0255  +2.4337 -0.46650	+0.0745 -0.0127  +0.0618 -0.01185	+ 3.54 + 2.7016 - 1.2212  + 5.0204 - 0.95233	+ 0.76 + 0.4656 - 0.1920  + 1.0336 - 0.19813	+0.80 -0.0462 +0.0972  +0.8510 -0.16312	- 3.2 - 0.6591 - 0.1447  - 4.0038 + 0.76747	+ 11.21 + 4.7400 - 2.9027  + 13.0473 - 2.50097

*Solution of*

	5	25	8	9	26	10	12	27
4	+6	-0.62	-2	-2	+0.42			
6	-0.8008	+0.2683						
24	-0.2766	+0.5101						
7	-0.0553	-0.0509						
	-1.1353	-1.1353						
	+3.7320	-1.0278	-2	-2	+0.42			
	$C_5$	+0.27540	+0.53591	+0.53591	+0.11254			
		+7.2568	+0.51	+0.51	-1.1220			
4	-0.0899							
6	-0.9408							
24	-0.0469							
7	-1.1353							
5	-0.2831	-0.5508	-0.5508	+0.1157				
	+4.7608	-0.0408	-0.0408	-1.0063				
	$C_{25}$	+0.00857	+0.00857	+0.21137				
		+6	+2	-1.82	-2			
	5	-1.0718	-1.0718	+0.2251				
	25	-0.0003	-0.0003	-0.0086				
		+4.9279	+0.9279	-1.6035	-2			
		$C_8$	-0.18830	+0.32539	+0.40585			
			+6	+0.25	+2			+8.45
		5	-1.0718	+0.2251				
		25	-0.0003	-0.0086				
		8	-0.1747	+0.3019	+0.3766			
			+4.7532	+0.7684	+2.3766			+8.45
			$C_9$	-0.16166	-0.5			-1.77775
				+6.7354	+0.08			-1.8590
			5	-0.0473				
			25	-0.2127				
			8	-0.5218	-0.6508			-1.3660
			9	-0.1242	-0.3842			
				+5.8294	-0.9550			-3.2250
				$C_{26}$	+0.16382			+0.55323
					+6			+8.45
				8	-0.8117			
				9	-1.1883			-4.2250
				26	-0.1564			-0.5283
					+3.8436			+3.6967
					$C_{10}$			-0.96178
						+6		+0.34
						$C_{12}$		-0.05667

normals—Continued

11	13	31	32	33	34	$\eta$	$\Sigma$
		+0.1937 -0.0447 -0.0255 -0.0288  +0.0947 -0.02538	+8.61 -1.2269 -1.6199 -2.4425 -2.3420  +0.9787 -0.26225	+1.75 -0.2697 -0.2792 -0.3841 -0.4822  +0.3348 -0.08971	+1.46 -0.4900 +0.0277 +0.1945 -0.3970  +0.7952 -0.21308	-2.9 -1.7135 +0.3952 -0.2893 +1.8678  -2.6398 +0.70734	+14.86 -1.4378 -2.8421 -5.8055 -6.0866  -1.3122 +0.35161
		-0.0649 +0.0824 -0.0235 -0.0288 +0.0261  -0.0087 +0.00183	+0.8640 +0.4110 +2.9876 -2.2501 -2.3420 +0.2695  -0.0600 +0.01260	+0.1260 +0.0903 +0.5148 -0.3538 -0.4822 +0.0922  -0.0127 +0.00267	-0.1377 +0.1642 +0.0510 +0.1792 -0.3970 +0.2190  -0.0233 -0.00489	-6.09 +0.5740 -0.7289 -0.2665 +1.8678 -0.7270  -5.3706 +1.12809	+4.2703 +0.4817 +5.2417 -5.3483 -6.0866 -0.3614  -1.8024 +0.37859
		+0.0508 -0.0001  +0.0507 -0.01029	+3.18 +0.5245 -0.0005  +3.7040 -0.75164	+0.49 +0.1794 -0.0001  +0.6693 -0.13582	-0.29 +0.4262 -0.0002  +0.1360 -0.02760	-2.3 -1.4147 -0.0460  -3.7607 +0.76314	+3.77 -0.7032 -0.0154  +3.0516 -0.351925
-2		+2 +0.0508 -0.0001 -0.0095	+2.02 +0.5245 -0.0005 -0.6975	+1.20 +0.1794 -0.0001 -0.1260	+1.79 +0.4262 -0.0002 -0.0256	-3.2 -1.4147 -0.0460 +0.7081	+19.02 -0.7032 -0.0154 -0.5746
-2 +0.42077		+2.0412 -0.42944	+1.8465 -0.38848	+1.2533 -0.263675	+2.1904 -0.46083	-3.9526 +0.83157	+17.7270 -3.72949
+0.05  +0.3233  +0.3733 -0.06404		-0.05 -0.0107 -0.0018 +0.0165 -0.3300  -0.3760 +0.06450	+3.9252 -0.1101 -0.0127 +1.2052 -0.2985  +4.7091 -0.80782	+0.5728 -0.0377 -0.0027 +0.2178 -0.2026  +0.5476 -0.03394	+0.0732 -0.0895 -0.0049 +0.0443 -0.3541  -0.3310 +0.05678	-2.60 +0.2971 -1.1352 -1.2237 +0.6390  -4.0228 +0.69009	+4.6556 +0.1477 -0.3810 +0.9930 -2.8657  +2.5496 -0.43737
-2 +1 +0.0612  -0.9388 +0.24425		+2 +0.0206 -1.0206 -0.0616  +0.9384 -0.24415	-4.83 +1.5033 -0.9233 +0.7714  -3.4785 +0.90504	-0.07 +0.2716 -0.6266 +0.0897  -0.3353 +0.08724	+1.29 +0.0552 -1.0952 -0.0542  +0.1956 -0.05089	-3.2 -1.5263 +1.9763 -0.6590  -3.4090 +0.88693	+7.72 +1.2385 -8.8635 +0.4177  +0.5126 -0.13336
+2 -0.33333	-2 +0.33333		+4.24 -0.70667	-0.05 -0.00833	-0.87 +0.145	-1.0 +0.16667	+8.66 -1.44333

	27	11	13	14	15	28	16	17
9	+149.8778	-15.67	+1.02	-2.98	+2.98	- 8.1354		
26	- 15.0220	+ 3.5555						
10	- 1.7842	+ 0.2065						
12	- 3.5554	+ 0.9029						
	- 0.0193	- 0.1133	+0.1133					
	+129.4969	-11.1184	+1.1333	-2.98	+2.98	- 8.1354		
	$C_{27}$	+ 0.0858584	-0.0087516	+0.0230121	-0.0230121	+ 0.0628231		
		+ 6	+2	+2	-2	+ 1.66		
9		- 0.8415						
26		- 0.0239						
10		- 0.2293						
12		- 0.6667	+0.6667					
27		- 0.9546	+0.0973	-0.2559	+0.2559	- 0.6985		
		+ 3.2840	+2.7640	+1.7441	-1.7441	+ 0.9615		
		$C_{11}$	-0.84166	-0.53109	+0.53109	- 0.29278		
			+6	+2	-2	+ 1.66		
12		-0.6667						
27		-0.0099	+0.0261	-0.0261	+ 0.0712	+ 0.0712		
11		-2.3263	-1.4679	+1.4679	- 0.8093	- 0.8093		
		+2.9971	+0.5582	-0.5582	+ 0.9219	+ 0.9219		
		$C_{13}$	-0.18625	+0.18625	- 0.30760	- 0.30760		
			+6	-2	+ 4.22	+ 2		
27		-0.0686	+0.0686	+0.0686	- 0.1872	- 0.1872		
11		-0.9263	+0.9263	+0.9263	- 0.5106	- 0.5106		
13		-0.1040	+0.1040	+0.1040	- 0.1717	- 0.1717		
		+4.9011	-0.9011	-0.9011	+ 3.3505	+ 3.3505	+ 2	
		$C_{14}$	+0.18386	+0.18386	- 0.68362	- 0.68362	- 0.40807	
			+6	-2	- 1.94	+ 2	+ 2	+2
27		-0.0686	+0.0686	+0.0686	+ 0.1872	+ 0.1872		
11		-0.9263	+0.9263	+0.9263	- 0.5106	- 0.5106		
13		-0.1040	+0.1040	+0.1040	+ 0.1717	+ 0.1717	+ 0.3677	
14		-0.1657	+0.1657	+0.1657	+ 0.6160	+ 0.6160	+ 2.3677	+2
		+4.7354	-0.4545	-0.4545	- 0.09598	- 0.09598	- 0.5	-0.42235
		$C_{15}$	+0.09598	+0.09598				
			+158.2846	+18.86	+5.48			
27		- 0.5111	- 0.5111	- 0.5111	- 0.5111	- 0.5111		
11		- 0.2815	- 0.2815	- 0.2815	- 0.2815	- 0.2815		
13		- 0.2836	- 0.2836	- 0.2836	- 0.2836	- 0.2836		
14		- 2.2905	- 2.2905	- 2.2905	- 2.2905	- 2.2905	- 1.3672	+0.1920
15		- 0.0436	- 0.0436	- 0.0436	- 0.0436	- 0.0436	+0.2273	
		+154.8743	+17.7201	+5.6720				
		$C_{28}$	- 0.114416	- 0.036623				
			+ 6	+2				
14		- 0.8161	- 0.8161	- 0.8161	- 0.8161	- 0.8161		
15		- 1.1839	- 1.1839	- 1.1839	- 1.1839	- 1.1839	-1	
28		- 2.0275	- 2.0275	- 2.0275	- 2.0275	- 2.0275	-0.6490	
		+ 1.9725	+0.3510	+0.3510				
		$C_{16}$	-0.17795	-0.17795				

normals—Continued

18	29	31	32	33	34	$\eta$	$\Sigma$
		+ 11.43 - 3.6287 - 0.2080 - 0.9025	-53.9453 - 3.2826 + 2.6052 + 3.3456 - 0.2403	-0.7272 -2.2281 +0.3029 +0.3225 +0.0028	+9.1030 -3.8940 -0.1831 -0.1881 +0.0493	+0.0 +7.0267 -2.2255 +3.2787 +0.0567	+108.3339 - 31.5142 + 1.4105 - 0.4930 - 0.4908
		+ 6.6908 - 0.0516676	-51.5174 + 0.3978273	-2.3271 +0.0178703	+4.8871 -0.0377391	+8.1366 -0.0628324	+ 77.2464 - 0.5965116
		- 4 + 0.8589 + 0.0241 + 0.2292  + 0.5745  - 2.3133 - 0.70442	+ 6.76 + 0.7770 - 0.3016 - 0.8496 - 1.4133 - 4.4232  + 0.5493 + 0.16727	-0.32 +0.5274 -0.0351 -0.0819 +0.0167 -0.1998  -0.0927 +0.02823	-2.52 +0.9217 +0.0212 +0.0478 +0.2900 +0.4196  -0.8197 +0.24960	+5.0 -1.6631 +0.2576 -0.8326 +0.3333 +0.6986  +3.7938 -1.15524	- 3.04 + 7.4590 - 0.1633 + 0.1252 - 2.8866 + 6.6323  + 8.1269 - 2.47470
		- 2  - 0.0586 + 1.9470  - 0.1116 - 0.03724	- 2.31 + 1.4133 + 0.4509 - 0.4623  - 0.9081 + 0.30299	-0.34 -0.0167 +0.0204 +0.0780  -0.2583 +0.08618	-0.36 -0.2900 -0.0428 +0.6899  -0.0029 +0.00097	+3.7 -0.3333 +0.0712 -3.1931  +0.1024 -0.03417	+ 7.37 + 2.8966 - 0.6760 - 6.8401  + 2.7405 + 0.91438
		- 2 + 0.1540 + 1.2286 + 0.0208  - 0.5966 + 0.12173	- 2.31 - 1.1855 - 0.2917 + 0.1691  - 3.6181 + 0.73822	-0.34 -0.0536 +0.0492 +0.0481  -0.2963 +0.06046	-0.36 +0.1125 +0.4353 +0.0005  +0.1883 -0.03842	+5.2 +0.1872 -2.0148 -0.0191  +3.3533 -0.68419	+ 11.43 + 1.7776 - 4.3161 - 0.5204  + 8.3811 - 1.71004
+2	+0.02	+ 4 - 0.1540 - 1.2286 - 0.0208 - 0.1097	+ 3.51 + 1.1855 + 0.2917 - 0.1691 - 0.6652	+0.48 +0.0536 -0.0492 -0.0481 -0.0545	+1.23 -0.1125 -0.4353 -0.0005 +0.0346	-2.5 -0.1872 +2.0148 +0.0191 +0.6165	+ 13.78 - 1.7776 + 4.3161 + 0.5104 + 1.5409
+2 -0.42235	+0.02 -0.00422	+ 2.4869 - 0.52517	+ 4.1529 - 0.87699	+0.3818 -0.08063	+0.7163 -0.151265	-0.0368 +0.00777	+ 18.3697 - 3.87923
+5.48	+3.2298	+ 3.82 + 0.4203 + 0.6773 + 0.0343 + 0.4078 + 0.2387	+ 6.8009 - 3.2365 - 0.1608 + 0.2793 + 2.4734 + 0.3986	-0.0428 -0.1462 +0.0271 +0.0795 +0.2026 +0.0366	-0.7425 +0.3070 +0.2400 +0.0009 -0.1287 +0.0688	+6.6 +0.5112 -1.1107 -0.0315 -2.2924 -0.0035	+205.2346 + 4.8529 - 2.3794 - 0.8430 - 5.7295 + 1.7631
+0.1920	+0.0019	+ 0.4078 + 0.2387	+ 2.4734 + 0.3986	+0.2026 +0.0366	-0.1287 +0.0688	-2.2924 -0.0035	- 5.7295 + 1.7631
+5.6720 -0.036623	+3.2317 -0.020867	+ 5.5984 - 0.036148	+ 6.5549 - 0.042324	+0.1568 -0.001012	-0.2545 +0.001643	+3.6731 -0.023717	+202.8988 - 1.310087
+2 -1 -0.6490	+0.02 -0.01 -0.3698	+ 2 + 0.2435 - 1.2435 - 0.6405	+ 0.67 + 1.4764 - 2.0764 - 0.7500	+0.02 +0.1209 -0.1909 -0.0179	+0.39 -0.0768 -0.3582 +0.0291	+2.8 -1.3684 +0.0184 -0.4203	+ 38.76 - 3.4201 - 9.1849 - 23.2149
+0.3510 -0.17795	-0.3598 +0.18241	+ 0.3595 - 0.13226	- 0.6800 + 0.34474	-0.0679 +0.03442	-0.0159 +0.00806	+1.0297 -0.52203	+ 2.9401 - 1.49054

Solution of

	17	18	29	19	20	21	22
15	+6	+2	+ 8.60	-2			
28	-0.8447	-0.8447	- 0.0084				
16	-0.2077	-0.2077	- 0.1184				
	-0.0625	-0.0625	+ 0.0640				
	+4.8851	+0.8851	+ 8.5372	-2			
	$C_{17}$	-0.18118	- 1.74760	+0.40941			
		+6	- 2.30	+2	-2		
15	-0.8447	- 0.0084					
28	-0.2077	- 0.1184					
16	-0.0625	+ 0.0640					
17	-0.1604	- 1.5468	+0.3624				
	+4.7247	- 3.9096	+2.3624	-2			
	$C_{18}$	+ 0.82748	-0.50001	+0.42331			
		+127.4272	-8.18	+0.95			
15	- 0.0001						
28	- 0.0674						
16	- 0.0656						
17	-14.9196	+3.4952					
18	- 3.2351	+1.9548			-1.6550		
	+109.1394	-2.73			-0.7050		
	$C_{20}$	+0.025014			+0.006460		
		+6	-2				
17	-0.8188						
18	-1.1812	+1					
29	-0.0683	-0.0176					
	+3.9317	-1.0176					
	$C_{19}$	+0.25882					
		+6					
18	-0.8466	+2					
29	-0.0046						
19	-0.2634						
	+4.8854	-2					
	$C_{20}$	-0.40938					
		+6					
20	-0.8188	+2					
	+5.1812	+2.8188					
	$C_{21}$	-0.54404					
		+6					
20	-0.8188						
21	-1.5335						
	+3.6477						
	$C_{22}$						

normals—Continued

30	31	32	33	34	$\eta$	$\Sigma$
	+2 -1.0503 -0.2050 -0.0640  +0.6807 -0.13934	+ 0.67 - 1.7540 - 0.2401 + 0.1210  - 1.2031 + 0.24628	+0.02 -0.1613 -0.0057 +0.0121  -0.1349 +0.02761	+0.39 -0.3025 +0.0093 +0.0028  +0.0996 -0.02039	- 5.1 + 0.0155 - 0.1345 - 0.1832  - 5.4022 + 1.10585	+22.06 - 7.7584 - 7.4308 - 0.5232  + 6.3475 - 1.29936
- 4.38	+4 -1.0503 -0.2050 -0.0640 -0.1233	- 3.00 - 1.7540 - 0.2401 + 0.1210 + 0.2180	+0.08 -0.1613 -0.0057 +0.0121 +0.0244	+0.85 -0.3025 +0.0093 +0.0028 -0.0180	- 1.7 + 0.0155 - 0.1345 - 0.1832 + 0.9788	+11.03 - 7.7584 - 7.4308 - 0.5232 - 1.1500
- 4.38 + 0.92704	+2.5574 -0.54128	- 4.6551 + 0.98527	-0.0505 +0.01069	+0.5416 +0.01463	- 1.0234 + 0.21661	- 5.8325 + 1.23447
+ 8.0144	-0.93 -0.0105 -0.1168 +0.0656 -1.1896 +2.1162	-37.6627 - 0.0175 - 0.1368 - 0.1240 + 2.1025 - 3.8520	-1.5212 -0.0016 -0.0033 -0.0124 +0.2358 -0.0418	+0.7206 -0.0030 +0.0053 -0.0029 -0.1741 +0.4482	-18.4 + 0.0002 - 0.0766 + 0.1878 + 9.4409 - 0.8468	+79.9881 - 0.0776 - 4.2339 + 0.5363 -11.0929 + 4.8263
- 3.6244						
+ 4.3900 - 0.040224	-0.0651 +0.000596	-39.6905 + 0.363668	-1.3445 +0.012319	+0.9941 -0.009109	- 9.6945 + 0.088827	+60.2939 - 0.552449
- 4.38 + 2.19 + 0.1098	+2 +0.2787 -1.2787 -0.0016	- 3.27 - 0.4926 + 2.3276 - 0.9928	+0.03 -0.0552 +0.0252 -0.0336	+0.23 +0.0408 -0.2708 +0.0249	+ 2.1 - 2.2117 + 0.5117 - 0.2425	- 7.47 + 2.5987 + 2.9163 + 1.5082
- 2.0802 + 0.52908	+0.9984 -0.25394	- 2.4278 + 0.61749	-0.0336 +0.00855	+0.0249 -0.00633	+ 0.1575 - 0.04006	- 0.4467 + 0.11361
+ 4.50 - 1.8541 + 0.0284 - 0.5384	-4 +1.0826 -0.0004 +0.2584	+ 0.19 - 1.9706 - 0.2564 - 0.6284	-0.35 -0.0214 -0.0087 -0.0087	-0.18 +0.2293 +0.0064 +0.0064	+ 2.3 - 0.4332 - 0.0626 + 0.0408	+ 5.41 - 2.4690 + 0.3895 - 0.1157
+ 2.1359 - 0.43720	-2.6594 +0.54436	- 2.6654 + 0.54558	-0.3888 +0.07958	+0.0621 -0.01271	+ 1.8450 - 0.37766	+ 3.2148 - 0.65804
-10.92 - 0.8744	+1 +1.0887	- 0.93 + 1.0912	-0.21 +0.1592	-0.01 -0.0254	+ 6.1 - 0.7553	+ 5.03 - 1.3161
-11.7944 + 2.27638	+2.0887 -0.40313	+ 0.1612 - 0.03111	-0.0508 +0.00980	-0.0354 +0.00683	+ 5.3447 - 1.03156	+ 3.7140 - 0.71682
- 3.75 + 0.8744 + 6.4166	+3 -1.0887 -1.1363	+ 3.12 - 1.0912 - 0.0577	+0.11 -0.1592 +0.0276	-0.06 +0.0254 +0.0193	+ 0.1 + 0.7553 - 2.9077	+ 8.52 + 1.3161 - 2.0206
+ 3.5410 - 0.97075	+0.7750 -0.21246	+ 1.9411 - 0.53214	-0.0216 +0.00592	-0.0153 +0.00419	- 2.0524 + 0.56266	+ 7.8155 - 2.14258

## Solution of normals—Continued

	30	31	32	33	34	$\eta$	$\Sigma$
18	+76.0764	- 9.74	+ 1.8338	- 0.5171	- 0.0430	- 8.2	+ 48.4945
29	- 4.0604	+ 2.3708	+ 4.3155	- 0.0468	+ 0.5021	+ 0.9487	- 5.4070
19	- 0.1766	+ 0.0026	+ 1.5965	+ 0.0541	- 0.0400	+ 0.3900	- 2.4253
20	- 1.1006	+ 0.5282	+ 1.2845	- 0.0178	+ 0.0132	+ 0.0833	- 0.2363
21	- 0.9338	+ 1.1627	+ 1.1653	+ 0.1700	- 0.0272	- 0.8066	- 1.4055
22	-26.8485	+ 4.7547	+ 0.3670	- 0.1156	- 0.0806	+12.1666	+ 8.4545
	- 3.4374	- 0.7523	- 1.8843	+ 0.0210	+ 0.0149	+ 1.9924	- 7.5869
	+39.5191	- 1.6733	- 2.5217	- 0.4522	+ 0.3394	+ 4.6770	+ 39.8883
	$C_{30}$	+ 0.042342	+ 0.063810	+ 0.011443	- 0.008588	- 0.118348	+ 1.009342
		+16	+ 6.82	+ 1.67	+ 3.69	- 7.3	+ 46.82
1	- 1.5	+ 3.33	+ 1.13	- 1.54	+ 2.75	+ 0.9050	- 15.3450
3	- 2.25	- 10.17	- 1.8262	- 1.1475	+ 5.4375	+ 0.8488	- 0.3478
23	- 0.0548	- 0.4201	- 0.0875	- 0.0638	+ 0.3218	+ 0.4591	- 2.6783
4	- 0.0469	+ 0.2968	+ 0.0652	+ 0.1185	+ 0.4145	+ 0.1546	- 0.0333
6	- 0.0072	- 0.2617	- 0.0451	+ 0.0945	+ 0.0638	+ 0.0098	- 0.0314
24	- 0.0118	- 1.1268	- 0.1772	+ 0.0897	+ 0.1335	+ 1.6974	- 7.6127
7	- 0.0007	- 0.0595	- 0.0122	- 0.0101	+ 0.0474	+ 0.1044	- 0.1252
5	- 0.0024	- 0.0248	- 0.0085	- 0.0202	+ 0.0070	+ 0.0033	- 0.0033
25		- 0.0001			- 0.0087	+ 0.0314	- 7.6127
8	- 0.0005	- 0.0381	- 0.0069	- 0.0014	+ 0.0387	+ 0.1044	- 0.1021
9	- 0.8766	- 0.7930	- 0.5382	- 0.9406	+ 1.6974	+ 0.1044	- 0.1252
26	- 0.0243	+ 0.3037	+ 0.0353	- 0.0213	+ 0.2595	+ 0.0033	- 0.0033
10	- 0.2291	- 0.8493	+ 0.0819	- 0.0478	+ 0.8323	+ 0.0033	- 0.0033
27	- 0.3457	+ 2.6618	- 0.1202	- 0.2525	+ 0.4204	+ 3.9911	+ 5.7248
11	- 1.6295	+ 0.3869	- 0.0653	- 0.5774	+ 2.6724	+ 0.1021	- 1.0202
13	- 0.0042	- 0.0338	- 0.0096	- 0.0001	+ 0.0038	+ 0.0033	- 0.0033
14	- 0.0726	- 0.4404	- 0.0361	+ 0.0229	+ 0.4082	+ 1.0202	- 9.6472
15	- 1.3060	- 2.1810	- 0.2905	- 0.3762	+ 0.0193	+ 7.3344	- 0.5-59
28	- 0.2024	- 0.2369	- 0.0057	+ 0.0092	- 0.1328	- 0.8845	- 3.1570
16	- 0.0655	+ 0.1239	+ 0.0124	+ 0.0029	+ 0.1577	+ 0.0359	- 0.1134
17	- 0.0948	+ 0.1676	+ 0.0188	+ 0.0139	+ 0.7527	+ 1.7500	- 1.4972
18	- 1.3843	+ 2.5197	+ 0.0273	- 0.2932	+ 0.5539	+ 1.6005	+ 1.6890
29		- 0.0237	- 0.0008	+ 0.0006	+ 0.0058	+ 0.0033	- 0.0033
19	- 0.2535	+ 0.6105	+ 0.0985	- 0.0063	- 0.0400	+ 0.1134	- 1.7500
20	- 1.4477	- 1.4509	+ 0.2116	+ 0.0338	+ 1.0043	+ 1.4972	- 1.6005
21	- 0.8420	- 0.0650	+ 0.0205	+ 0.0143	+ 2.1546	+ 1.6890	- 9.0541
22	- 0.1647	- 0.4124	+ 0.0046	+ 0.0033	+ 0.4361	+ 2.27353	+ 2.90951
30	- 0.0769	- 0.1068	- 0.0191	+ 0.0144	+ 0.1980		
	+ 3.1119	+ 0.2312	- 0.0558	- 1.3082	+ 7.0750	+ 9.0541	
	$C_{31}$	+ 0.07430	+ 0.01793	+ 0.42039	+ 2.27353	+ 2.90951	
		+351.4744	+49.3161	-13.0822	- 5.2	+478.9301	
1	- 7.3926	- 2.5086	+ 3.4188	+ 6.1050	- 2.0091	+ 1.9049	
2	- 0.1387	- 0.0253	- 0.4881	+ 0.9406	+ 1.9049	- 69.3594	
3	-45.9684	- 8.2546	- 5.1867	+24.5775	+ 6.5041	- 2.0300	
23	- 3.2189	- 0.6708	- 0.4889	+ 2.4655	- 16.6447	-256.6033	
4	- 1.3798	- 0.4131	- 0.7508	- 2.6254	- 12.7883	- 12.5558	
6	- 9.4868	- 1.0349	+ 0.1621	+ 2.3146	+ 3.8530	+ 0.3441	
24	-107.9561	-16.9750	+ 8.5969	-12.7883	+ 3.8530	+ 0.2227	
7	- 4.8313	- 0.9947	- 0.8189	+ 3.8530	+ 2.8267	- 2.2937	
5	- 0.2567	- 0.0678	- 0.2085	+ 0.6923	+ 1.5355	- 6.8866	
25	- 0.0008	- 0.0602	- 0.0003	+ 0.6677	+ 3.2497	- 2.0596	
8	- 2.7841	- 0.5331	- 0.1022	+ 2.8267	+ 3.0853	+ 0.4639	
9	- 0.7173	- 0.4869	- 0.8509	+ 1.5355	+ 0.7067	- 6.1198	
26	- 3.8041	- 0.4424	+ 0.2674	+ 3.2497	+ 3.2370	+ 30.7307	
10	- 3.1483	- 0.3035	+ 0.1770	+ 3.0853	+ 0.6346	+ 1.3594	
12	- 2.9963	+ 0.0353	+ 0.6148	+ 0.7067	+ 0.0310	+ 0.8303	
27	-20.4950	- 0.9258	+ 1.9442	+ 3.2370	+ 2.4755	+ 6.1871	
11	- 0.0919	+ 0.0135	+ 0.1371	+ 0.6346	+ 0.9323	- 16.1100	
13	- 0.2751	- 0.0783	- 0.0009	+ 0.0310	+ 0.1555	- 8.5875	
14	- 2.6710	- 0.2187	+ 0.1390	+ 2.4755	+ 3.3505	+ 1.6136	
15	- 3.6421	- 0.3348	- 0.6282	+ 0.9323	+ 1.3305	+ 1.5633	
28	- 0.2774	- 0.0066	+ 0.0108	+ 0.1555	+ 1.0083	+ 5.7406	
16	- 0.2344	- 0.0234	+ 0.0055	+ 3.3505	- 3.5256	+ 21.9270	
17	- 0.2963	- 0.0332	+ 0.0245	+ 1.3305	+ 0.0973	- 0.2758	
18	- 4.5865	- 0.0498	+ 0.5336	+ 1.0083	+ 1.0066	+ 1.7539	
29	-14.4342	- 0.4890	+ 0.3615	- 3.5256	+ 0.1663	- 4.1589	
19	- 1.4991	- 0.0207	+ 0.0154	+ 0.0973	+ 1.0922	+ 2.5453	
20	- 1.4542	- 0.2121	+ 0.0339	+ 1.0066	+ 0.2984	- 0.6727	
21	- 0.0050	+ 0.0016	+ 0.0011	+ 0.1663	+ 0.5257		
22	- 1.0329	+ 0.0115	+ 0.0081	+ 1.0922			
30	- 0.1609	- 0.0289	+ 0.0217	+ 0.2984			
31	- 0.0172	+ 0.0041	+ 0.0972	+ 0.5257			
	+105.7210	+13.6619	- 6.0470	+14.5692	+127.9051		
	$C_{32}$	- 0.129226	+ 0.057198	- 0.137808	- 1.209836		

## Solution of normals—Continued

	33	34	$\eta$	$\Sigma$
	+8.7558	-1.0341	-2.5334	+79.9111
1	-0.8513	+1.1601	-2.0717	-0.6818
2	-0.0046	-0.0889	+0.1714	+0.3471
3	-1.4823	-0.9314	+4.4134	-12.4550
23	-0.1398	-0.1019	+0.5138	-1.3555
4	-0.0908	-0.1650	-0.5770	-0.4841
6	-0.2817	+0.0279	+0.3989	-2.8685
24	-2.6691	+1.3518	-2.0108	-40.3482
7	-0.2048	-0.1686	+0.7933	-2.5851
5	-0.0300	-0.0713	+0.2368	+0.1177
25	-	-0.0001	-0.0143	-0.0048
8	-0.0909	-0.0185	+0.5108	-0.4145
9	-0.3305	-0.3776	+1.0422	-4.6742
26	-0.0514	+0.0311	+0.3779	-0.2395
10	-0.0293	+0.0171	-0.2974	+0.0447
12	-0.0004	-0.0072	-0.0083	+0.0721
27	-0.0416	+0.0873	+0.1454	+1.3804
11	-0.0026	-0.0231	+0.1071	+0.2294
13	-0.0223	-0.0002	+0.0088	+0.2352
14	-0.0179	+0.0114	+0.2027	+0.5067
15	-0.0308	-0.0578	+0.0030	-1.4811
28	-0.0002	+0.0003	-0.0037	-0.2053
16	-0.0023	-0.0005	+0.0354	+0.1012
17	-0.0037	+0.0027	-0.1492	-0.1753
18	-0.0005	+0.0058	-0.0109	-0.0623
29	-0.0166	+0.0122	-0.1194	+0.7428
19	-0.0003	+0.0002	+0.0013	-0.0038
20	-0.0309	+0.0049	+0.1468	+0.2558
21	-0.0005	-0.0003	+0.0524	+0.0364
22	-0.0001	-0.0001	-0.0122	+0.0463
30	-0.0052	+0.0039	+0.0535	+0.4564
31	-0.0011	-0.0261	+0.1410	+0.1804
32	-1.7655	+0.7814	-1.8827	-16.5287
	+0.5568	+0.2254	-0.3351	+0.4471
	$C_{33}$	-0.40481	+0.60183	-0.80293
		+9.4562	-20.0499	-18.6051
1	-1.5811	+2.8233	+0.9291	+0.9291
2	-1.7177	+3.3105	+6.7041	+6.7041
3	-0.5825	+2.7731	-7.8260	-7.8260
23	-0.0743	+0.3745	-0.9880	-0.9880
4	-0.2999	-1.0486	-0.8799	-0.8799
6	-0.0028	-0.0395	+0.2844	+0.2844
24	-0.6846	+1.0184	+20.4342	+20.4342
7	-0.1388	+0.6531	-2.1283	-2.1283
5	-0.1694	+0.5625	+0.2796	+0.2796
25	-0.0001	-0.0263	-0.0088	-0.0088
8	-0.0038	+0.1038	-0.0842	-0.0842
9	-1.0094	+1.8215	-8.1691	-8.1691
26	-0.0188	-0.2284	+0.1448	+0.1448
10	-0.0100	+0.1735	-0.0261	-0.0261
12	-0.1262	-0.1450	+1.2557	+1.2557
27	-0.1844	-0.3071	-2.9152	-2.9152
11	-0.2046	+0.9469	+2.0285	+2.0285
13	-	+0.0001	+0.0027	+0.0027
14	-0.0072	-0.1288	-0.3220	-0.3220
15	-0.1084	+0.0056	-2.7787	-2.7787
28	-0.0004	+0.0060	+0.3334	+0.3334
16	-0.0001	+0.0083	+0.0237	+0.0237
17	-0.0020	+0.1102	-0.1294	-0.1294
18	-0.0621	+0.1173	+0.6686	+0.6686
29	-0.0091	+0.0883	-0.5492	-0.5492
19	-0.0002	-0.0010	+0.0028	+0.0028
20	-0.0008	-0.0234	-0.0409	-0.0409
21	-0.0002	+0.0365	+0.0254	+0.0254
22	-0.0001	-0.0086	+0.0327	+0.0327
30	-0.0029	-0.0402	-0.3426	-0.3426
31	-0.5500	+2.9743	+3.8063	+3.8063
32	-0.3459	+0.8333	+7.3159	+7.3159
33	-0.0912	+0.1357	-0.1810	-0.1810
	+1.4672	-3.1701	-1.7029	-1.7029
	$C_{34}$	+2.10065	+1.10065	+1.10065

### Back solution

34	33	32	31	30	22	21	20
+2.16065	+0.60183 -0.87465	-0.13781 +0.12358 +0.03526	-2.27353 +0.00832 -0.00489 -0.00156	-0.11835 -0.01856 -0.00312 +0.00134 -0.05868	+0.5627 +0.0091 -0.0016 -0.0112 +0.2914 +0.1910	-1.0316 +0.0148 -0.0027 -0.0007 +0.5530 -0.4479 -0.5666	-0.3777 -0.0275 -0.0217 +0.0115 -0.7407 +0.0860 +0.4233 +0.6065
+2.16065	-0.27282	+0.02103	-1.37166	-0.19077	+1.0414	-1.4817	-0.0433
19	29	18	17	16	28	15	14
-0.0401 -0.0137 -0.0023 +0.0130 +0.3483 -0.1041 -0.6112	+0.08883 -0.01968 -0.00336 +0.00765 -0.00082 -0.00791 -0.00028 +0.00475	+0.2166 -0.2477 -0.0029 +0.0207 +0.7425 -0.1824 -0.0183 -0.00650 +0.0703	+1.1058 -0.0441 -0.0075 +0.0052 +0.1911 +0.0777 -0.1485 -0.0913	-0.5220 +0.0174 -0.0094 +0.6072 +0.2500 +0.0155 -0.0897 -0.1987	-0.02372 +0.00355 +0.00028 -0.00089 +0.04958 -0.00177 -0.01845 -0.03986 +0.06003	+0.0078 -0.3268 +0.0220 +0.0184 +0.7204 -0.0004 -0.2128 -0.4597 +0.2624 +0.0028	-0.6842 -0.0830 -0.0165 +0.0155 -0.1670 +0.2141 -0.0197 -0.0005
+0.1859	+0.08500	+0.5038	+1.0884	-0.5247	+0.02875	-0.0027	-0.7413
13	11	27	12	10	26	9	8
-0.0342 +0.0021 -0.0235 +0.0064 -0.0511 -0.0688 -0.0005 +0.1381	-1.1552 +0.5393 -0.0077 -0.0035 -0.9662 -0.0084 -0.0014 +0.3337 -0.0240	-0.06283 -0.08154 -0.00488 +0.00837 +0.07087 +0.00181 +0.00066 +0.01706 -0.00025 -0.10590	+0.1667 +0.3133 -0.0023 -0.0149 +0.0095 +0.4111 +0.0108	+0.8869 -0.1100 -0.0238 +0.0190 +0.3349 -0.3013 +0.1840	+0.69009 +0.12268 +0.02563 -0.01699 -0.08847 +0.07899 -0.10586 +0.16213	+0.8316 -0.9957 +0.0719 -0.0082 +0.5890 -0.5190 +0.3402 -0.4948 -0.1404	+0.7631 -0.0596 +0.0371 -0.0158 +0.0141 +0.4017 +0.2825 +0.0613
+0.0285	-1.2334	-0.19135	+0.8942	+0.9897	+0.80820	-0.3254	+1.4844
25	5	7	24	6	4	23	3
+1.12809 +0.01057 -0.00073 +0.00026 -0.00251 +0.18351 -0.00279 +0.01272	+0.7073 -0.4604 +0.0245 -0.0055 +0.0348 -0.0077 -0.1744 +0.7955 +0.3660	+0.7675 -0.3524 +0.0541 -0.0202 +0.0163 -0.6200 -0.5552 -0.7099	-0.10618 +0.15423 +0.03845 -0.01885 +0.01283 -0.02483 -0.02413 +0.00720	+0.3296 +0.0499 +0.0635 -0.0284 +0.0511 +0.5654 -0.2745 -0.2731 -0.0264	+0.8508 +0.5294 -0.0308 +0.0129 +0.1328 -0.1783 +0.4765 -0.0271 +0.1830	+1.15411 -0.49453 +0.08567 -0.03169 +0.26972 -0.03462 -0.11342 +0.83524	-1.8125 +0.8264 -0.1661 +0.0713 -1.0287 +0.0766 +0.9746 +0.2662
+1.32912	+1.1901		+0.03872	+0.4571	+1.9492		-0.7922
2	1						
-1.0938 +1.2262 -0.0080 +0.0034 -0.3169 +0.3961	+0.9167 -1.1091 -0.1028 +0.0233 +0.6858 -0.0738 -0.2641 -0.0690						
+0.2070	+0.0070						

Probable error of an observed direction =  $\pm 0.6745 \sqrt{\frac{103.76}{34}} = \pm 1''.2$

*Computation of corrections*

1	2	3	4	5	6	7	8
-0.007 -0.207 -0.459 +1.372	+0.007 +0.792 +0.393 -1.372 -0.003 +0.038 +2.503	+0.267 -0.792 +0.687 +0.003 -0.038 -2.593	-0.007 +0.019 -0.106 -1.275	-0.207 -0.207 -0.2	+0.007 +0.207 -0.019 +0.106 +1.275	-0.174 -0.094 +0.188 +0.886	-1.949 +0.073 -1.876 -1.9
+0.699 +0.7	+0.038 +2.503 +2.448 +2.4	-3.146 -3.2	-1.369 -1.4		+1.576 +1.6	+0.806 +0.7	
9	10	11	12	13	14	15	16
+0.792 +1.949 +0.008 +0.101 -1.372 +0.094 -0.188 -0.886	-0.007 -0.792 +0.200 +1.372 +0.049 -0.183 -0.756	+0.007 -0.209 -0.049 +0.183 +0.756 +0.688 +0.7	-0.207 +0.792 -0.359 +0.048 -0.180 +3.003	+0.207 +0.952 +1.159 +1.2	-0.792 -1.949 -0.593 -0.205 +1.372 -0.160 +0.371 -0.735	+0.317 +0.172 -0.341 -1.620 -1.472 -1.4	+1.949 -1.190 -0.457 -0.112 -0.359 -0.061 +0.150 -0.648
+0.498 +0.5	-0.117 -0.1		+3.097 +3.1		-2.691 -2.7		-0.728 -0.7
17	18	19	20	21	22	23	24
+1.190 -1.484 +0.325 +0.984 -0.260	+0.457 -0.625 -0.168 -0.1	+1.484 -0.990 +0.451 +0.945 +1.0	-0.894 +0.804 -0.090 -0.1	-0.325 +0.990 +1.233 -0.191 -1.617 -1.372 -0.089 +0.025 +1.448	-1.233 +0.894 +0.813 +0.089 -0.025 -1.448	-1.190 +0.710 -0.532 -0.052 +0.134 -0.735	-0.457 -0.710 +1.781 +0.614 +0.7
+0.755 +0.8				-0.089 +0.025 +1.448 +0.102 +0.1	-0.910 -0.9	-1.665 -1.6	
25	26	27	28	29	30	31	32
-1.949 +1.190 +0.457 -0.029 -1.249 +0.113 -0.254 +0.151	+1.949 +0.141 +2.090 +2.1	-0.112 -0.061 +0.120 +0.583 +0.530 +0.5	+0.325 -1.042 -0.089 +0.207 -0.367 -0.966 -1.0	-1.484 +1.667 +0.183 +0.2	+0.710 -1.874 -1.164 -1.2	-1.190 +1.484 -0.325 +1.661 -0.625 +0.067 -0.134 -0.627	+1.190 -0.710 +0.213 +0.023 -0.074 +0.994 +1.636 +1.6
-1.570 -1.5						+0.311 +0.3	
33	34	35	36	37	38	39	40
-0.457 -0.457 -0.4	+0.457 +0.710 +1.167 +1.2	-0.710 -0.710 -0.7	-1.484 +0.990 -0.494 -0.5	+1.484 +1.484 +1.5	-0.990 -0.990 -1.0	+0.003 +0.017 -0.048 +0.055 -0.346 -0.319 -0.3	+1.233 -0.028 +0.741 -0.003 +0.031 -1.372 +0.048 -0.055 +0.346 +0.941 +0.9

## Computation of corrections—Continued

41	42	43	44	45	46	47	48
-0.741 -0.048	+0.325 -0.990 -1.233	+0.028	+0.990 +0.564	-0.325 -0.417 -0.012	-0.028 +0.725	-0.894 +0.028 -0.612	+0.894 -0.113
-0.789 -0.8	-0.148 +1.372 +0.012 +0.005 -1.340	+0.028 +0.0	+1.554 +1.6	-0.005 +1.340	+0.697 +0.7	-1.478 -1.5	+0.781 +0.8
	-1.997 -2.0			+0.581 +0.6			
49	50	51	52	53	54	55	56
+1.233 -0.894 +0.002 -0.038 +0.432	+0.741 +0.525 -0.121	+0.894 -0.028 -0.572	-1.233 +0.028 -0.741	-1.088 -0.434	-0.504 +0.541 -0.051	-0.003 -0.525 +1.088	+0.741
	+1.145 +1.2	+0.294 +0.3	+0.003 +0.570 +0.078 +1.372 +0.038 -0.432	-1.522 -1.5	+0.027 -0.151	+0.504 +0.043 -0.107 -1.372 +0.051 -0.027 +0.151	+0.741 +0.7
+0.735 +0.8			-0.317 -0.3		-0.138 -0.1	-0.197 -0.2	
57	58	59	60	61	62	63	64
+0.525	-0.741 -0.525	+0.003 +0.525	-0.525 +0.263	-0.003 -0.149	+1.088 -0.190	+0.504 +0.190	+1.482 -1.118
+0.525 +0.5	-1.266 -1.3	-1.088 -0.504 -0.115 -0.109 +1.372 +0.037 -0.022 -0.691	-0.262 -0.2	-0.037 +0.022 +0.691	+0.295	+0.043 -0.186 +0.771 -1.372 -0.009 -0.027 +0.151	+0.364 +0.4
		-0.592 -0.6		+0.524 +0.6	+1.193 +1.2	+0.065 +0.1	
65	66	67	68	69	70	71	72
-0.043 -1.482 +0.346 +0.009 +0.027 -0.151	-0.190 -0.190 -0.2	-1.088 +0.190	+1.088	-1.041 +0.317 -0.024	+0.043 +1.041 -0.226 -1.372 +0.024	-0.504 -0.190 -0.043 -0.105 -0.091 +1.372	+0.504 +0.425 -0.060 +0.019 +0.246
-1.294 -1.3		-0.9	+1.1	-0.748 -0.8	-0.490 -0.5	+0.060 -0.019 -0.246	+1.234 +1.2
						+0.134 +0.1	
73	74	75	76	77	78	79	
+0.190 -0.320	+0.043 +1.482 -0.030	-0.043 -1.041 -0.626	-1.482 +1.041 +0.655	+1.482 -1.041 +0.004	-1.482	+1.041 -0.004	
-0.130 -0.2	+0.025 -0.030 -0.130	+1.372 -0.025 +0.030 +0.130	-1.372	+0.445 +0.4	-1.482 -1.5	+1.037 +1.0	
	+1.360 +1.3	-0.203 -0.2	-1.158 -1.2				

*Final solution of triangles*

Symbol	Station	Observed angle	Correc- tion	Spher- ical angle	Spher- ical excess	Plane an- gle	Logarithm
		° ' "	"	"	"	° ' "	
-10+11	Turn-Dundas	42 00 30.0	+ 0.8	30.8	0.1	30.7	4.266771
- 1+ 2	Tower	24 17 25.4	+ 1.7	27.1	0.2	26.9	0.174417
- 4+ 6	Dundas	113 41 59.6	+ 3.0	62.6	0.2	42 02.4	9.614231
			+ 5.5		0.5		9.961733
	Tower-Dundas						4.055419
	Tower-Turn						4.402921
-12+13	Turn-Dundas	25 52 38.1	- 1.9	36.2	0.4	35.8	4.266771
- 1+ 3	Lazaro	110 36 08.7	- 3.9	04.8	0.5	04.3	0.360081
- 5+ 6	Turn	43 31 18.5	+ 1.8	20.3	0.4	19.9	9.971300
	Dundas		- 4.0		1.3		9.837989
	Lazaro-Dundas						4.598152
	Lazaro-Turn						4.464841
-12+14	Turn-Tower	42 30 21.5	- 5.8	15.7	0.6	15.1	4.402921
- 2+ 3	Lazaro	86 18 43.3	- 5.6	37.7	0.7	37.0	0.170282
- 9+10	Turn	51 11 09.1	- 0.6	08.5	0.6	07.9	9.999099
	Tower		-12.0		1.9		9.891638
	Lazaro-Tower						4.572302
	Lazaro-Turn						4.464841
-13+14	Dundas-Tower	16 37 43.4	- 3.9	39.5	0.4	39.1	4.055419
- 4+ 5	Lazaro	70 10 41.1	+ 1.2	42.3	0.4	41.9	0.543408
- 9+11	Dundas	93 11 39.1	+ 0.2	39.3	0.3	39.0	9.973475
	Tower		- 2.5		1.1		9.999325
	Lazaro-Tower						4.572302
	Lazaro-Dundas						4.598152
-14+15	Lazaro-Tower	47 10 38.8	+ 1.3	37.7	2.2	21 40 35.5	4.572302
- 7+ 9	Tow Hill	111 09 07.2	- 0.2	08.5	2.2	06.3	0.432543
	Lazaro	20.5		20.3	2.1	18.2	9.865314
	Tower				6.5		9.969699
	Tow Hill-Tower						4.870159
	Tow Hill-Lazaro						4.974544
-25+26	Lazaro-Tower	30 04 51.8	+ 3.6	55.4	1.7	53.7	4.572302
-14+16	Nichols	101 38 55.1	+ 2.0	57.1	1.7	55.4	0.299961
- 8+ 9	Lazaro	48 16 10.2	+ 2.4	12.6	1.7	10.9	9.990982
	Tower		+ 8.0		5.1		9.872905
	Nichols-Tower						4.863225
	Nichols-Lazaro						4.745168+1
-25+27	Lazaro-Tow Hill	89 23 18.6	+ 2.0	20.6	3.6	17.0	4.974544
-15+16	Nichols	54 28 47.9	+ 0.7	48.6	3.6	45.0	0.000025
	Lazaro	64.3		61.6	3.6	36 07 58.0	9.910573
	Tow Hill				10.8		9.770601
	Nichols-Tow Hill						4.885142
	Nichols-Lazaro						4.745170+1

## Final solution of triangles—Continued

Symbol	Station	Observed angle	Correc- tion	Spher- ical angle	Spher- ical excess	Plane an- gle	Logarithm
		° ' "	"	"	"	° ' "	
-26+27	Tower-Tow Hill	59 18 26.8	- 1.6	25.2	4.1	21.1	4. 870159
- 7+ 8	Nichols	62 53 10.3	- 2.6	07.7	4.0	03.7	0. 065550
	Tower	35.1		39.3	4.1	57 48 35.2	9. 949433
	Tow Hill						9. 927516
	Nichols-Tow Hill				12.2		4. 885142
	Nichols-Tower						4. 863225
-31+32	Lazaro-Nichols	116 59 49.5	+ 1.3	50.8	0.8	50.0	4. 745169
-16+17	Ken	22 32 57.3	+ 1.5	58.8	0.7	58.1	0. 050108
-23+25	Lazaro	40 27 12.5	+ 0.1	12.6	0.7	11.9	9. 583744
	Nichols						9. 812130
	Ken-Nichols		+ 2.9		2.2		4. 379021
	Ken-Lazaro						4. 607407
-33+34	Lazaro-Nichols	128 55 09.3	+ 1.6	10.9	0.4	10.5	4. 745169
-16+18	Seal	38 29 18.8	+ 0.6	19.4	0.5	18.9	0. 109005
-24+25	Lazaro	12 35 33.3	- 2.2	31.1	0.5	30.6	9. 794041
	Nichols						9. 338465
	Seal-Nichols		+ 0.0		1.4		4. 648215
	Seal-Lazaro						4. 192639
-33+35	Lazaro-Ken	154 32 21.8	- 0.3	21.5	0.2	21.3	4. 607407
-17+18	Seal	15 56 21.5	- 0.9	20.6	0.1	20.5	0. 366640
-30+31	Lazaro	9 31 16.8	+ 1.5	18.3	0.1	18.2	9. 438723
	Ken						9. 218592
	Seal-Ken		+ 0.3		0.4		4. 412770 <sup>+</sup>
	Seal-Lazaro						4. 192639
-34+35	Nichols-Ken	25 37 12.5	- 1.9	10.6	0.4	10.2	4. 379021
-23+24	Seal	27 51 39.2	+ 2.3	41.5	0.4	41.1	0. 364122
-30+32	Nichols	126 31 06.3	+ 2.8	09.1	0.4	08.7	9. 669628
	Ken						9. 905072
	Seal-Ken		+ 3.2		1.2		4. 412771
	Seal-Nichols						4. 648215
-36+37	Lazaro-Ken	128 20 51.4	+ 2.0	53.4	0.3	53.1	4. 607407
-17+19	Mid	35 17 43.3	+ 0.2	43.5	0.3	43.2	0. 105542
-29+31	Lazaro	16 21 23.9	+ 0.1	24.0	0.3	23.7	9. 761771
	Ken						9. 449655
	Mid-Ken		+ 2.3		0.9		4. 474720
	Mid-Lazaro						4. 162604
-42+44	Lazaro-Mid	51 05 11.2	+ 3.6	14.8	0.2	14.6	4. 162604
-19+21	Round	43 39 34.4	- 0.9	33.5	0.2	33.3	0. 108962
+36-38	Lazaro	85 15 11.7	+ 0.5	12.2	0.1	12.1	9. 839081
	Mid						9. 998508
	Round-Mid		+ 3.2		0.5		4. 110647
	Round-Lazaro						4. 270074

*Final solution of triangles—Continued*

Symbol	Station	Observed angle	Correc- tion	Spher- ical angle	Spher- ical excess	Plane an- gle	Logarithm
		° ' "	"	"	"	° ' "	
-42+45	Lazaro-Ken	74 42 34.2	+ 2.6	36.8	0.6	36.2	4.607407
-17+21	Round	78 57 17.7	- 0.7	17.0	0.7	16.3	0.015651
-28+31	Lazaro	26 20 06.8	+ 1.3	08.1	0.6	07.5	9.991879
	Ken						9.647016
	Round-Ken		+ 3.2		1.9		4.614937
	Round-Lazaro						4.270074
-44+45	Mid-Ken	23 37 23.0	- 1.0	22.0	0.2	21.8	4.474720
-37+38	Round	146 23 56.9	- 2.5	54.4	0.1	54.3	0.397167
-28+29	Mid	9 58 42.9	+ 1.2	44.1	0.2	43.9	9.743050
	Ken						9.238760
	Round-Ken		- 2.3		0.5		4.614937
	Round-Mid						4.110647
-49+52	Lazaro-Round	89 47 52.8	- 1.1	51.7	0.2	51.5	4.270074
-21+22	Cat	26 22 55.0	- 1.0	54.0	0.1	53.9	0.000003
-40+42	Lazaro	63 49 17.6	- 2.9	14.7	0.1	14.6	9.647723
	Round						9.952995
	Cat-Round		- 5.0		0.4		3.917800
	Cat-Lazaro						4.223072
-46+47	Round-Cat	33 20 40.5	- 2.2	38.3	0.1	38.2	3.917800
-40+43	Spur	111 30 09.4	- 0.9	08.5	0.0	08.5	0.259903
-51+52	Round	35 09 14.0	- 0.6	13.4	0.1	13.3	9.968671
	Cat						9.760250
	Spur-Cat		- 3.7		0.2		4.146374
	Spur-Round						3.937953
-46+48	Round-Lazaro	105 41 48.0	+ 0.1	48.1	0.1	48.0	4.270074
-42+43	Spur	47 40 51.8	+ 2.0	53.8	0.1	53.7	0.016506
-20+21	Round	26 37 18.2	+ 0.2	18.4	0.1	18.3	9.868888
	Lazaro						9.651373
	Spur-Lazaro		+ 2.3		0.3		4.155468
	Spur-Round						3.937953
-47+48	Cat-Lazaro	72 21 07.5	+ 2.3	09.8	0.1	09.7	4.223072
-49+51	Spur	54 38 38.8	- 0.5	38.3	0.2	38.1	0.020934
-20+22	Cat	53 00 13.2	- 0.8	12.4	0.2	12.2	9.911462
	Lazaro						9.902368
	Spur-Lazaro		+ 1.0		0.5		4.155468
	Spur-Cat						4.146374
-59+61	Cat-Round	49 58 12.0	+ 1.2	13.2	0.1	13.1	3.917800
-52+55	Beaver	87 34 53.9	+ 0.1	54.0	0.0	54.0	0.115935
-39+40	Cat	42 26 51.8	+ 1.2	53.0	0.1	52.9	9.999613
	Round						9.829253
	Beaver-Round		+ 2.5		0.2		4.033348
	Beaver-Cat						3.862988

*Final solution of triangles—Continued*

Symbol	Station	Observed angle	Correc- tion	Spher- ical angle	Spher- ical excess	Plane an- gle	Logarithm
		° ' "	"	"	"	° ' "	
—56+57	Round-Beaver	52 14 18.1	— 0.2	17.9	0.1	17.8	4.033348
—39+41	Snipe	105 37 20.7	— 0.5	20.2	0.0	20.2	0.102063
—60+61	Round	22 08 21.3	+ 0.8	22.1	0.1	22.0	9.983652
	Beaver						9.576182
	Snipe-Beaver		+ 0.1		0.2		4.119063
	Snipe-Round						3.711593
—56+58	Round-Cat	79 10 11.9	— 2.0	09.9	0.0	09.9	3.917800
—40+41	Snipe	63 10 28.9	— 1.7	27.2	0.0	27.2	0.007806
—50+52	Round	37 39 24.5	— 1.5	23.0	0.1	22.9	9.950551
	Cat						9.785987
	Snipe-Cat		— 5.2		0.1		3.876157
	Snipe-Round						3.711593
—57+58	Beaver-Cat	26 55 53.8	— 1.8	52.0	0.0	52.0	3.862988
—59+60	Snipe	27 49 50.7	+ 0.4	51.1	0.0	51.1	0.343980
—50+55	Beaver	125 14 18.4	— 1.4	17.0	0.1	16.9	9.669189
	Cat						9.912095
	Snipe-Cat		— 2.8		0.1		3.876157
	Snipe-Beaver						4.119063
—67+68	Beaver-Cat	62 13 29.4	+ 2.0	31.4	0.1	31.3	3.862988
—59+62	Khwain	58 43 17.2	+ 1.8	19.0	0.0	19.0	0.053161
—53+55	Beaver	59 03 08.4	+ 1.3	09.7	0.0	09.7	9.931792
	Cat						9.933305
	Khwain-Cat		+ 5.1		0.1		3.847941
	Khwain-Beaver						3.849454
—71+72	Beaver-Cat	36 34 55.5	+ 1.1	56.6	0.0	56.6	3.862988
—59+63	Lim	102 39 45.5	+ 0.7	46.2	0.0	46.2	0.224770
—54+55	Beaver	40 45 17.4	— 0.1	17.3	0.1	17.2	9.989306
	Cat						9.814795
	Lim-Cat		+ 1.7		0.1		4.077064
	Lim-Beaver						3.902553
—71+73	Beaver-Khwain	59 25 24.7	— 0.3	24.4	0.0	24.4	3.849454
—62+63	Lim	43 56 28.3	— 1.1	27.2	0.0	27.2	0.065022
—66+67	Beaver	76 38 09.2	— 0.7	08.5	0.1	08.4	9.841307
	Khwain						9.988077
	Lim-Khwain		— 2.1		0.1		3.755783
	Lim-Beaver						3.902553
—72+73	Cat-Khwain	22 50 29.2	— 1.4	27.8	0.1	27.7	3.847941
—53+54	Lim	18 17 51.0	+ 1.4	52.4	0.0	52.4	0.410972
—66+68	Cat	138 51 38.6	+ 1.3	39.9	0.0	39.9	9.498870
	Khwain						9.818151
	Lim-Khwain		+ 1.3		0.1		3.755783
	Lim-Cat						4.077064

*Final solution of triangles—Continued*

Symbol	Station	Observed angle	Correc- tion	Spher- ical angle	Spher- ical excess	Plane an- gle	Logarithm
		° ' "	"	"	"	° ' "	
-74+75	Beaver-Lim	60 40 06.2	- 1.5	04.7	0.0	04.7	3.902553
-63+65	South Twin	78 24 38.4	- 1.4	37.0	0.1	36.9	0.059585
-70+71	Beaver	40 55 17.8	+ 0.6	18.4	0.0	36.9	9.991054
	Lim					18.4	9.816260
			- 2.3		0.1		
	South Twin-Lim						3.953192
	South Twin-Beaver						3.778398
-77+78	South Twin-Beaver	35 38 30.7	- 1.9	28.8	0.0	28.8	3.778398
-74+76	Ham	94 10 29.2	- 2.5	26.7	0.1	28.6	0.234548
-64+65	South Twin	50 11 06.3	- 1.7	04.6	0.0	26.6	9.998847
	Beaver					04.6	9.885424
			- 6.1		0.1		
	Ham-Beaver						4.011793
	Ham-South Twin						3.898370 <sup>+1</sup>
							3.898371
-77+79	South Twin-Lim	85 04 23.4	+ 0.6	24.0	0.1	23.9	3.953192
-75+76	Ham	33 30 23.0	- 1.0	22.0	0.0	22.0	0.001608
-69+70	South Twin	61 25 13.8	+ 0.3	14.1	0.0	22.0	9.741959
	Lim					14.1	9.943571
			- 0.1		0.1		
	Ham-Lim						3.696759
	Ham-South Twin						3.898371
-78+79	Beaver-Lim	49 25 52.7	+ 2.5	55.2	0.0	55.2	3.902553
-63+64	Ham	28 13 32.1	+ 0.3	32.4	0.0	32.4	0.119395
-69+71	Beaver	102 20 31.6	+ 0.9	32.5	0.1	32.4	9.674811
	Lim					32.4	9.989845
			+ 3.7		0.1		
	Ham-Lim						3.696759
	Ham-Beaver						4.011793

Final position computation,

STATION TOWER

$\alpha$	Turn to Dundas					$^{\circ}$	$'$	$''$
Second angle	Dundas and Tower					357	33	07.7
$\alpha$						+	24	27.1
$\Delta\alpha$	Turn to Tower					21	50	34.8
						—	7	07.6
$\alpha'$	Tower to Turn					180		
						201	43	27.2
						42	00	30.8
	First angle of triangle							
$\phi$	$^{\circ}$	$'$	$''$					
$\Delta\phi$	54	48	06.742	Turn	$\lambda$	130	56	04.052
	—	12	39.415		$\Delta\lambda$	+	8	43.963
$\phi'$	54	35	27.327	Tower	$\lambda'$	131	04	48.015
			—1					
$s$	4.402921	$s^2$	8.80585	$(\partial\phi)^2$	$D$	5.7609	$-\frac{h}{s^2}$	2.8803
$\cos \alpha$	9.9676448	$\sin^2 \alpha$	9.14128				$s^2 \sin^2 \alpha$	7.9471
$B$	8.5097251	$C$	1.55459			2.3672	$E$	6.4574
$h$	2.8802909		9.50172			8.1281		7.2848
	"		"					
1st term	+759.0859	3d term	+0.0134					
2d term	+ 0.3175	4th term	—0.0019					
	+759.4034							
3d and 4th terms	+ 0.0115		—6					
$-\Delta\phi$	+759.4149	$s$	4.402921	Arg.	$s$	—11	$\frac{\Delta\lambda}{\sin \frac{1}{2}(\phi+\phi')}$	2.7193010
		$A'$	9.5706188				$\frac{\Delta\lambda}{\sec \frac{1}{2}(\Delta\phi)}$	9.9117440
		$\sec \phi'$	8.5087480			+ 5		7
			0.2370138					
$\frac{1}{2}(\phi+\phi')$	54	41	47	2.7193010	Corr.	— 6		2.631057
				"				"
		$\Delta\lambda$	+523.9635				$-\Delta\alpha$	+427.61

## STATION LAZARO

$\alpha$	Turn to Tower						$^{\circ}$	$'$	$''$
Second angle	Tower and Lazaro						21	50	34.8
$\alpha$							+	86	37.7
$\Delta\alpha$	Turn to Lazaro						108	09	12.5
							—	21	10.8
$\alpha'$	Lazaro to Turn						180		
							287	48	01.7
	First angle of triangle						42	30	15.7
$\phi$	$^{\circ}$	$'$	$''$						
$\Delta\phi$	54	48	06.742	Turn		$\lambda$	130	56	04.052
	+	4	51.079			$\Delta\lambda$	+	25	54.366
$\phi'$	54	52	57.821	Lazaro		$\lambda'$	131	21	58.418
			—1						—1
$s$	4.464841	$s^2$	8.92963	$(\partial\phi)^2$ D	4.9281 2.3672	$-\frac{h}{s^2}$	2.4681		
$\cos \alpha$	9.4935464	$\sin^2 \alpha$	9.95564			$s^2 \sin^2 \alpha$	8.8853		
B	8.5097251	C	1.55459			E	6.4574		
h	2.4681125		0.43986				7.8108		
	"		"						
1st term	—293.8411	3d term	+0.0020						
2d term	+ 2.7534	4th term	+0.0065						
	—291.0877								
3d and 4th terms	+ 0.0085		+27						
		$s$	4.464841	Arg. $s$ $\Delta\lambda$	—15 +42	$\frac{\Delta\lambda}{\sin \frac{1}{2}(\phi+\phi')}$ $\sec \frac{1}{2}(\Delta\phi)$	3.1915533 9.9125251		
$-\Delta\phi$	—291.0792	$A'$	9.9778267						
		$\sec \phi'$	8.5087409						
	$^{\circ}$		0.2401420						
$\frac{1}{2}(\phi+\phi')$	54	50	32.3	3.1915533	Corr.	+27	3.1040784		
			"				"		
		$\Delta\lambda$	+1554.3661				$-\Delta\alpha$	+1270.8	

## primary triangulation

## STATION TOWER

$\alpha$ Third angle	Dundas to Turn Tower and Turn						$^{\circ}$ 177 -113	$'$ 33 42	$''$ 43.6 02.6
$\alpha$ $\Delta\alpha$	Dundas to Tower						63 —	51 7	41.0 43.0
$\alpha'$	Tower to Dundas						180 243	43	58.0
$\phi$ $\Delta\phi$	$^{\circ}$ 54 —	$'$ 38 2	$''$ 09.559 42.233	Dundas	$\lambda$ $\Delta\lambda$		130 +	55 9	20.042 27.973
$\phi'$	54	35	27.326	Tower	$\lambda'$		131	04	48.015
$s$ cos $\alpha$ B	4.055419 9.6439895 8.5097372	$s^2$ $\sin^2 \alpha$ C	8.11087 9.90630 1.55194	$(\delta\phi)^2$ D	4.4203 2.3681		$-\frac{h}{s^2 \sin^2 \alpha}$ E		2.2091 8.0172 6.4528
h	2.2091457 "		9.56911 "		6.7884				6.6791
1st term	+161.8623	3d term	+0.0006						
2d term	+ 0.3708	4th term	-0.0005						
3d and 4th terms	+162.2331 +0.0001		+3 4.055419 9.9531462 8.5087480 0.2370138	Arg. $s$ $\Delta\lambda$	-2 +5		$\frac{\Delta\lambda}{\sin \frac{1}{2}(\phi+\phi')}$ $\sec \frac{1}{2}(\Delta\phi)$		2.7543283 9.9112981
$\frac{1}{2}(\phi+\phi')$	54 36 48.4 "		2.7543273 "	Corr.	+3				2.6656264 "
		$\Delta\lambda$	+567.9725				$-\Delta\alpha$		+463.05

## STATION LAZARO

$\alpha$ Third angle	Tower to Turn Lazaro and Turn						$^{\circ}$ 201 — 51	$'$ 43 11	$''$ 27.2 08.5
$\alpha$ $\Delta\alpha$	Tower to Lazaro						150 —	32 14	18.7 01.3
$\alpha'$	Lazaro to Tower						180 330	18	17.4
$\phi$ $\Delta\phi$	$^{\circ}$ 54 +	$'$ 35 17	$''$ 27.326 30.494	Tower	$\lambda$ $\Delta\lambda$		131 +	04 17	48.015 10.401
$\phi'$	54	52	57.820	Lazaro	$\lambda'$		131	21	58.416 +1
$s$ cos $\alpha$ B	4.572302 9.9398618 8.5097404	$s^2$ $\sin^2 \alpha$ C	9.14459 9.38353 1.55122	$(\delta\phi)^2$ D	6.0428 2.3683		$-\frac{h}{s^2 \sin^2 \alpha}$ E		3.0219 8.5281 6.4516
h	3.0219042 "		0.07934 "		8.4111				8.0016
1st term	-1051.7298	3d term	+0.0253						
2d term	+ 1.2004	4th term	+0.0100						
3d and 4th terms	-1050.5294 + 0.0358		-7 4.572302 9.6918222 8.5087409 0.2401420	Arg. $s$ $\Delta\lambda$	-25 +18		$\frac{\Delta\lambda}{\sin \frac{1}{2}(\phi+\phi')}$ $\sec \frac{1}{2}(\Delta\phi)$		3.0130064 9.9119609
$\frac{1}{2}(\phi+\phi')$	54 44 12.6 "		3.0130064 "	Corr.	- 7				2.9249673 "
		$\Delta\lambda$	+1030.4012				$-\Delta\alpha$		+841.3

*Final position computation,*

## STATION TOW HILL

$\alpha$ Second angle	Lazaro to Tower Tower and Tow Hill				$^{\circ}$ 330 + 47	$'$ 18 10	$''$ 17.4 08.5
$\alpha$ $\Delta\alpha$	Lazaro to Tow Hill				17 —	28 21	25.9 07.5
$\alpha'$	Tow Hill to Lazaro				180 197 21	07 40	18.4 37.7
	First angle of triangle						
$\phi$ $\Delta\phi$ $\phi'$	$^{\circ}$ 54 —	$'$ 52 48	$''$ 57.820 32.022	Lazaro	$\lambda$ $\Delta\lambda$	131 +	21 25 58.417 57.250
	54	04	25.798	Tow Hill	$\lambda'$	131	47 55.667 —2
$s$ $\cos \alpha$ B	4.974544 9.9794818 8.5097191	$s^2$ $\sin^2 \alpha$ C	9.94909 8.95504 1.55589	$(\delta\phi)^2$ D	6.9283 2.3667	$-\frac{h}{s^2 \sin^2 \alpha}$ E	3.4637 8.9043 6.4597
h	3.4637449		0.46002		9.2950		8.8277
1st term	+2909.0080	3d term	+0.1972				
2d term	+ 2.8842	4th term	-0.0673				
	+2911.8922						
3d and 4th } terms	+ 0.1299	$s$ $\sin \alpha$ A'	-117 4.974544 9.4775129 8.5087606 0.2315526	Arg. $s$ $\Delta\lambda$	-159 + 42	$\sin \frac{\Delta\lambda}{2(\phi+\phi')}$ $\sec \frac{1}{2}(\Delta\phi)$	3.1923584 9.9105687 108
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 54 28 41.9	$\sec \phi'$	3.1923584 "	Corr.	-117		3.1029379 "
		$\Delta\lambda$	+1557.2502			$-\Delta\alpha$	+1267.5

## STATION NICHOLS

$\alpha$ Second angle	Lazaro to Tow Hill Tow Hill and Nichols					$^{\circ}$ 17 + 54	$'$ 28 28	$''$ 25.9 48.6
$\alpha$ $\Delta\alpha$	Lazaro to Nichols					71 —	57 40	14.5 14.3
$\alpha'$	Nichols to Lazaro					180 251 89	17 23	00.2 20.6
	First angle of triangle							
$\phi$ $\Delta\phi$ $\phi'$	$^{\circ}$ 54 —	$'$ 52 9	$''$ 57.820 27.129	Lazaro	$\lambda$ $\Delta\lambda$	131 +	21 49	58.417 14.369
	54	43	30.691	Nichols	$\lambda'$	132	11	12.786 +1
$s$ $\cos \alpha$ B	4.745169 9.4910534 8.5097191	$s^2$ $\sin^2 \alpha$ C	9.49034 9.95619 1.55589	$(\partial\phi)^2$ D	5.5072 2.3667	$-\frac{h}{s^2 \sin^2 \alpha}$ E	2.7458 9.4465 6.4597	
h	2.7459415		1.00242		7.8739		8.6520	
1st term	+	3d term	+					
2d term	557.1107 + 10.0559	4th term	0.0075 -0.0449					
3d and 4th } terms	+567.1666							
$-\Delta\phi$	— 0.0374	$s$ $\sin \alpha$ A'	+92 4.745169 9.9780929 8.5087447 0.2334490	Arg. $s$ $\Delta\lambda$	— 56 +148	$\sin \frac{1}{2}(\phi+\phi')$ $\sec \frac{1}{2}(\Delta\phi)$	3.4704648 9.9123203	
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 54 48 14.3	$\sec \phi'$	3.4704648 "	Corr.	+ 92		3.3827851 "	
		$\Delta\lambda$	+2954.3694			$-\Delta\alpha$	+2414.27	

primary triangulation—Continued

## STATION TOW HILL

$\alpha$ Third angle	Tower to Lazaro Tow Hill and Lazaro				$^{\circ}$ 150 —111	$'$ 32 09	$''$ 18.7 20.3	
	Tower to Tow Hill							
	Tow Hill to Tower							
$\phi$ $\Delta\phi$	$^{\circ}$ 54 —	$'$ 35 31	$''$ 27.326 01.527	Tower	$\lambda$ $\Delta\lambda$	131 +	04 43	48.015 07.648
	$\phi'$	54	04	25.799 —1				
$s$ $\cos \alpha$ B	4.870159 9.8881363 8.5097404	$s^2$ $\sin^2 \alpha$ C	9.74032 9.60486 1.55122	$(\delta\phi)^2$ D	6.5397 2.3683	$-\text{h}$ $s^2 \sin^2 \alpha$ E	3.2680 9.3452 6.4516	
h	3.2680357 "		0.89640 "		8.9080		9.0648	
1st term	+1853.6842	3d term	+0.0809					
2d term	+ 7.8777	4th term	—0.1160					
3d and 4th terms	+1861.5619							
	— 0.0351		+16					
	+1861.5268	$s$ $\sin \alpha$ A'	4.870159 9.8024314	Arg. $s$	— 98	$\sin \frac{\Delta\lambda}{2}(\phi+\phi')$	3.4129052 9.9097770	
	$^{\circ}$ $'$ $''$ 54 19 56.6	$\sec \phi'$	0.2315526	$\Delta\lambda$	+114	$\sec \frac{1}{2}(\Delta\phi)$	44	
$\frac{1}{2}(\phi+\phi')$			3.4129052 "	Corr.	+ 16		3.3226866 "	
		$\Delta\lambda$	+2587.6482			$-\Delta\alpha$	+2102.3	

## STATION NICHOLS

$\alpha$ Third angle	Tow Hill to Lazaro Nichols and Lazaro				$^{\circ}$ 197 — 36	$'$ 07 08	$''$ 18.4 01.6	
$\alpha$ $\Delta\alpha$	Tow Hill to Nichols				160 —	59 18	16.8 56.0	
$\alpha'$	Nichols to Tow Hill				180 340	40	20.8	
$\phi$ $\Delta\phi$	$^{\circ}$ 54 +	$'$ 04 39	$''$ 25.798 04.894	Tow Hill	$\lambda$ $\Delta\lambda$	131 +	47 23	55.665 17.123
$\phi'$	54	43	30.692 —1	Nichols	$\lambda'$	132	11	12.788 —1
$s$ $\cos \alpha$ B	4.885142 9.9756388 8.5097780	$s^2$ $\sin^2 \alpha$ C	9.77028 9.02582 1.54301	$(\partial\phi)^2$ D	6.7403 2.3709	$-\text{h}$ $s^2 \sin^2 \alpha$ E	3.3706 8.7959 6.4373	
h	3.3705588 "		0.33911 "		9.1112		8.6038	
1st term	—2347.2470	3d term	+0.1292					
2d term	+ 2.1833	4th term	+0.0402					
	—2345.0637							
3d and 4th terms	+ 0.1694		—70					
$-\Delta\phi$	—2344.8943	$s$ $\sin \alpha$ A' $\sec \phi'$	4.885142 9.5129060 8.5087447 0.2384490	Arg. $s$ $\Delta\lambda$	—104 + 34	$\sin \frac{1}{2}(\phi+\phi')$ $\sec \frac{1}{2}(\Delta\phi)$	3.1452347 9.9101427 70	
$\frac{1}{2}(\phi+\phi')$	54 23 58.9		3.1452347 "	Corr.	— 70		3.0553844 "	
		$\Delta\lambda$	+1397.1232			$-\Delta\alpha$	+1136.02	

*Final position computation,*

## STATION KEN

$\alpha$ Second angle	Lazaro to Nichols Nichols and Ken					$^{\circ}$ 71 + 22	$'$ 57 32	$''$ 14.5 58.8
$\alpha$ $\Delta\alpha$	Lazaro to Ken					94 —	30 30	13.3 53.7
$\alpha'$	Ken to Lazaro				First angle of triangle	180 273 116	59 59	19.6 50.8
$\alpha$ $\Delta\phi$	$^{\circ}$ 54 +	$'$ 52 1	$''$ 57.820 36.965	Lazaro		$\lambda$ $\Delta\lambda$	131 +	21 37
$\phi'$	54	54	34.785	Ken	$\lambda'$	131	59	44.326
$s$ $\cos \alpha$ B	4.607407 8.8949989 8.5097191	$s^2$ $\sin^2 \alpha$ C	9.21480 9.99731 1.55589	$(\partial\phi)^2$ D	4.0250 2.3667	$-\frac{h}{s^2 \sin^2 \alpha}$ E		2.0125 9.2121 6.4597
h	2.0121250		0.76800		6.3917			7.6843
1st term	—102.8312	3d term	+0.0002					
2d term	+ 5.8614	4th term	+0.0048					
3d and 4th terms }	— 96.9698							
	+ 0.0050	$s$ $\sin \alpha$ A'	4.607407 9.9986569 8.5087403 0.2404325	Arg. $s$ $\Delta\lambda$	—29 +87	$\sin \frac{\Delta\lambda}{s^2} \sec \frac{1}{2}(\phi+\phi')$		3.355242 9.912798
	— 96.9648	$\sec \phi'$						
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 54 $'$ 53 $''$ 46.4		3.3552425 " $\Delta\lambda$ +2265.9094	Corr.	+58			3.268040 " +1853.70

## STATION ROUND

$\alpha$ Second angle	Lazaro and Ken Ken and Round								$^{\circ}$ 94 + 78	$'$ 30 57	$''$ 13.3 17.0
$\alpha$ $\Delta\alpha$	Lazaro to Round								173 —	27 1	30.3 37.8
$\alpha'$	Round to Lazaro				First angle of triangle				180 353 74	25 42	52.5 36.8
$\phi$ $\Delta\phi$	$^{\circ}$ 54 +	$'$ 52 9	$''$ 57.820 58.327	Lazaro	$\lambda$ $\Delta\lambda$			131 +	21 1	58.417 59.499	
$\phi'$	55	02	56.147	Round	$\lambda'$			131	23	57.916 +1	
$s$ $\cos \alpha$ B	4.270074 9.9971633 8.5097191	$s^2$ $\sin^2 \alpha$ C	8.54013 8.11255 1.55589	$(\partial\phi)^2$ D	5.5538 2.3667	$-\frac{h}{s^2 \sin^2 \alpha}$ E		2.7769 6.6527 6.4597			
h	2.7769564 "	8.20857 "		7.9205				5.8893			
1st term	-598.3514	3d term	+0.0083								
2d term	+ 0.0161	4th term	+0.0001								
3d and 4th } terms	-598.3353										
	+ 0.0084	$s$ $\sin \alpha$ A'	4.270074 9.0566163 8.5087369 0.2419389	Arg. $s$ $\Delta\lambda$	-6 +0	$\sin \frac{\Delta\lambda}{s^2} \sec \frac{1}{2}(\phi+\phi')$		2.077365 9.913183			
	-598.3269	$\sec \phi'$	2.0773655 "	Corr.	-6			1.990548 "			
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 54 $'$ 57 $''$ 57	2.0773655 " $\Delta\lambda$ +119.4993						- $\Delta\alpha$ +97.85			

primary triangulation—Continued

## STATION KEN

$\alpha$ Third angle	Nichols to Lazaro Ken and Lazaro					$^{\circ}$ 251 — 40	$'$ 17 27	$''$ 00. 2 12. 6
$\alpha$ $\Delta\alpha$	Nichols to Ken					210 +	49 9	47. 6 22. 7
$\alpha'$	Ken to Nichols					180 30	59	10. 3 + . 1
$\phi$ $\Delta\phi$	$^{\circ}$ 54 +	$'$ 43 11	$''$ 30. 691 04. 093	Nichols	$\lambda$ $\Delta\lambda$	132 —	11 11	12. 787 28. 462
$\phi'$	54	54	34. 784 +1	Ken	$\lambda'$	131	59	44. 325 +1
$s$ $\cos \alpha$ B	4. 379021 9. 9338378 8. 5097307	$s^2$ $\sin^2 \alpha$ C	8. 75804 9. 41937 1. 55337	$(\partial\phi)^2$ D	5. 6450 2. 3676	—h $s^2 \sin^2 \alpha$ E	2. 8226 8. 1775 6. 4553	
h	2. 8225895		9. 73078		8. 0126			7. 4554
1st term	—664. 6446	3d term	+0. 0103					
2d term	+ 0. 5380	4th term	+0. 0029					
3d and 4th } terms	—664. 1066							
— $\Delta\phi$	+ 0. 0132	$s$ $\sin \alpha$ A'	—2 4. 379021 9. 7096861 8. 5087403 0. 2404325	Arg. $s$ $\Delta\lambda$	—10 + 8	$\Delta\lambda$ $\sin \frac{1}{2}(\phi+\phi')$ $\sec \frac{1}{2}(\Delta\phi)$	2. 837880 9. 912392	
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 54 49 02. 9	$\sec \phi'$	2. 8378797 " —688. 4616	Corr.	— 2		2. 750272 " —562. 69	
		$\Delta\lambda$				— $\Delta\alpha$		

## STATION ROUND

$\alpha$ Third angle	Ken to Lazaro Round and Lazaro					$^{\circ}$ 273 — 26	$'$ 59 20	$''$ 19. 6 08. 1
$\alpha$ $\Delta\alpha$	Ken to Round					247 +	39 29	11. 5 17. 8
$\alpha'$	Round to Ken					180 68	08	29. 3
$\phi$ $\Delta\phi$	$^{\circ}$ 54 +	$'$ 54 8	$''$ 34. 785 21. 362	Ken	$\lambda$ $\Delta\lambda$	131 —	59 35	44. 326 46. 407
$\phi'$	55	02	56. 147	Round	$\lambda'$	131	23	57. 919 —2
$s$ $\cos \alpha$ B	4. 614937 9. 5800256 8. 5097172	$s^2$ $\sin^2 \alpha$ C	9. 22987 9. 93219 1. 55631	$(\partial\phi)^2$ D	5. 4091 2. 3666	—h $s^2 \sin^2 \alpha$ E	2. 7046 9. 1621 6. 4604	
h	2. 7046798		0. 71837		7. 7757			8. 3271
1st term	—506. 6171	3d term	+0. 0058					
2d term	+ 5. 2285	4th term	+0. 0212					
3d and 4th } terms	—501. 3886							
— $\Delta\phi$	+ 0. 0270	$s$ $\sin \alpha$ A'	+49 4. 614937 9. 9660945 8. 5087369 0. 2419389	Arg. $s$ $\Delta\lambda$	—30 +79	$\Delta\lambda$ $\sin \frac{1}{2}(\phi+\phi')$ $\sec \frac{1}{2}(\Delta\phi)$	3. 331712 9. 913254	
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 54 58 46. 0	$\sec \phi'$	3. 3317122 " —2146. 4074	Corr.	+49		3. 244966 " —1757. 79	
		$\Delta\lambda$				— $\Delta\alpha$		

*Final position computation,*

## STATION CAT

$\alpha$ Second angle	Lazaro to Round Round and Cat						$^{\circ}$ 173 + 26	$'$ 27 22	$''$ 30.3 54.0
$\alpha$ $\Delta\alpha$	Lazaro to Cat						199 +	50 4	24.3 21.4
$\alpha'$	Cat to Lazaro				First angle of triangle		180 19 89	54 47	45.7 51.7
$\phi$ $\Delta\phi$	$^{\circ}$ 54 +	$'$ 52 8	$''$ 57.820 28.290	Lazaro	$\lambda$ $\Delta\lambda$		131 —	21 5	58.417 19.289
$\phi'$	55	01	26.110	Cat	$\lambda'$		131	16	39.128 +1
$s$ $\cos \alpha$ B	4.223072 9.9734251 8.5097191	$s^2$ $\sin^2 \alpha$ C	8.44612 9.06164 1.55589	$(\delta\phi)^2$ D	5.4124 2.3667	$-\frac{h}{s^2 \sin^2 \alpha}$ E			2.7062 7.5078 6.4597
h	2.7062162		9.06365		7.7791				6.6737
1st term	"		"						
2d term	-508.4124 + 0.1158	3d term	+0.0060 +0.0005						
3d and 4th terms	-508.2966 + 0.0065								
$-\Delta\phi$	-508.2901	$s$ $\sin \alpha$ $\Delta'$ $\sec \phi'$	4.223072 9.5307069 8.5087375 0.2416677	Arg. $s$ $\Delta\lambda$	-5 +2	$\frac{\Delta\lambda}{\sin \frac{1}{2}(\phi+\phi') \sec \frac{1}{2}(\Delta\phi)}$			2.504184 9.913117
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 54 57 42.0		2.5041838	Corr.	-3				2.417301
		$\Delta\lambda$	-319.2889						" -261.4

## STATION BEAVER

$\alpha$ Second angle	Cat to Round Round and Beaver						$^{\circ}$ 109 + 87	$'$ 42 34	$''$ 37.4 54.0
$\alpha$ $\Delta\alpha$	Cat to Beaver						197 +	17 1	31.4 40.2
$\alpha'$	Beaver to Cat				First angle of triangle		180 17 49	19 58	11.6 13.2
$\phi$ $\Delta\phi$	$^{\circ}$ 55 +	$'$ 01 3	$''$ 26.110 45.204	Cat	$\lambda$ $\Delta\lambda$		131 —	16 2	39.129 02.231
$\phi'$	55	05	11.314	Beaver	$\lambda'$		131	14	36.898
$s$ $\cos \alpha$ B	3.862988 9.9799133 8.5097090	$s^2$ $\sin^2 \alpha$ C	7.7260 8.9465 1.5584	$(\delta\phi)^2$ D	4.705 2.366	$-\frac{h}{s^2 \sin^2 \alpha}$ E			2.353 6.672 6.464
h	2.3526103		8.2309		7.071				5.489
1st term	"		"						
2d term	-225.2218 + 0.0170	3d term	+0.0012						
3d and 4th terms	-225.2048 + 0.0012								
$-\Delta\phi$	-225.2036	$s$ $\sin \alpha$ $\Delta'$ $\sec \phi'$	3.862988 9.4731109 8.5087360 0.2423463	Arg. $s$ $\Delta\lambda$	-1 0	$\frac{\Delta\lambda}{\sin \frac{1}{2}(\phi+\phi') \sec \frac{1}{2}(\Delta\phi)}$			2.087181 9.913657
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 55 03 18.7		2.0871811	Corr.	-1				2.000838
		$\Delta\lambda$	-122.2309						" -100.19

primary triangulation—Continued

## STATION CAT

$\alpha$ Third angle	Round to Lazaro Cat and Lazaro					$^{\circ}$ 353 — 63	$'$ 25 49	$''$ 52.5 14.7
$\alpha$ $\Delta\alpha$	Round to Cat					289 +	36 5	37.8 59.6
$\alpha'$	Cat to Round					180 109	42	37.4
$\phi$ $\Delta\phi$	$^{\circ}$ 55 —	$'$ 02 1	$''$ 56.147 30.037	Round	$\lambda$ $\Delta\lambda$	131 —	23 7	57.917 18.788
$\phi'$	55	01	26.110	Cat	$\lambda'$	131	16	39.129
$s$ $\cos \alpha$ B	3.917800 9.5258533 8.5097071	$s^2$ $\sin^2 \alpha$ C	7.8356 9.9481 1.5586	$(\partial\phi)^2$ D	3.9069 2.3658	$-\frac{h}{s^2} \sin^2 \alpha$ E		1.9535 7.7837 6.4643
h	1.9533604		9.3423		6.2727			6.2015
1st term	"	3d term	"					
2d term	+89.8174 + 0.2199	4th term	+0.0002 -0.0002					
3d and 4th terms	+90.0373 0		+2 3.917800 9.9740491 8.5087375 0.2416677	Arg. $s$ $\Delta\lambda$	-1 +3	$\frac{\Delta\lambda}{\sec \frac{1}{2}(\phi+\phi')}$ $\sec \frac{1}{2}(\Delta\phi)$		2.642254 9.913558
$\frac{1}{2}(\phi+\phi')$	55 02 11.1	$\frac{s}{\sin \alpha}$ $\frac{A'}{\sec \phi'}$	2.6422545 "	Corr.	+2			2.555812 "
		$\Delta\lambda$	-438.7877			$-\Delta\alpha$		-359.59

## STATION BEAVER

$\alpha$ Third angle	Round to Cat Beaver and Cat					$^{\circ}$ 289 — 42	$'$ 36 26	$''$ 37.8 53.0
$\alpha$ $\Delta\alpha$	Round to Beaver					247 +	09 7	44.8 39.9
$\alpha'$	Beaver to Round					180 67	17	24.7 + .1
$\phi$ $\Delta\phi$	$^{\circ}$ 55 +	$'$ 02 2	$''$ 56.147 15.166	Round	$\lambda$ $\Delta\lambda$	131 —	23 9	57.917 21.019
$\phi'$	55	05	11.313 +1	Beaver	$\lambda'$	131	14	36.898
$s$ $\cos \alpha$ B	4.033348 9.5889657 8.5097071	$s^2$ $\sin^2 \alpha$ C	8.0667 9.9291 1.5586	$(\partial\phi)^2$ D	4.264 2.366	$-\frac{h}{s^2} \sin^2 \alpha$ E		2.132 7.996 6.464
h	2.1320208		9.5544		6.630			6.592
1st term	"	3d term	"					
2d term	-135.5254 + 0.3584	4th term	+0.0004 +0.0004					
3d and 4th terms	-135.1670 + 0.0008		+3 4.033348 9.9645467 8.5087360 0.2423463	Arg. $s$ $\Delta\lambda$	-2 +5	$\frac{\Delta\lambda}{\sec \frac{1}{2}(\phi+\phi')}$ $\sec \frac{1}{2}(\Delta\phi)$		2.748977 9.913723
$\frac{1}{2}(\phi+\phi')$	55 04 03.7	$\frac{s}{\sin \alpha}$ $\frac{A'}{\sec \phi'}$	2.7489773 "	Corr.	+3			2.662700 "
		$\Delta\lambda$	-561.0186			$-\Delta\alpha$		-459.94

*Final position computation,*

## STATION LIM

$\alpha$ Second angle	Beaver to Cat Cat and Lim					$^{\circ}$ 17 +102	$'$ 19 39	$''$ 11.6 46.2
$\alpha$ $\Delta\alpha$	Beaver to Lim					119 —	58 5	57.8 20.3
$\alpha'$	Lim to Beaver					180 299 36	53 34	37.5 56.6
First angle of triangle								
$\phi$ $\Delta\phi$	$^{\circ}$ 55 +	$'$ 05 2	$''$ 11.314 08.948	Beaver	$\lambda$ $\Delta\lambda$	131 +	14 6	36.898 30.508
$\phi'$	55	07	20.262	Lim	$\lambda'$	131	21	07.406 +1
$s$ $\cos \alpha$ B	3.902553 9.6987431 8.5097044	$s^2$ $\sin^2 \alpha$ C	7.8051 9.8751 1.5591	$(\partial\phi)^2$ D	4.222 2.366	$-\frac{h}{s^2 \sin^2 \alpha}$ E	2.111 7.680 6.465	
h	2.1110005		9.2393		6.588		6.256	
1st term	" -129.1221	3d term	" +0.0004					
2d term	0.1735	4th term	+0.0002					
	-128.9486							
3d and 4th terms	+ 0.0006		+2 3.902553 9.9376062 8.5087351 0.2427356	Arg. $s$ $\Delta\lambda$	-1 +3	$\sin \frac{\Delta\lambda}{2}(\phi+\phi')$ $\sec \frac{1}{2}(\Delta\phi)$	2.591630 9.913918	
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 55 $'$ 06 $''$ 15.8	$s$ $\sin \alpha$ A' $\sec \phi'$	2.5916301 " +390.5082	Corr.	+2		2.505548 " +320.3	
		$\Delta\lambda$				$-\Delta\alpha$		

## STATION SOUTH TWIN

$\alpha$ Second angle	Beaver to Lim Lim and South Twin					$^{\circ}$ 119 + 78	$'$ 58 24	$''$ 57.8 37.0
$\alpha$ $\Delta\alpha$	Beaver to South Twin					198 +	23 1	34.8 27.7
$\alpha'$	South Twin to Beaver					180 18 60	25 40	02.5 04.7
	First angle of triangle							
$\phi$ $\Delta\phi$	$^{\circ}$ 55 +	$'$ 05 3	$''$ 11.314 04.203	Beaver	$\lambda$ $\Delta\lambda$	131 —	14 1	36.898 46.925
$\phi'$	55	08	15.517 15.517	South Twin	$\lambda'$	131	12	49.973 +1 49.974
$s$ $\cos \alpha$ B	3.778398 9.9772271 8.5097044	$s^2$ $\sin^2 \alpha$ C	7.5568 8.9984 1.5591	$(\partial\phi)^2$ D	4.530 2.366	$-\frac{h}{s^2 \sin^2 \alpha}$ E	2.265 6.555 6.465	
h	2.2653295		8.1143		6.896		5.285	
1st term	" —184.2169	3d term	" +0.0008					
2d term	+ 0.0130	4th term						
3d and 4th terms	—184.2039 + 0.0008		—1 3.778398 9.4990449 8.5087347 0.2429024	Arg. $s$ $\Delta\lambda$	—1 0	$\sin \frac{\Delta\lambda}{2}(\phi+\phi')$ $\sec \frac{1}{2}(\Delta\phi)$	2.029080 9.913958	
$\frac{1}{2}(\phi+\phi')$	55 06 43.4		2.0290799 " —106.9252	Corr.	—1		1.943038 " —87.7	
		$\Delta\lambda$				$-\Delta\alpha$		

## primary triangulation—Continued

## STATION LIM

$\alpha$ Third angle	Cat to Beaver Lim and Beaver						° 197 — 40	' 17 45	'' 31.4 17.3
$\alpha$ $\Delta\alpha$	Cat to Lim						156 —	32 3	14.1 40.0
$\alpha'$	Lim to Cat						180 336	28	34.1
$\phi$ $\Delta\phi$	° 55 +	' 01 5	'' 26.110 54.152	Cat		$\lambda$ $\Delta\lambda$	131 +	16 4	39.129 28.278
$\phi'$	55	07	20.262	Lim		$\lambda'$	131	21	07.407
$s$ $\cos \alpha$ B	4.077064 9.9625205 8.5097090	$s^2$ $\sin^2 \alpha$ C	8.1541 9.1999 1.5584	$(\partial\phi)^2$ D		5.099 2.366	—h $s^2 \sin^2 \alpha$ E	2.549 7.354 6.464	
h	2.5492935		8.9124			7.465		6.367	
1st term	—354.2367	3d term	+0.0029						
2d term	+ 0.0817	4th term	+0.0002						
	—354.1550								
3d and 4th terms }	+ 0.0031	$s$ $\sin \alpha$ A'	4.077064 9.6000497 8.5087351 0.2427356	Arg. $s$ $\Delta\lambda$		—2 +2	$\Delta\lambda$ $\sin \frac{1}{2}(\phi+\phi')$ sec $\frac{1}{2}(\Delta\phi)$	2.428584 9.913752	
— $\Delta\phi$	—354.1519	sec $\phi'$							
$\frac{1}{2}(\phi+\phi')$	° ' '' 55 04 23.2		2.4285844	Corr.		0		2.342336	
		$\Delta\lambda$	+268.2776				— $\Delta\alpha$	+219.96	

## STATION SOUTH TWIN

$\alpha$ Third angle	Lim to Beaver South Twin and Beaver					$^{\circ}$ 299 — 40	$'$ 53 55	$''$ 37. 5 18. 4
$\alpha$ $\Delta\alpha$	Lim to South Twin					258 +	58 6	19. 1 48. 1
$\alpha'$	South Twin to Lim					180 79	05	07. 2
$\phi$ $\Delta\phi$	$^{\circ}$ 55 +	$'$ 07	$''$ 20. 262 55. 255	Lim	$\lambda$ $\Delta\lambda$	131 —	21 8	07. 407 17. 433
$\phi'$	55	08	15. 517	South Twin	$\lambda'$	131	12	49. 974
$s$ $\cos \alpha$ B	3. 953192 9. 2816903 8. 5097018	$s^2$ $\sin^2 \alpha$ C	7. 9064 9. 9838 1. 5597	$(\partial\phi)^2$ D	3. 488 2. 365	$-h$ $s^2 \sin^2 \alpha$ E	1. 744 7. 890 6. 467	
h	1. 7445841		9. 4499		5. 853		6. 101	
1st term 2d term	" —55. 5372 + 0. 2818	3d term 4th term	" +0. 0001 +0. 0001					
3d and 4th terms	—55. 2554 + 0. 0002		+2 3. 953192 9. 9919052 8. 5087347 0. 2429024	Arg. $s$ $\Delta\lambda$	—2 +4	$\Delta\lambda$ $\sin \frac{1}{2}(\phi+\phi')$ $\sec \frac{1}{2}(\Delta\phi)$	2. 696734 9. 914053	
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 55 $'$ 07 $''$ 47. 9		2. 6967345	Corr.	+2		2. 610787	
		$\Delta\lambda$	" —497. 4329			$-\Delta\alpha$	" —408. 12	

*Final position computation,*

## STATION SEAL

$\alpha$ Second angle	Nichols to Ken Ken and Seal					$^{\circ}$ 210 + 27	$'$ 49 51	$''$ 47.6 41.5
$\alpha$ $\Delta\alpha$	Nichols to Seal					238 +	41 29	29.1 04.7
$\alpha'$	Seal to Nichols					180 59	10	33.8 + .1
	First angle of triangle					25	37	10.6
$\phi$ $\Delta\phi$	$^{\circ}$ 54 +	$'$ 43 12	$''$ 30.691 22.365	Nichols	$\lambda$ $\Delta\lambda$	132 -	11 35	12.787 34.411
$\phi'$	54	55	53.056 -1	Seal	$\lambda'$	131	35	38.376 +1
$s$ $\cos \alpha$ B	4.648215 9.7157086 8.5097307	$s^2$ $\sin^2 \alpha$ C	9.2964 9.8633 1.5534	$(\partial\phi)^2$ D	5.747 2.368	$-h$ $s^2 \sin^2 \alpha$ E	2.874 9.160 6.455	
h	2.8736543		0.7131		8.115			8.489
1st term	"		"					
2d term	-747.5741 + 5.1654	3d term	+0.0130 +0.0308					
3d and 4th terms	-742.4087 + 0.0438							
$-\Delta\phi$	-742.3649	$s$ $\sin \alpha$ $\Lambda'$ $\sec \phi'$	4.648215 9.931652 8.508740 0.240667	Arg. $s$ $\Delta\lambda$	-4 +8	$\sin \frac{\Delta\lambda}{2}(\phi+\phi')$ $\sec \frac{\Delta\lambda}{2}(\Delta\phi)$	3.320278 9.912450	
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 54 49 41.9		3.329278 " $\Delta\lambda$ -2134.4108	Corr.	+4		3.241728 " $-\Delta\alpha$ -1744.72	

## STATION MID

$\alpha$ Second angle	Round to Lazaro Lazaro and Mid					$^{\circ}$ 353 + 51	$'$ 25 05	$''$ 52.5 14.8
$\alpha$ $\Delta\alpha$	Round to Mid					44 -	31 6	07.3 56.5
$\alpha'$	Mid to Round					180 224 85	24 15	10.8 12.2
	First angle of triangle							
$\phi$ $\Delta\phi$	$^{\circ}$ 55 -	$'$ 02 4	$''$ 56.147 57.777	Round	$\lambda$ $\Delta\lambda$	131 +	23 8	57.917 28.436
$\phi'$	54	57	58.370	Mid	$\lambda'$	131	32	26.353 -1
$s$ $\cos \alpha$ B	4.110647 9.853103 8.509707	$s^2$ $\sin^2 \alpha$ C	8.2213 9.6916 1.5586	$(\partial\phi)^2$ D	4.947 2.366	$-h$ $s^2 \sin^2 \alpha$ E	2.473 7.913 6.464	
h	2.473457		9.4715		7.313			6.850
1st term	"		"					
2d term	+297.4795 + 0.2961	3d term	+0.0021 -0.0007					
3d and 4th terms	+297.7756 + 0.0014							
$-\Delta\phi$	+297.7770	$s$ $\sin \alpha$ $\Lambda'$ $\sec \phi'$	4.110647 9.845806 8.508740 0.241043	Arg. $s$ $\Delta\lambda$		$\sin \frac{\Delta\lambda}{2}(\phi+\phi')$ $\sec \frac{\Delta\lambda}{2}(\Delta\phi)$	2.706236 9.913406	
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 55 00 27.3		2.706236 " $\Delta\lambda$ +508.4356	Corr.			2.619641 " $-\Delta\alpha$ +416.52	

## primary triangulation—Continued

## STATION SEAL

$\alpha$ Third angle	Ken to Nichols Seal and Nichols					$^{\circ}$ 30 -126	$'$ 59 31	$''$ 10.4 09.1
$\alpha$ $\Delta\alpha$	Ken to Seal					264 +	28 19	01.3 43.3
$\alpha'$	Seal to Ken					180 84	47	44.6 - .1
$\phi$ $\Delta\phi$	$^{\circ}$ 54 +	$'$ 54 1	$''$ 34.785 18.270	Ken	$\lambda$ $\Delta\lambda$	131 -	59 24	44.326 05.949
$\phi'$	54	55	53.055	Seal	$\lambda'$	131	35	38.377
$s$ $\cos \alpha$ B	4.412771 8.984161 8.509717	$s^2$ $\sin^2 \alpha$ C	8.8255 9.9960 1.5563	$(\partial\phi)^2$ D	3.813 2.367	$-h$ $s^2 \sin^2 \alpha$ E	1.907 8.821 6.460	
h	1.906649		0.3778		6.180		7.188	
1st term	"	3d term	"					
2d term	-80.6583 + 2.3867	4th term	+0.0002 +0.0015					
3d and 4th terms	-78.2716 + 0.0017	$s$ $\sin \alpha$ A'	+3 4.412771 9.997972 8.508740 0.240667	Arg. $s$ $\Delta\lambda$	-1 +4	$\Delta\lambda$ $\sin \frac{1}{2}(\phi+\phi')$ $\sec \frac{1}{2}(\Delta\phi)$	3.160153 9.912942	
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 54 $'$ 55 $''$ 13.9	$\sec \phi'$	3.160153	Corr.	+3		3.073095	
		$\Delta\lambda$	" -1445.949			$-\Delta\alpha$	" -1183.30	

## STATION MID

$\alpha$ Third angle  $\alpha$ $\Delta\alpha$  $\alpha'$  $\phi$ $\Delta\phi$  $\phi'$	Lazaro to Round Mid and Round				$\circ$ 173 — 43  129 —  180 309  131 +	$'$ 27 39  47 8  39  21 10	$''$ 30.3 33.5  56.8 33.9  22.9 + .1  58.417 27.934
	Lazaro to Mid						
	Mid to Lazaro						
	$\circ$ 54 +	$'$ 52 5	$''$ 57.820 00.550	Lazaro	$\lambda$ $\Delta\lambda$		
	54	57	58.370	Mid	$\lambda'$	131	32 26.351 +1
$s$ $\cos \alpha$ B	4.162604 9.806246 8.509719	$s^2$ $\sin^2 \alpha$ C	8.3252 9.7710 1.5559	$(\partial\phi)^2$ D	4.957 2.367	$-h$ $s^2 \sin^2 \alpha$ E	2.479 8.096 6.460
h	2.478569		9.6521		7.324		7.035
1st term	"	3d term	"				
2d term	—301.0017 + 0.4488	4th term	+0.0021 +0.0011				
3d and 4th terms	—300.5529 + 0.0032						
	—300.5497	$s$ $\sin \alpha$ A	4.162604 9.885527 8.508740 0.241043	Arg. $s$ $\Delta\lambda$		$\Delta\lambda$ $\sin \frac{1}{2}(\phi+\phi')$ $\sec \frac{1}{2}(\Delta\phi)$	2.797914 9.912963
	$\frac{1}{2}(\phi+\phi')$ 54 55 28.1	$\sec \phi'$	2.797914	Corr.			2.710877
		$\Delta\lambda$	" +627.9340			$-\Delta\alpha$	" +513.90

*Final position computation,*

## STATION SPUR

$\alpha$ Second angle	Round to Cat Cat and Spur						$^{\circ}$ 289 +111	$'$ 36 30	$''$ 37.8 08.5
$\alpha$ $\Delta\alpha$	Round to Spur						41 —	06 4	46.3 22.7
$\alpha'$	Spur to Round				First angle of triangle		180 221 33	02 20	23.6 38.3
$\phi$ $\Delta\phi$	$^{\circ}$ 55 —	$'$ 02 3	$''$ 56.147 31.319	Round	$\lambda$ $\Delta\lambda$		131 +	23 5	57.917 20.567
$\phi'$	54	59	24.828 —1	Spur	$\lambda'$		131	29	18.484
$s$ $\cos \alpha$ B	3.937953 9.877035 8.509707	$s^2$ $\sin^2 \alpha$ C	7.8759 9.6358 1.5586	$(\delta\phi)^2$ D	4.649 2.366		$-\frac{h}{s^2} \sin^2 \alpha$ E		2.325 7.512 6.464
h	2.324695		9.0703		7.015				6.301
1st term	"		"						
2d term	+211.2006 + 0.1176	3d term 4th term	+0.0010 —0.0002						
3d and 4th terms	+211.3182 + 0.0008								
$-\Delta\phi$	+211.3190	$s$ $\sin \alpha$ $\Lambda'$ $\sec \phi'$	3.937953 9.817925 8.508738 0.241303	Arg. $s$ $\Delta\lambda$			$\sin \frac{\Delta\lambda}{2} (\phi + \phi')$ $\sec \frac{1}{2}(\Delta\phi)$		2.505919 9.913468
$\frac{1}{2}(\phi + \phi')$	$^{\circ}$ 55 01 10.5		2.505919 "	Corr.					2.419387 "
		$\Delta\lambda$	+320.5672				$-\Delta\alpha$		+262.66

## STATION SNIPE

$\alpha$ Second angle	Round to Cat Cat and Snipe						$^{\circ}$ 289 + 63	$'$ 36 10	$''$ 37.8 27.2
$\alpha$ $\Delta\alpha$	Round to Snipe						352 +	47	05.0 29.8
$\alpha'$	Snipe to Round				First angle of triangle		180 172 79	47 10	34.8 09.9
$\phi$ $\Delta\phi$	$^{\circ}$ 55 —	$'$ 02 2	$''$ 56.147 45.140	Round	$\lambda$ $\Delta\lambda$		131 —	23	57.917 36.371
$\phi'$	55	00	11.007	Snipe	$\lambda'$		131	23	21.546
$s$ $\cos \alpha$ B	3.711593 9.996547 8.509707	$s^2$ $\sin^2 \alpha$ C	7.4232 8.1980 1.5586	$(\delta\phi)^2$ D	4.436 2.366		$-\frac{h}{s^2} \sin^2 \alpha$ E		2.218 5.621 6.464
h	2.217847		7.1798		6.802				4.303
1st term	"		"						
2d term	+165.1380 + 0.0015	3d term 4th term	+0.0006						
3d and 4th terms	+165.1395 + 0.0006								
$-\Delta\phi$	+165.1401	$s$ $\sin \alpha$ $\Lambda'$ $\sec \phi'$	3.711593 9.098982 8.508738 0.241442	Arg. $s$ $\Delta\lambda$			$\sin \frac{\Delta\lambda}{2} (\phi + \phi')$ $\sec \frac{1}{2}(\Delta\phi)$		1.560755 9.913502
$\frac{1}{2}(\phi + \phi')$	$^{\circ}$ 55 01 33.6		1.560755 "	Corr.					1.474257 "
		$\Delta\lambda$	—36.3710				$-\Delta\alpha$		—29.80

primary triangulation—Continued

## STATION SPUR

$\alpha$ Third angle	Cat to Round Spur and Round					$^{\circ}$ 109 — 35	$'$ 42 09	$''$ 37.4 13.4
$\alpha$ $\Delta\alpha$	Cat to Spur					74 —	33 10	24.0 22.1
$\alpha'$	Spur to Cat					180 254	23	01.9
$\phi$ $\Delta\phi$	$^{\circ}$ 55 —	$'$ 01 2	$''$ 26.110 01.283	Cat	$\lambda$ $\Delta\lambda$	131 +	16 12	39.129 39.355
$\phi'$	54	59	24.827	Spur	$\lambda'$	131	29	18.484
$s$ $\cos \alpha$ B	4.146374 9.425347 8.509709	$s^2$ $\sin^2 \alpha$ C	8.2927 9.9681 1.5582	$(\partial\phi)^2$ D	4.163 2.366	$-\frac{h}{s^2} \sin^2 \alpha$ E		2.081 8.261 6.464
h	2.081430 "		9.8190 "		6.529			6.806
1st term	+120.6230	3d term	+0.0003					
2d term	+ 0.6592	4th term	-0.0006					
3d and 4th terms	+121.2832 -0.0003		+1 4.146374 9.954029 8.508738 0.241303	Arg. $s$ $\Delta\lambda$	+1	$\sin \frac{\Delta\lambda}{s} (\phi + \phi')$ $\sec \frac{1}{2}(\Delta\phi)$		2.880445 9.913402
$-\Delta\phi$	+121.2829	$s$ $\sin \alpha$ A' $\sec \phi'$						
$\frac{1}{2}(\phi + \phi')$	$^{\circ}$ 55 00 25.5		2.880445 "	Corr.	+1			2.793847 "
		$\Delta\lambda$	+759.3552			$-\Delta\alpha$		+622.07

## STATION SNIPE

$\alpha$ Third angle	Cat to Round Snipe and Round					$^{\circ}$ 109 — 37	$'$ 42 39	$''$ 37.4 23.0
$\alpha$ $\Delta\alpha$	Cat to Snipe					72 —	03 5	14.4 29.7
$\alpha'$	Snipe to Cat					180 251	57	44.7
$\phi$ $\Delta\phi$	$^{\circ}$ 55 —	$'$ 01 1	$''$ 26.110 15.103	Cat	$\lambda$ $\Delta\lambda$	131 +	16 6	39.129 42.417
$\phi'$	55	00	11.007	Snipe	$\lambda'$	131	23	21.546
$s$ $\cos \alpha$ B	3.876157 9.488721 8.509709	$s^2$ $\sin^2 \alpha$ C	7.7523 9.9567 1.5582	$(\partial\phi)^2$ D	3.949 2.366	$-\frac{h}{s^2} \sin^2 \alpha$ E		1.975 7.709 6.464
h	1.874587 "		9.2672 "		6.315			6.148
1st term	+74.9181	3d term	+0.0002					
2d term	+ 0.1850	4th term	-0.0001					
3d and 4th terms	+75.1031 + 0.0001							
$-\Delta\phi$	+75.1032	$s$ $\sin \alpha$ A' $\sec \phi'$	3.876157 9.978339 8.508738 0.241442	Arg. $s$ $\Delta\lambda$		$\sin \frac{\Delta\lambda}{s} (\phi + \phi')$ $\sec \frac{1}{2}(\Delta\phi)$		2.604676 9.913436
$\frac{1}{2}(\phi + \phi')$	$^{\circ}$ 55 00 48.6		2.604676 "	Corr.				2.518112 "
		$\Delta\lambda$	+402.4167			$-\Delta\alpha$		+329.69

## Final position computation, primary triangulation—Continued

## STATION KHWAİN

$\alpha$ Second angle	Beaver to Cat Cat and Khwaın						° 17 + 58	' 19 43	" 11.6 19.0
$\alpha$ $\Delta\alpha$	Beaver to Khwaın						—	76 02 5	30.6 17.1
$\alpha'$	Khwaın to Beaver			First angle of triangle			180 255 62	57 13	13.5 31.4
$\phi$ $\Delta\phi$	° 55 —	' 05	" 11.314 55.322	Beaver	$\lambda$ $\Delta\lambda$		131 +	14 6	36.898 26.681
$\phi'$	55	04	15.992 —1	Khwaın	$\lambda'$		131	21	03.579
$s$ $\cos \alpha$ B	3.849454 9.3824014 8.5097044	$s^2$ $\sin^2 \alpha$ C	7.6989 9.9740 1.5591	$(\delta\phi)^2$ D	3.483 2.366		$-\frac{h}{s^2} \sin^2 \alpha$ E		1.742 7.673 6.465
h	1.7415598		9.2320		5.849				5.880
1st term	"		"						
2d term	+ 55.1518 + 0.1706	3d term	+ 0.0001 — 0.0001						
3d and 4th terms	+ 55.3224 0.0000								
$-\Delta\phi$	+ 55.3224	$s$ A' sec $\phi'$	3.849454 9.986983 8.508736 0.242180	Arg. $s$ $\Delta\lambda$			$\sin \frac{\Delta\lambda}{2} (\phi + \phi')$ sec $\frac{1}{2}(\Delta\phi)$		2.587353 9.913782
$\frac{1}{2}(\phi + \phi')$	° ' " 55 04 43.7		2.587353	Corr.					2.501135
		$\Delta\lambda$	" + 386.6812				$-\Delta\alpha$		" + 317.06

## STATION KHWAİN\*

$\alpha$ Third angle	Cat to Beaver Khwaın and Beaver						° 197 — 59	' 17 03	" 31.4 09.7
$\alpha$ $\Delta\alpha$	Cat to Khwaın						138 —	14 3	21.7 36.8
$\alpha'$	Khwaın to Cat						180 318	10	44.9
$\phi$ $\Delta\phi$	° 55 +	' 01 2	" 26.110 49.881	Cat	$\lambda$ $\Delta\lambda$		131 +	16 4	39.129 24.450
$\phi'$	55	04	15.991	Khwaın	$\lambda'$		131	21	03.579
$s$ $\cos \alpha$ B	3.847941 9.872700 8.509709	$s^2$ $\sin^2 \alpha$ C	7.6959 9.6470 1.5584	$(\delta\phi)^2$ D	4.461 2.366		$-\frac{h}{s^2} \sin^2 \alpha$ E		2.230 7.343 6.464
h	2.230350		8.9013		6.827				6.037
1st term	"		"						
2d term	— 169.9613 + 0.0797	3d term	+ 0.0007 + 0.0001						
3d and 4th terms	— 169.8816 + 0.0008								
$-\Delta\phi$	— 169.8808	$s$ A' sec $\phi'$	3.847941 9.823487 8.508736 0.242180	Arg. $s$ $\Delta\lambda$			$\sin \frac{\Delta\lambda}{2} (\phi + \phi')$ sec $\frac{1}{2}(\Delta\phi)$		2.422344 9.913616
$\frac{1}{2}(\phi + \phi')$	° ' " 55 02 51.1		2.422344	Corr.					2.335960
		$\Delta\lambda$	" + 264.4502				$-\Delta\alpha$		" + 216.75

\* This is right-hand portion of computation above.

*List of geographic positions—Felice Strait, Alaska, southeast Alaska datum*

Station	Latitude and longitude			Sec- onds in meters	Azimuth			Back azimuth			To station	Distance	Loga- rithm
	°	'	"		°	'	"	°	'	"		<i>Meters</i>	
Tower 1907	54 35 27.826			845.0	201 43 27.2			21 50 34.8			Turn Dundas	25288.4	4.402921
	131 04 48.015			862.3	243 43 58.0			63 51 41.0				11361.1	4.055419
Lazaro 1907	54 52 57.820			1788.0	287 48 01.7			108 09 12.5			Turn Dundas Tower	29163.6	4.464841
	131 21 58.417			1041.5	313 40 37.9			134 02 23.3				39641.7	4.598152
					330 18 17.4			150 32 18.7				37351.0	4.572302
Tow Hill 1908	54 04 25.798			797.6	197 07 18.4			17 28 25.9			Lazaro Tower	94307.0	4.974544
	131 47 55.665			1012.2	218 47 56.1			39 22 58.4				74158.2	4.870159
Nichols 1907	54 43 30.691			949.0	251 17 00.2			71 57 14.5			Lazaro Tower	55612.1	4.745169
	132 11 12.787			228.9	281 21 55.6			102 16 06.1			Tow Hill	72983.6	4.863225
					340 40 20.8			160 59 16.8				76761.2	4.885142
Ken 1907	54 54 34.785			1075.7	273 59 19.6			94 30 13.3			Lazaro Nichols	40495.5	4.607407
	131 59 44.326			789.7	30 59 10.4			210 49 47.6				23934.3	4.379021
Seal 1907	54 55 53.055			1640.6	290 15 23.0			110 26 33.9			Lazaro Nichols	15582.6	4.192639
	131 35 38.377			653.4	59 10 33.9			238 41 29.1			Ken	44485.1	4.648215
					84 47 44.5			264 28 01.3				25868.5	4.412771
Mid 1914	54 57 58.370			1805.0	309 39 23.0			129 47 56.8			Lazaro Ken	14541.3	4.162604
	131 32 26.352			468.8	78 00 16.4			257 37 55.6				29834.6	4.474720
Round 1914	55 02 56.147			1736.3	353 25 52.5			173 27 30.3			Lazaro Mid	18624.0	4.270074
	131 23 57.917			1028.3	44 31 07.3			224 24 10.8			Ken	12901.7	4.110647
					68 08 29.3			247 39 11.5				41208.8	4.614987
Spur 1914	54 59 24.827			767.7	221 02 23.6			41 06 46.3			Round Lazaro	8668.7	3.937953
	131 29 18.484			328.7	326 44 11.7			146 50 11.9				14304.3	4.155468
Cat 1914	55 01 26.110			807.4	19 54 45.7			199 50 24.3			Lazaro Spur	16713.7	4.223072
	131 16 39.129			695.2	74 33 24.0			254 23 01.9			Round	14007.9	4.146374
					109 42 37.4			289 36 37.8				8275.6	3.917800
Snipe 1914	55 00 11.007			340.4	172 47 34.8			352 47 05.0			Round Cat	5147.5	3.711593
	131 23 21.546			383.0	251 57 44.7			72 03 14.4				7518.9	3.876157
Beaver 1914	55 05 11.314			349.9	17 19 11.6			197 17 31.4			Cat Snipe	7294.4	3.862988
	131 14 36.898			654.5	45 09 02.7			225 01 52.7			Round	13154.2	4.119063
					67 17 21.8			247 09 44.8			Ham	10798.1	4.033348
					148 12 30.2			328 08 19.3				10275.3	4.011793
					198 23 34.8			18 25 02.5			South Twin	6003.4	3.778398
Khwain 1914	55 04 15.991			494.5	255 57 13.5			76 02 30.6			Beaver Cat	7070.6	3.849454
	131 21 03.579			63.5	318 10 44.9			138 14 21.7				7046.0	3.847941
Lim 1914	55 07 20.262			626.6	197 33 05.0			17 34 14.5			Ham South Twin	4974.6	3.696759
	131 21 07.407			131.3	258 58 19.1			79 05 07.2			Beaver	8978.3	3.953192
					299 53 37.5			119 58 57.8				7990.1	3.902553
					336 28 34.1			156 32 14.1			Cat	11941.6	4.077064
					359 19 01.9			179 19 05.0			Khwain	5698.8	3.755783

## ADJUSTMENT OF TRIANGULATION BY THE METHOD OF VARIATION OF GEOGRAPHIC COORDINATES

## DEVELOPMENT OF FORMULAS

A scheme of triangulation may be adjusted not only by means of equations of condition \* but also by means of observation equations in which the number of independent unknowns is just sufficient to

\* There is some confusion in usage as to the term *equation of condition*, or *condition equation*. In this publication the meaning is restricted to that of an equation expressing some condition which is imposed *a priori* and independently of anything arising from the observations themselves, and which must be rigorously satisfied by the adopted results. An equation which expresses the results of an observation, and which will, in general, be satisfied only approximately by the adopted results, is not herein termed an equation of condition, but an *observation equation*.

determine the entire triangulation. These independent unknowns may very conveniently be taken as the small corrections to the assumed approximate geographic coordinates (that is, the latitudes and longitudes) of the points in the triangulation. To form the observation equations the relation must be found that connects the small change in the direction of a line with the small arbitrary changes in the geographic coordinates of its ends. The following derivation of the formulas is based on the formulas for the computation of geographic positions given in U. S. Coast and Geodetic Survey Special Publication No. 8 and on the notation there used. A "δ" before the symbol of a quantity denotes a small arbitrary change in that quantity.  $\phi$  and  $\lambda$  are, respectively, the latitude and longitude of  $A_1$ , the initial point of the position computation, which may also be thought of as the occupied point, while  $\phi'$  and  $\lambda'$  are the latitude and longitude of  $B_1$ , the terminal point in the position computation, which may also be thought of as the point sighted on. By definition also,

$$\Delta\phi = \phi' - \phi$$

$$\Delta\lambda = \lambda' - \lambda$$

$$h = sB \cos \alpha$$

$\alpha$  is the azimuth at  $A_1$  of the line  $A_1B_1$  reckoned from the south toward the west.

$$\Delta\lambda = sA' \sec \phi' \sin \alpha$$

$$\cos \alpha = \frac{h}{sB}$$

$$\sin \alpha = \frac{\Delta\lambda}{sA' \sec \phi'}$$

$$\cot \alpha = \frac{A' \sec \phi'}{B} \frac{h}{\Delta\lambda}$$

The meaning of  $A'$  and  $B$  is explained in Special Publication No. 8.

By differentiating the preceding equation and neglecting the effects of changes in  $A'$ ,  $B$ , and  $\sec \phi'$  there results:

$$-\operatorname{cosec}^2 \alpha \, d\alpha = \frac{A' \sec \phi'}{B} \left[ \frac{\Delta\lambda \delta h - h \delta(\Delta\lambda)}{(\Delta\lambda)^2} \right]$$

$$\text{Multiplying by } -\sin^2 \alpha = -\frac{(\Delta\lambda)^2}{s^2 A'^2 \sec^2 \phi'}$$

and dividing by arc  $1''$  in order to express  $d\alpha$  in seconds instead of in radians gives,

$$\begin{aligned} d\alpha \text{ in seconds} &= \frac{1}{s^2 BA' \sec \phi' \text{ arc } 1''} [h\delta(\Delta\lambda) - \Delta\lambda\delta h] \\ &= \frac{sB \cos \alpha}{s^2 BA' \sec \phi' \text{ arc } 1''} \delta(\Delta\lambda) - \frac{sA' \sin \alpha \sec \phi'}{s^2 BA' \sec \phi' \text{ arc } 1''} \delta h \\ &= \frac{\sin \alpha \cos \alpha}{sA' \sec \phi' \sin \alpha \text{ arc } 1''} \delta(\Delta\lambda) - \frac{\sin \alpha \cos \alpha}{sB \cos \alpha \text{ arc } 1''} \delta h \\ &= \frac{\sin \alpha \cos \alpha}{\text{arc } 1''} \left[ \frac{\delta(\Delta\lambda)}{\Delta\lambda} - \frac{\delta h}{h} \right] \end{aligned}$$

By neglecting the variations in all the terms of the expression given for  $\Delta\phi$  in Special Publication No. 8 except the first or principal term,  $h$ , there results,

$$\delta(\Delta\phi) = -\delta h = \delta\phi' - \delta\phi$$

Evidently, also,

$$\delta(\Delta\lambda) = \delta\lambda' - \delta\lambda$$

It thus appears that, to the degree of approximation here adopted, it is the difference in the changes of coordinates at the ends of a line that turns the line in azimuth. The formulas for computing  $d\alpha$  become,

$$\begin{aligned} d\alpha \text{ in sec.} &= \frac{1}{s^2 BA' \sec \phi' \text{ arc } 1''} [\Delta\lambda(\delta\phi' - \delta\phi) + h(\delta\lambda' - \delta\lambda)] \\ &= \frac{\sin \alpha \cos \alpha}{\text{arc } 1''} \left[ \frac{\delta\phi' - \delta\phi}{h} + \frac{\delta\lambda' - \delta\lambda}{\Delta\lambda} \right] \end{aligned}$$

In practice  $-\Delta\phi$  may be used for  $h$ , but if a position computation has been made over the line,  $\log h$  will be immediately available. The change in the azimuth  $\alpha'$  at  $B_1$  of the line  $B_1 A_1$  for given changes in the coordinates of  $A_1$  and  $B_1$  may usually be taken the same as the change in  $\alpha$ , the azimuth at  $A_1$  of the line  $A_1 B_1$ .\* If the point  $A_1$  is fixed  $\delta\phi$  and  $\delta\lambda$  are zero, and if  $B_1$  is fixed  $\delta\phi'$  and  $\delta\lambda'$  are zero.

This formula will now be applied to three examples, first, the adjustment of a quadrilateral, next the adjustment of three new points connected with a number of fixed points, and, lastly, to a figure involving a closure in geographic position. The steps to be taken and the precautions to be observed will be explained as they arise in the course of the examples.

\* For more exact formula to be used with longer lines, see Dr. F. R. Helmert's *Höhere Geodäsie*, vol. 1, pp. 495 and 496. For such lines some of the approximations made in the derivation here given are no longer permissible.

In all cases treated by this method, however complicated they may be, a start is made with the assumed positions of the points to be determined and the assumed azimuths and lengths of the lines sighted over. These positions, azimuths, and lengths must be consistent with each other and not too far from the final result so that the corrections to the assumed quantities are in fact small, as is implied in the development of the formulas. Otherwise it is not important how these preliminary quantities are found.

#### ADJUSTMENT OF A QUADRILATERAL WITH TWO POINTS FIXED

As a simple example a quadrilateral,  $A_1, A_2, A_3, A_4$ , with two points,  $A_1$  and  $A_2$ , fixed is adjusted. The coordinates of  $A_1$  and  $A_2$  and the length and direction of the line  $A_1-A_2$  are fixed as shown in the first lines of the position computation that follows. The angles of the preliminary computation of the triangles are obtained from the list of directions. To obtain the preliminary positions, directions and lengths, the triangles  $A_1, A_2, A_3$ , and  $A_2, A_3, A_4$  were made to close by correcting each angle by approximately one-third of the error of closure as indicated in the triangle computation. This determined the entire quadrilateral. In each of the other triangles two sides and an included angle became known and thus their remaining parts were computed.

#### *List of observed directions\**

AT $A_2$		AT $A_3$	
Station	Direction †	Station	Direction †
Initial	0 00 00.0+ $z_1$	Initial	0 00 00.0+ $z_3$
$A_1$	0 00 00.0+ $v_1$	$A_4$	0 00 00.0+ $v_7$
$A_3$	101 44 45.1+ $v_2$	$A_2$	31 03 42.5+ $v_8$
$A_4$	133 53 46.3+ $v_3$	$A_1$	61 47 35.0+ $v_9$

AT $A_1$		AT $A_4$	
Initial	Direction †	Initial	Direction †
$A_3$	0 00 00.0+ $z_2$	$A_2$	0 00 00.0+ $z_4$
$A_4$	0 00 00.0+ $v_4$	$A_3$	0 00 00.0+ $v_{10}$
$A_2$	26 40 22.5+ $v_5$	$A_1$	25 15 16.2+ $v_{11}$
$A_3$	47 31 20.2+ $v_6$	$A_4$	116 47 20.0+ $v_{12}$

\*See fig. 1 on p. 16.

† Each observed value has its symbolic correction affixed.

*Preliminary computation of triangles*

Station	Observed angle			Correc- tion	Spher- ical angle	Spher- ical excess	Plane angle	Loga- rithm
	°	'	"	"	"	"	"	
$A_2-A_1$	30	43	52.5	+0.8	53.3			3.772745
$A_3$	101	44	45.1	+0.7	45.8	0.1	45.7	0.291566
$A_1$	47	31	20.2	+0.8	21.0			9.990809
				+2.3		0.1		9.867787
$A_3-A_1$								4.055120
$A_3-A_2$								3.932098
$A_2-A_1^*$								3.772745
$A_4$	25	15	16.2		17.3			0.369934
$A_2$	133	53	46.3		45.8	0.1	45.7	9.857694
$A_1$	20	50	56.7		57.0			9.551339
				+0.9		0.1		4.000373
$A_4-A_1$								2.694018
$A_4-A_2$								
$A_2-A_3$								3.932098
$A_4$	116	47	20.0	-1.2	18.8	0.1	18.7	0.049306
$A_2$	32	09	01.2	-1.2	00.0			9.726024
$A_3$	31	03	42.5	-1.2	41.3			9.712614
				-3.6		0.1		
$A_4-A_3$								3.707428
$A_4-A_2$								3.694018
$A_1-A_3^*$								4.055120
$A_4$	91	32	03.8		01.5	0.1	01.4	0.000156
$A_1$	26	40	23.5		24.0			9.652153
$A_3$	61	47	35.0		34.6			9.945097
				-2.2		0.1		
$A_4-A_3$								3.707429-1
$A_4-A_1$								4.000373

\* This triangle is computed from two sides and the included angle.

91865°—15—7

*Preliminary position computation,*STATION  $A_3$ 

$\alpha$ Second angle	$A_2$ to $A_1$ $A_1$ and $A_3$				$^{\circ}$ 156 +101	$'$ 20 44	$''$ 26.6 45.8	
	$A_2$ to $A_3$				258 +	05 8	12.4 05.9	
	$A_3$ to $A_2$				180 78 30	00 13 43	00.00 18.3 53.3	
	First angle of triangle							
$\phi$ $\Delta\phi$	$^{\circ}$ 60 +	$'$ 56	$''$ 01.089 56.720	$A_2$	$\lambda$ $\Delta\lambda$	149 —	34 9	19.237 15.877
$\phi'$	60	56	57.809	$A_3$	$\lambda'$	149	25	03.360
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 60	$'$ 56	$''$ 29	$\frac{s}{\cos B}$	$\frac{s^2}{C}$	7.86420 9.98109 1.65750	$h^2$ D	3.512 2.322
1st term	—57.0388			h	1.756170	9.05279		5.834
2d and 3d terms	+ 0.3184					+0.3183 +0.0001		
— $\Delta\phi$	—56.7204							
	$\frac{s}{\sin \alpha}$ $A'$ $\sec \phi'$	3.932098 9.990544 8.508600 0.313737		$\sin \frac{\Delta\lambda}{\frac{1}{2}(\phi+\phi')}$	2.744979 9.941572			
		2.744979			2.686551			
		"			"			
	$\Delta\lambda$	—555.8774		— $\Delta\alpha$	—485.90			

STATION  $A_4$ 

$\alpha$ Second angle	$A_2$ to $A_3$ $A_3$ and $A_4$					$^{\circ}$ 258 + 32	$'$ 05 09	$''$ 12.4 00.0		
$\alpha$ $\Delta\alpha$	$A_2$ to $A_4$					290 +	14 4	12.4 29.0		
$\alpha'$	$A_4$ to $A_2$					180 110 116	00 18 47	00.00 41.4 18.8		
First angle of triangle										
$\phi$ $\Delta\phi$	$^{\circ}$ 60 —	$'$ 56	$''$ 01.089 55.340	$A_2$	$\lambda$ $\Delta\lambda$	149 —	34 5	19.237 07.794		
$\phi'$	60	55	05.749	$A_4$	$\lambda'$	149	29	11.443		
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 60	$'$ 55	$''$ 33	$\sin \alpha$ B	$\frac{s^2}{C}$ 3.694018 9.538951 8.509299	$\sin^2 \alpha$ 7.38804 9.94466 1.65750	$h^2$ D	$^{\circ}$ 3.484 2.322		
1st term	+55.2418			h	1.742268	8.99020		5.806		
2d and 3d terms	+ 0.0979									
— $\Delta\phi$	+55.3397									
$\frac{s}{\sin \alpha}$ $A'$ $\sec \phi'$	3.694018 9.972328 8.508601 0.313313			$\sin \frac{1}{2}(\phi+\phi')$	2.488260 9.941507					
	2.488260				2.429767					
	"				"					
$\Delta\lambda$	—307.7939			— $\Delta\alpha$	—269.0					



## FORMATION OF OBSERVATION EQUATIONS

The observation equations used in making the adjustments are formed on the assumptions of the direction method.\* Each pointing of the telescope is treated as an independent observation and the sum of the squares of the corrections to the separate pointings is to be made a minimum. A single pointing, however, taken by itself determines nothing, for if each of the pointings at a station be changed by the same amount the set of pointings has the same significance as before. The effect is simply a change in the zero direction, which is a purely arbitrary matter. If a set of corrections to directions at a point has been determined by any method and the mean of these corrections is not zero, the sum of the squares of these corrections can always be diminished by subtracting from each correction the mean of all of the corrections so that the algebraic sum of the reduced corrections is zero. Hence in any set of directions adjusted by the method of least squares the algebraic sum of the corrections at a point is zero.† To allow for this change of zero direction, or for the constant correction to all directions at a point, an unknown constant correction, “ $z$ ,” is introduced into all equations expressing the results of observations at a point, a different “ $z$ ” for each point where observations are taken.

The observation equation may be written,

$$\text{Assumed azimuth} + d\alpha - \text{observed azimuth} + z - v = 0.$$

The coefficients of the  $\delta\phi$ 's and  $\delta\lambda$ 's come from the last equation on page 93. As a sample, take those in the expression for  $v_3$ .  $A_1$  corresponds to the  $A_1$  and  $A_3$  to the  $B_1$  of the explanation of the formulas.  $\sin \alpha$ ,  $\cos \alpha$ ,  $h$ , and  $\Delta\lambda$  come from the position computation on page 97.

$\log \sin \alpha$	9. 9762 <i>n</i>		4. 7984 <i>n</i>		4. 7984 <i>n</i>
$\log \cos \alpha$	9. 5078	$\log h$	2. 0722	$\log \Delta\lambda$	2. 8537 <i>n</i>
$\text{colog arc } 1''$	5. 3144				
			2. 7262 <i>n</i>		1. 9447
	4. 7984 <i>n</i>	Number	-532	Number	+88

The observed angles in the following formation of equations come from the list of directions on page 94.

Azimuth $A_2$ to $A_1$ (initial direction).....	156	20	26. 6
Observed angle initial direction to $A_1$ .....	0	00	00. 0- $z_1+v_1$
Observed azimuth $A_2$ to $A_1$ .....	156	20	26. 6- $z_1+v_1$
Assumed azimuth $A_2$ to $A_1$ .....	156	20	26. 6+ $d\alpha$
Assumed azimuth-observed azimuth.....			0=0. 0+ $d\alpha+z_1-v_1$

\* See Wright and Hayford, Adjustment of Observations, Chap. VII.

† This does not necessarily hold good when a line whose direction has already been fixed enters into the set.

Azimuth  $A_2$  to  $A_1$  is fixed. Therefore  $d\alpha = 0$  and  $v_1 = z_1$

Azimuth $A_2$ to $A_1$ (initial direction).....	156	20	26.6
Observed angle initial direction to $A_3$ .....	101	44	45.1 - $z_1 + v_2$
Observed azimuth $A_2$ to $A_3$ .....	258	05	11.7 - $z_1 + v_2$
Assumed azimuth $A_2$ to $A_3$ .....	258	05	12.4 + $d\alpha$
Assumed azimuth—observed azimuth.....			0 = +0.7 + $d\alpha + z_1 - v_2$

$d\alpha = -730\delta\phi_3 - 75\delta\lambda_3$ . Therefore  $v_2 = z_1 - 730\delta\phi_3 - 75\delta\lambda_3 + 0.7$ .

Azimuth $A_2$ to $A_1$ (initial direction).....	156	20	26.6
Observed angle initial direction to $A_4$ .....	133	53	46.3 - $z_1 + v_3$
Observed azimuth $A_2$ to $A_4$ .....	290	14	12.9 - $z_1 + v_3$
Assumed azimuth $A_2$ to $A_4$ .....	290	14	12.4 + $d\alpha$
Assumed azimuth—observed azimuth.....			0 = -0.5 + $d\alpha + z_1 - v_3$

$d\alpha = -1212\delta\phi_4 + 218\delta\lambda_4$ . Therefore  $v_3 = z_1 - 1212\delta\phi_4 + 218\delta\lambda_4 - 0.5$

In the same way at  $A_1$   $v_4 = z_2 - 532\delta\phi_3 + 88\delta\lambda_3 - 0.8$

$v_5 = z_2 - 447\delta\phi_4 + 221\delta\lambda_4 - 0.3$

$v_6 = z_2$

Azimuth $A_3$ to $A_4$ (initial direction).....	47	09	37.0
Observed angle initial direction to $A_4$ .....	0	00	00.0 - $z_3 + v_7$
Observed azimuth $A_3$ to $A_4$ .....	47	09	37.0 - $z_3 + v_7$
Assumed azimuth $A_3$ to $A_4$ .....	47	09	37.0 + $d\alpha$
Assumed azimuth—observed azimuth.....			0 = 0.0 + $d\alpha + z_3 - v_7$

$d\alpha = +918(\delta\phi_4 - \delta\phi_3) + 414(\delta\lambda_4 - \delta\lambda_3)$ . Therefore  $v_7 = z_3 - 918\delta\phi_3 - 414\delta\lambda_3 + 918\delta\phi_4 + 414\delta\lambda_4 + 0.0$

Similarly

$$v_8 = z_3 - 730\delta\phi_3 - 75\delta\lambda_3 - 1.2$$

$$v_9 = z_3 - 532\delta\phi_3 + 88\delta\lambda_3 - 0.4$$

$$v_{10} = z_4 - 1211\delta\phi_4 + 218\delta\lambda_4 + 0.0$$

$$v_{11} = z_4 - 447\delta\phi_4 + 221\delta\lambda_4 + 1.1$$

$$v_{12} = z_4 - 918\delta\phi_3 - 414\delta\lambda_3 + 918\delta\phi_4 + 414\delta\lambda_4 - 1.2$$

We have then the set of observation equations:

$$v_1 = z_1$$

$$v_2 = z_1 - 730\delta\phi_3 - 75\delta\lambda_3 + 0.7$$

$$v_3 = z_1 - 1212\delta\phi_4 + 218\delta\lambda_4 - 0.5$$

$$v_4 = z_2 - 532\delta\phi_3 + 88\delta\lambda_3 - 0.8$$

$$v_5 = z_2 - 447\delta\phi_4 + 221\delta\lambda_4 - 0.3$$

$$v_6 = z_2$$

$$v_7 = z_3 - 918\delta\phi_3 - 414\delta\lambda_3 + 918\delta\phi_4 + 414\delta\lambda_4 + 0.0$$

$$v_8 = z_3 - 730\delta\phi_3 - 75\delta\lambda_3 - 1.2$$

$$v_9 = z_3 - 532\delta\phi_3 + 88\delta\lambda_3 - 0.4$$

$$v_{10} = z_4 - 1211\delta\phi_4 + 218\delta\lambda_4 + 0.0$$

$$v_{11} = z_4 - 447\delta\phi_4 + 221\delta\lambda_4 + 1.1$$

$$v_{12} = z_4 - 918\delta\phi_3 - 414\delta\lambda_3 + 918\delta\phi_4 + 414\delta\lambda_4 - 1.2$$

These equations contain  $z$ 's which are of no particular interest in themselves. The normal equations might be formed and the  $z$ 's eliminated in the regular way, but this work is made easier by the following mechanical rule, the effect of which is to form at once the

reduced normal equations with the  $z$ 's eliminated. For the proof of the rule and further particulars see Jordan's *Handbuch der Vermessungskunde*, Vol. I, pages 151-171, of the third edition. Each direction is assumed to have equal weight. Write the observation equations dropping the  $z$ 's and giving each unit weight. Add together as they stand all observation equations containing the  $z$  for any particular point. Drop the  $z$  term out and treat the resulting equation as a new observation equation with a negative weight equal to  $-1/r$ , where  $r$  is the total number of directions, both fixed and to be determined, that have been observed at the point in question. To reduce the new fictitious observation to unit weight it

must be multiplied through by  $\sqrt{\frac{-1}{r}} = \frac{1}{r}\sqrt{r}i$  where  $i = \sqrt{-1}$ .

Table 1 below shows the coefficients of the unknowns, the coefficients formed by adding the equations containing any particular  $z$ , and the weights. Table 2 shows these equations divided through by 100 for convenience. This has no effect on the relative weights. The table also contains the fictitious observation equa-

tions obtained by multiplying the sum equation by  $\sqrt{\frac{-1}{r}}$ . From

Table 2 the normal equations which do not contain the  $z$ 's are formed in the ordinary way for observation equations of equal weight, using the  $i$ 's strictly according to algebraic laws. Thus in the first line of Table 2  $(-4.23i)^2$  contributes to the first diagonal coefficient not  $+17.8929$  but  $+17.8929i^2$ , or  $-17.8929$ , and  $(-4.23i) \times + (1.26i)$  contributes toward the side coefficient not  $-5.3298$ , but  $-5.3298i^2$  or  $+5.3298$ .

Table for formation of normals, No. 1

		$\delta\phi_3$	$\delta\lambda_3$	$\delta\phi_4$	$\delta\lambda_4$	$l$	$p$	$\sqrt{p}$
Sum	$z_1$							
	1						1	1
	1	- 730	- 75	-1212	+218	+0.7	1	1
	1					-0.5	1	1
Sum	3	- 730	- 75	-1212	+218	+0.2	$-\frac{1}{3}$	0.58i
	$z_2$							
	1	- 532	+ 88			-0.8	1	1
	1			- 447	+221	-0.3	1	1
Sum	3	- 532	+ 88	- 447	+221	-1.1	$-\frac{1}{3}$	0.58i
	$z_3$							
	1	- 918	-414	+ 918	+414	+0.0	1	1
	1	- 730	- 75			-1.2	1	1
Sum	3	- 532	+ 88			-0.4	1	1
	3	-2180	-401	+ 918	+414	-1.6	$-\frac{1}{3}$	0.58i
	$z_4$							
	1			-1211	+218	+0.0	1	1
Sum	1			- 447	+221	+1.1	1	1
	1	- 918	-414	+ 918	+414	-1.2	1	1
	3	- 918	-414	- 740	+853	-0.1	$-\frac{1}{3}$	0.58i

Table for formation of normals, No. 2

	$\delta\phi_3$	$\delta\lambda_3$	$\delta\phi_4$	$\delta\lambda_4$	$l$	$\Sigma$
2	- 7.30	-0.75			+0.007	- 8.043
3			-12.12	+2.18	-0.005	- 9.945
$z_1$	- 4.23i	-0.44i	- 7.03i	+1.26i	+0.00116i	-10.43884i
4	- 5.32	+0.88			-0.008	- 4.448
5			- 4.47	+2.21	-0.003	- 2.263
$z_2$	- 3.09i	+0.51i	- 2.59i	+1.28i	-0.00638i	- 3.89638i
7	- 9.18	-4.14	+ 9.18	+4.14	+0.0	0.0
8	- 7.30	-0.75			-0.012	- 8.062
9	- 5.32	+0.88			-0.004	- 4.441
$z_3$	-12.64i	-2.33i	+ 5.32i	+2.40i	-0.00928i	- 7.25928i
10			-12.11	+2.18	+0.0	- 9.93
11			- 4.47	+2.21	+0.011	- 2.249
12	- 9.18	-4.14	+ 9.18	+4.14	-0.012	- 0.012
$z_4$	- 5.32i	-2.40i	- 4.29i	+4.95i	-0.00058i	- 7.06058i

## Normal equations

	$\delta\phi_3$	$\delta\lambda_3$	$\delta\phi_4$	$\delta\lambda_4$	$\eta$	$\Sigma$
1	+116.2166	+35.0927	-161.8628	-10.0554	+0.0753	- 20.5336
2		+25.3104	- 75.6831	-16.9056	+0.0236	- 32.1620
3			+399.2176	+24.0721	-0.0468	+185.6970
4				+20.0637	-0.01105	+ 17.16375

The forward and back solution of the normals, conducted according to the Doolittle method, is next shown.

To compute the  $v$ 's from the observation equations a knowledge of the  $z$ 's is required. Substitute the  $\delta\phi$ 's and  $\delta\lambda$ 's in the right-hand side of the sum equation formed from the observation equations that contain the  $z$  in question as if the  $z$  were not there and divide the result of the substitution by the weight  $-r$ . As a check the sum of the  $v$ 's about a point should equal zero. The computation of the  $v$ 's is shown in the table on page 102. Below each  $v$  as computed to 3 decimals is given its value as adopted and reduced to 1 decimal.

Following the computation of the  $v$ 's there is given a computation of the triangles using the adjusted directions.

## Solution of normals

$\delta\phi_3$	$\delta\lambda_3$	$\delta\phi_4$	$\delta\lambda_4$	$\eta$	$\Sigma$
+116.2166 $\delta\phi_3$  1	+35.0927	-161.8628	-10.0544	+0.0753	- 20.5336
	- 0.301959	+ 1.392768	+ 0.086523	-0.000648	+ 0.176684
	+25.3104	- 75.6831	-16.9056	+0.0236	- 32.1620
	-10.5966	+ 48.8759	+ 3.0363	-0.0227	+ 6.2003
	+14.7138	- 26.8072	-13.8693	+0.0009	- 25.9618
	$\delta\lambda_3$	+ 1.821909	+ 0.942605	-0.000061	+ 1.764452
	1 2	+399.2176	+24.0721	-0.0468	+185.6970
		-225.4373	-14.0048	+0.1049	- 28.5985
		- 48.8403	-25.2686	+0.0016	- 47.3000
		+124.9400	-15.2013	+0.0597	+109.7984
		$\delta\phi_4$	+ 0.121669	-0.000478	- 0.878809
		1 2 3	+20.0637	-0.01105	+ 17.16375
			- 0.8700	+0.00652	- 1.77663
			-13.0733	+0.00085	- 24.47172
			- 1.8495	+0.00726	+ 13.35904
			+ 4.2709	+0.00358	+ 4.27448
		$\delta\lambda_4$		-0.000838	- 1.000838

*Back solution*

$\delta\lambda_4$	$\delta\phi_4$	$\delta\lambda_3$	$\delta\phi_3$
-0.00084	-0.00048	-0.00006	-0.00065
-0.00084	-0.00010	-0.00079	-0.00007
	-0.00058	-0.00106	-0.00081
		-0.00191	+0.00058
			-0.00095

*Computation of corrections*

1= $z_1$	2	3	$z_1$
-0.519	+0.6935	+0.7030	+0.6935
-0.5	+0.1432	-0.1831	+0.1432
	+0.7	-0.5	+0.7080
	-0.519	-0.519	-0.1831
	+1.018	-0.499	+0.2
	+1.0	-0.5	+1.5566 $\div$ -3
			-0.519
4	5	6= $z_2$	$z_2$
+0.5054	+0.2593	+0.230	+0.5054
-0.1681	-0.1856	+0.3	-0.1681
-0.8	-0.3		+0.2593
+0.230	+0.230		-0.1856
			-1.1
-0.233	+0.004		-0.689 $\div$ -3
-0.2	0.0		+0.230
7	8	9	$z_3$
+0.8721	+0.6935	+0.5054	+2.0710
+0.7907	+0.1432	-0.1681	+0.7659
-0.5324	-1.2	-0.4	-0.5324
-0.3478	-0.119	-0.119	-0.3478
-0.119			-1.6
	-0.482	-0.182	
+0.664	-0.5	-0.2	+0.3567 $\div$ -3
+0.7			-0.119
10	11	12	$z_4$
+0.7024	+0.2593	+0.8721	+0.8721
-0.1831	-0.1856	+0.7907	+0.7907
-0.425	+1.1	-0.5324	+0.4292
	-0.425	-0.3478	-0.7165
+0.094		-1.2	-0.1
+0.1	+0.749	-0.425	
	+0.7	-0.842	+1.2755 $\div$ -3
		-0.8	-0.425

*Adjusted computation of triangles*

Symbol	Station	Observed angle	Correc- tion	Spher- ical angle	Spher- ical excess	Plane angle	Loga- rithm
		° ' "	"	"	"	"	
- 8+ 9	$A_2-A_1$						3.772745
- 1+ 2	$A_3$	30 43 52.5	+0.3	52.8	0.0	52.8	0.291568
- 4+ 6	$A_2$	101 44 45.1	+1.5	46.6	0.1	46.5	9.990809
	$A_1$	47 31 20.2	+0.5	20.7	0.0	20.7	9.867787
			+2.3		0.1		
	$A_3-A_1$						4.055122
	$A_3-A_2$						3.932100
	$A_2-A_1$						3.772745
-10+11	$A_4$	25 15 16.2	+0.6	16.8	0.0	16.8	0.369936
- 1+ 3	$A_2$	133 53 46.3	0.0	46.3	0.1	46.2	9.857693
- 5+ 6	$A_1$	20 50 56.7	+0.3	57.0	0.0	57.0	9.551339
			+0.9		0.1		
	$A_4-A_1$						4.000374
	$A_4-A_2$						3.694020
	$A_2-A_3$						3.932100
-10+12	$A_4$	116 47 20.0	-0.9	19.1	0.1	19.0	0.049306
- 2+ 3	$A_2$	32 08 61.2	-1.5	59.7	0.0	59.7	9.726023
- 7+ 8	$A_3$	31 03 42.5	-1.2	41.3	0.0	41.3	9.712614
			-3.6		0.1		
	$A_4-A_3$						3.707429
	$A_4-A_2$						3.694020
	$A_1-A_3$						4.055122
-11+12	$A_4$	91 32 03.8	-1.5	02.3	0.1	02.2	0.000156
- 4+ 5	$A_1$	26 40 23.5	+0.2	23.7	0.0	23.7	9.652151
- 7+ 9	$A_3$	61 47 35.0	-0.9	34.1	0.0	34.1	9.945096
			-2.2		0.1		
	$A_4-A_3$						3.707429
	$A_4-A_1$						4.000374

ADJUSTMENT OF THREE NEW POINTS CONNECTED WITH SEVERAL  
FIXED POINTS BY VARIATION OF GEOGRAPHIC COORDINATES

## GENERAL STATEMENT

The method of adjustment by geographic coordinates seems to be especially suitable for the adjustment of a few new points depending upon a number of fixed points. The number of normal equations in such case is  $2n$ ,  $n$  being the number of new points. In the figure used the number of condition equations would be 15, which would form a very intricate set of normals. By the method of coordinates the number of normal equations is only six.

The adjustment of figure 6 is carried out in two different ways, the first one being more rigorous but a trifle longer than the second. The first method corresponds in its treatment of observed directions to the method developed in Jordan's *Vermessungskunde*, volume 1, pages 144-173, of the third edition. The second method resembles somewhat the method given by Jordan on pages 173-179 for the approximate treatment of the  $z$ 's and corresponds in its treatment of fixed directions to the ordinary practice of the Coast and Geodetic Survey for subsidiary triangulation as treated by the method of condition equations.

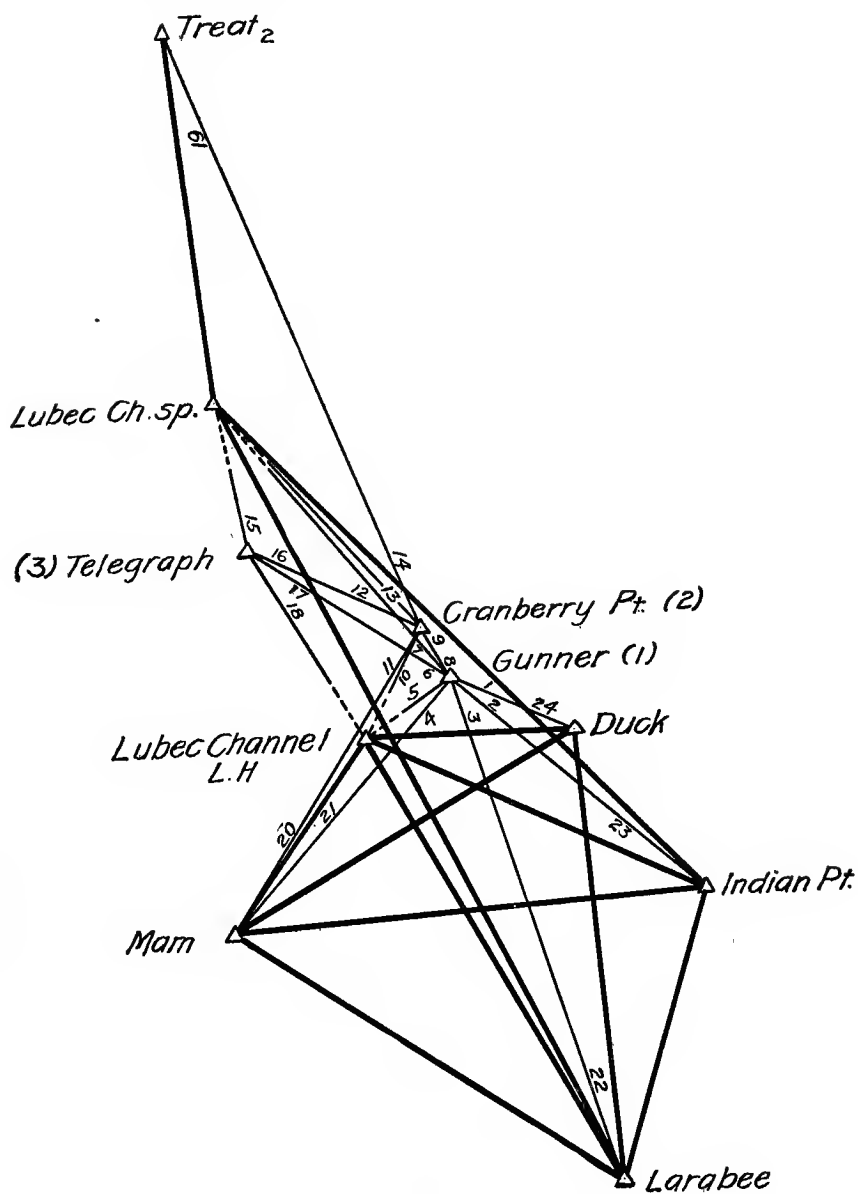


FIG. 6.

The solution by the method of condition equations was carried out for figure 6 and gave almost the same results, the greatest difference in the correction to a direction being  $0.2''$ . This difference was quite to be expected in view of the different formulas and the fact that the fixed positions, distances, and azimuths may not be strictly consistent with each other to the last figure given.

#### FIRST METHOD

The first method is fundamentally the same as the method used in the adjustment of the quadrilateral previously given, but the greater complication of the figure, particularly the great number of fixed lines, brings to light points that need mention. The groundwork of the adjustment by either method is shown in the tables of observed directions and of fixed positions, azimuths, and lengths which follow. In the list of directions the names of stations that are sighted on over fixed lines are shown in heavy type. For these stations the directions corrected from a previous adjustment are also shown. In forming the table of triangles for the preliminary computation these corrected directions were taken with the directly observed directions of new points in order to obtain such of the angles in the column "Observed angle" as have a fixed line for one of its sides.\* No particular procedure to obtain the consistent set of positions, azimuths, and lengths necessary to form the observation equations is essential to the method. In this particular case the corrections to the angles of the triangles Gunner-Larrabee-Mam, Cranberry Point-Gunner-Lubec Channel Lighthonse, and Telegraph-Cranberry Point-Gunner were arbitrarily assumed as shown in the table of preliminary computation of triangles. These assumptions, with the lines already fixed, determined enough parts in every one of the other triangles to make possible its solution with results as shown in the table.

Following the table of triangles the necessary preliminary computation of positions is included.

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\* This corresponds to the idea followed out in the second method of solution, but in the preliminary computation this is of no consequence, as is shown in the next sentence.

*Lists of directions*

## AT INDIAN POINT

Stations observed	Observed directions	Seconds after adjustment*
	° ' "	"
Larrabee	0 00 00.0	01.5
Mam	66 55 01.0	01.2
Lubec Channel Light-house	95 24 38.4	41.2
Gunner	114 00 37.5	
Lubec Church Spire	117 33 55.5	71.9

## AT MAM

Indian Point	0 00 00.0	55.4
Larrabee	34 52 43.6	46.7
Cranberry Point	317 37 23.5	
Lubec Channel Light-house	318 56 54.1	55.2
Gunner	322 01 44.8	
Duck	336 18 52.1	52.5

## AT GUNNER

Indian Point	0 00 00.0	
Larrabee	31 57 41.4	
Mam	94 56 05.9	
Lubec Channel Light-house	101 48 54.9	
Telegraph	168 28 59.0	
Lubec Church Spire	186 30 29.8	
Cranberry Point	191 54 53.6	
Duck	346 14 12.0	

AT TREAT<sub>2</sub>

Cranberry Point	0 00 00.0	
Lubec Church Spire	17 34 38.1	45.4

## AT LARRABEE

Stations observed	Observed directions	Seconds after adjustment*
	° ' "	"
Indian Point	0 00 00.0	56.7
Mam	281 47 48.4	47.9
Lubec Channel Light-house	312 58 58.5	55.0
Lubec Church Spire	315 12 19.0	24.9
Gunner	325 58 24.9	
Duck	336 12 51.0	50.6

## AT DUCK

Lubec Channel Light-house	0 00 00.0	59.9
Gunner	33 11 41.8	
Larrabee	269 09 40.7	40.9
Mam (computed)	336 10	44.0

## AT CRANBERRY POINT

Gunner	0 00 00.0	
Lubec Channel Light-house	75 45 12.8	
Mam	78 37 05.9	
Telegraph	153 39 19.7	
Lubec Church Spire	174 08 32.3	
Treat <sub>2</sub>	191 52 35.2	

## AT TELEGRAPH

Cranberry Point	0 00 00.0	
Gunner	2 54 53.2	
Lubec Channel Light-house	29 52 44.5	
Lubec Church Spire	231 21 00.1	

\* Refers to final values of heavy lines in Fig. 6, p. 104, obtained from a previous adjustment.

*List of fixed positions*

Station	Latitude and longitude	Azimuth	Back azimuth	To station	Logarithm of distance
	° ' "	° ' "	° ' "		
Lubec Church Spire	44 51 38.470 66 59 17.418				
Lubec Channel Light-house	44 50 31.652 66 58 38.299				
Treat <sub>2</sub>	44 52 44.333 66 59 25.919	354 45 17.5	174 45 23.5	Lubec Church Spire	3.309982
Indian Point	44 50 03.537 66 57 12.788	114 48 33.8 136 58 04.6	294 47 33.5 316 56 36.7	Lubec Channel Light-house Lubec Church Spire	3.315762 3.603123
Larrabee	44 49 10.841 66 57 38.857	152 22 33.8 154 36 03.7 199 23 35.7	332 21 51.9 334 34 54.3 19 23 54.1	Lubec Channel Light-house Lubec Church Spire Indian Point	3.449573 3.702872 3.236668
Duck	44 50 33.886 66 57 47.822	355 36 23.1 86 26 42.1	175 36 29.4 266 26 06.5	Larrabee Lubec Channel Light-house	3.410111 3.045619
Mam	44 49 57.369 66 59 26.892	225 14 19.0 242 36 16.3 266 17 19.2 301 10 10.5	45 14 53.3 62 37 26.2 86 18 53.8 121 11 26.7	Lubec Channel Light-house Duck Indian Point Larrabee	3.176968 3.389282 3.470097 3.443126

*Preliminary computation of triangles*

Symbol	Station	Observed angle	Correc- tion	Spheri- cal angle	Spheri- cal excess	Plane angle	Loga- rithm
		° ' "	"	"		° ' "	
-1+ 3 +24 -22	Duck-Larrabee						3.410111
	Gunner	45 43 29.4	+ 4.8	34.2			0.145080
	Duck	124 01 60.9	- 1.9	59.0			9.918405
	Larrabee	10 14 25.7	+ 1.1	26.8			9.249896
			+ 4.0				
	Gunner-Larabee						3.473596
	Gunner-Duck						2.805087
-1+ 4 +24 -21	Duck-Mam						3.339292
	Gunner	108 41 53.9	+ 3.6	57.5			0.023552
	Duck	57 00 57.8	- 1.9	55.9			9.923668
	Mam	14 17 07.7	- 1.1	06.6			9.392254
			+ 0.6				
	Gunner-Mam						3.336502
	Gunner-Duck						2.805088 <sup>-1</sup>
	Duck-Lubec Chan- nel Lighthouse						3.045619
-1+ 5 +24	Gunner	115 34 42.9	+ 7.1	50.0			0.044803
	Duck	33 11 41.9	- 1.9	40.0			9.738370
	Lubec Channel Lighthouse		- 5.2	30.0		31 13 30.0	9.714665
	Gunner-Lubec Channel Light- house						2.828792 <sup>+1</sup>
	Gunner-Duck						2 805087
	Indian Point-Lar- abee						3.236668
-2+ 3 +23 -22	Gunner	31 57 41.4	+ 7.0	48.4			0.276234
	Indian Point	114 00 36.0	+ 2.5	38.5			9.960694
	Larrabee	34 01 32.0	+ 1.1	33.1			9.747852
			+10.6				
	Gunner-Larrabee						3.473596
	Gunner-Indian Point						3.260754
-2+ 4 +23 -21	Indian Point-Mam						3.470097
	Gunner	94 56 05.9	+ 5.8	11.7			0.001614
	Indian Point	47 05 36.3	+ 2.5	38.8			9.864792
	Mam	37 58 10.6	- 1.1	03.5			9.789044
			+ 7.2				
	Gunner-Mam						3.336503 <sup>-1</sup>
	Gunner-Indian Point						3.280755 <sup>-1</sup>
	Indian Point-Lu- bec Channel Lighthouse						3.315782
-2+ 5 +23	Gunner	101 48 54.9	+ 9.3	64.2			0.003304
	Indian Point	18 35 56.3	+ 2.5	58.8			9.503728
	Lubec Channel Lighthouse			57.0		59 34 57.0	9.935688
	Gunner-Lubec Channel Light- house						2.828794 <sup>-1</sup>
	Gunner-Indian Point						3.260754
-3+ 4 +22 -21	Larrabee-Mam						3.443126
	Gunner	62 58 24.5	- 1.2	23.3			0.050223
	Larrabee	44 10 37.0	- 1.1	35.9			9.843153
	Mam	72 51 01.9	- 1.1	00.8			9.980247
			- 3.4				
	Gunner-Mam						3.336502
	Gunner-Larrabee						3.473596

*Preliminary computation of triangles—Continued*

Symbol	Station	Observed angle	Correc- tion	Spheri- cal angle	Spheri- cal excess	Plane angle	Loga- rithm
		° ' "	"	"		° ' "	
	Larabee-Lubec Channel Light- house						3.449573
-3+5 +22	Gunner Larabee	69 51 13.5 12 59 29.9	+ 2.3 - 1.1	15.8 28.8			0.027417 9.351803
	Lubec Channel Lighthouse	16.6		15.4		97 09 15.4	9.996606
	Gunner-Lubec Channel Light- house						2.828793
	Gunner-Larabee						3.473596
	Larabee-Lubec church spire						3.702872
-3+7 +22	Gunner Larabee	154 32 48.4 10 45 60.0	+20.7 - 1.1	69.1 58.9		33 09.1	0.366851 9.271388
	Lubec church spire	71.6		52.0		14 40 52.0	9.403873 3.341111
	Gunner-Lubec church spire						3.473596
	Gunner-Larabee						3.176968
	Mam-Lubec Chan- nel Lighthouse						0.921500 8.730324
-4+5 +21	Gunner Mam	6 52 49.0 3 04 49.6	+ 3.5 + 1.1	52.5 50.7			9.238033
	Lubec Channel Lighthouse	21.4		16.8		170 02 16.8	2.828792 <sup>+1</sup>
	Gunner-Lubec Channel Light- house						3.336501 <sup>+1</sup>
	Gunner-Mam						3.603122
	Lubec church spire-Indian Point						0.945080 8.712552
+2-7	Gunner Lubec church spire	173 29 30.2 56 55.4	-27.7	02.5 85.6		2 57 25.6	8.792909 3.260754
-23	Indian Point Gunner-Indian Point	3 33 34.4	- 2.5	31.9			3.341111
	Gunner-Lubec church spire						2.828793
	Gunner-Lubec Channel Light- house						0.013566 9.999999
-9+10 -5+8	Cranberry Point Gunner	75 45 12.8 90 05 58.7	0.0 0.0	12.8 58.7			9.388114
	Lubec Channel Lighthouse	48.5		48.5		14 08 48.5	2.842358
	Cranberry Point- Lubec Channel Lighthouse						2.230473
	Cranberry Point- Gunner						3.336502 0.008631
-9+11 -4+8	Gunner Cranberry Point	78 36 65.9 96 58 47.7	-13.1 + 3.5	52.8 51.2			9.996768 8.885341
-20+21	Mam	4 24 21.3	- 5.3	16.0			
			-14.9				
	Cranberry Point- Mam						3.341901
	Cranberry Point- Gunner						2.230474 <sup>+1</sup>
	Gunner-Lubec church spire						3.341111
-9+13 -7+8	Cranberry Point Gunner	174 08 32.3 5 24 23.8	+11.5 -18.4	43.8 05.4			0.991389 8.973748
	Lubec church spire	08.9		10.8		0 27 10.8	7.897971 3.306248
	Cranberry Point- church spire						2.230471 <sup>+2</sup>
	Cranberry Point- Gunner						

*Preliminary computation of triangles—Continued*

Symbol	Station	Observed angle	Correc- tion	Spheri- cal angle	Spheri- cal excess	Plane angle	Loga- rithm
		° ' "	"	"		° ' "	
-10+11	Lubec Channel Lighthouse-Mam Cranberry Point	2 51 53.1	-13.1	40.0			3.178968 1.301768
	Lubec Channel Lighthouse	35.2		54.7		175 48 54.7	8.863166
-20	Mam Cranberry Point- Mam	1 19 31.7	- 6.4	25.3			8.363626 3.341902 <sup>-1</sup>
	Cranberry Point- Lubec Channel Lighthouse						2.842362 <sup>-4</sup>
-13+14	Lubec church spire-Treat <sub>2</sub>						3.309982
	Cranberry Point	17 43 62.9	- 4.7	58.2			0.516300
-19	Lubec church spire	19.0		28.0		144 41 28.0	9.761916
	Treat <sub>2</sub>	17 34 38.1	- 4.3	33.8			9.479966 3.588198
	Cranberry Point- Treat <sub>2</sub>						3.306248
	Cranberry Point- Lubec church spire						3.306248
-15+16	Lubec church spire-Cranberry Point						0.107274
	Telegraph	128 38 59.9	-52.6	07.3		30 52 25.9	9.710244
-12+13	Lubec church spire	51 47.5		85.9			9.544138
	Cranberry Point	20 29 12.6	+14.2	26.8			3.123766
	Telegraph-Cran- berry Point						2.957660
	Telegraph-Lubec church spire						3.341111
-15+17	Lubec church spire-Gunner						0.125876
	Telegraph	131 33 53.1	-54.8	58.3		32 58.3	9.704449
- 6+ 7	Lubec church spire	24 36.1		75.1		30 25 15.1	9.490673
	Gunner	18 01 30.8	+15.8	46.6			3.171436 2.957660
	Telegraph-Gunner						2.230473
	Telegraph-Lubec church spire						1.293796
-16+17	Cranberry Point- Gunner	2 54 53.2	- 2.2	51.0			9.647167
- 9+12	Telegraph	153 39 19.7	- 2.7	17.0			9.599497
- 6+ 8	Cranberry Point	23 25 54.6	- 2.6	52.0			
			- 7.5				3.171436 3.123766
	Telegraph-Gunner						2.842358
	Telegraph-Cran- berry Point						0.302657
-16+18	Cranberry Point- Lubec Channel Lighthouse						9.990244
-10+12	Telegraph	29 52 44.5	- 9.6	34.9			9.978751
	Cranberry Point	77 54 06.9	- 2.7	04.2		72 13 20.9	3.135259 <sup>+1</sup>
	Lubec Channel Lighthouse	08.6		20.9			3.123766
	Telegraph-Lubec Channel Light- house						2.828793
	Telegraph-Cran- berry Point						0.343516
-17+18	Gunner-Lubec Channel Light- house	26 57 51.3	- 7.4	43.9			9.962951
- 5+ 6	Telegraph	66 40 04.1	+ 2.6	06.7		86 22 09.4	9.999127
	Gunner	04.6		09.4			3.135260
	Lubec Channel Lighthouse						3.171436
	Telegraph-Lubec Channel Light- house						
	Telegraph-Gunner						

*Preliminary position computation,*

## STATION GUNNER

$\alpha$ Second angle	Duck to Lubec Channel Lighthouse Lubec Channel Lighthouse and Gunner					$^{\circ}$ + 86 33	$'$ 26 11	$''$ 42.1 40.0
$\alpha$ $\Delta\alpha$	Duck to Gunner					119	38	22.1 17.8
$\alpha'$	Gunner to Duck					180 299 115	00 38 34	00.00 04.3 50.0
	First angle of triangle							
$\phi$ $\Delta\phi$	$^{\circ}$ 44 +	$'$ 50	$''$ 33.886 10.227	Duck	$\lambda$ $\Delta\lambda$	66 +	57	47.822 25.265
$\phi'$	44	50	44.113	Gunner	$\lambda'$	66	58	13.087
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 44	$'$ 50	$''$ 39	$s$ $\cos \alpha$ B	$s^2$ $\sin^2 \alpha$ C	5.6102 9.8782 1.4016		
1st term	$''$ -10.2275		$h$	1.009769		6.8900		
2d term	+ 0.0008							
$-\Delta\phi$	-10.2267							
	$s$ $\sin \alpha$ $A'$ $\sec \phi'$	2.805087 9.939097 8.508994 0.149348	$\sin \frac{\Delta\lambda}{2(\phi+\phi')}$	1.402528 9.848301				
		1.402526		1.250827				
		$''$		$''$				
	$\Delta\lambda$	+25.2654	$-\Delta\alpha$	+17.82				

## STATION CRANBERRY POINT

$\alpha$ Second angle	Gunner to Lubec Channel Lighthouse Lubec Channel Lighthouse and Cranberry Point					$^{\circ}$ + 55 90	$'$ 12 05	$''$ 54.3 58.7
$\alpha$ $\Delta\alpha$	Gunner to Cranberry Point					145	18	53.0 03.1
$\alpha'$	Cranberry Point to Gunner					180 325 75	00 18 45	00.00 49.9 12.8
	First angle of triangle							
$\phi$ $\Delta\phi$	$^{\circ}$ 44 +	$'$ 50	$''$ 44.113 04.529	Gunner	$\lambda$ $\Delta\lambda$	66 +	58	13.087 04.405
$\phi'$	44	50	48.642	Cranberry Point	$\lambda'$	66	58	17.492
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 44	$'$ 50	$''$ 46	$s$ $\cos \alpha$ B	$s^2$ $\sin^2 \alpha$ C	4.4609 9.5103 1.4016		
1st term	$''$ -4.5288		$h$	0.655978		5.3728		
2d term	0.0000							
$-\Delta\phi$	-4.5288							
	$s$ $\sin \alpha$ $A'$ $\sec \phi'$	2.230473 9.755164 8.508994 0.149357	$\sin \frac{\Delta\lambda}{2(\phi+\phi')}$	0.643988 9.848315				
		0.643988		0.492303				
		$''$		$''$				
	$\Delta\lambda$	+4.4054	$-\Delta\alpha$	+3.11				

## secondary triangulation

## STATION GUNNER

$\alpha$ Third angle	Lubec Channel Lighthouse to Duck Gunner and Duck					$^{\circ}$ 266 — 31	$'$ 26 13	$''$ 06.5 30.0
$\alpha$ $\Delta\alpha$	Lubec Channel Lighthouse to Gunner					235 +	12	36.5 17.8
$\alpha'$	Gunner to Lubec Channel Lighthouse					180 55	00 12	00.00 54.3
$\phi$	$^{\circ}$ 44	$'$ 50	$''$ 31.652	Lubec Channel Lighthouse	$\lambda$	66	58	38.299
$\Delta\phi$	+		12.461		$\Delta\lambda$	—		25.212
$\phi'$	44	50	44.113	Gunner	$\lambda'$	66	58	13.087
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 44	$'$ 50	$''$ 36	$s$ $\cos \alpha$ B	$s^2$ $\sin^2 \alpha$ C	5.6576 9.8289 1.4016		
1st term	$''$ —12.4618	$h$	1.095581			6.8881		
2d term	+ 0.0008							
$-\Delta\phi$	—12.4610							
	$s$ $\sin \alpha$ $\Lambda'$ $\sec \phi'$	2.828793 9.914474 8.508994 0.149348	$\Delta\lambda$ $\sin \frac{1}{2}(\phi+\phi')$	1.401609 9.848294				
		1.401609		1.249903				
	$\Delta\lambda$	$''$ —25.2121	$-\Delta\alpha$	$''$ —17.78				

## STATION CRANBERRY POINT

$\alpha$ Third angle	Lubec Channel Lighthouse to Gunner Cranberry Point and Gunner					$^{\circ}$ 235 — 14	$'$ 12 08	$''$ 36.5 48.5
$\alpha$ $\Delta\alpha$	Lubec Channel Lighthouse to Cranberry Point					221 +	03	48.0 14.7
$\alpha'$	Cranberry Point to Lubec Channel Lighthouse					180 41	00 04	00.00 02.7
$\phi$	$^{\circ}$ 44	$'$ 50	$''$ 31.652	Lubec Channel Lighthouse	$\lambda$	66	58	38.299
$\Delta\phi$	+		16.990		$\Delta\lambda$	—		20.807
$\phi'$	44	50	48.642	Cranberry Point	$\lambda'$	66	58	17.492
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 44	$'$ 50	$''$ 40	$s$ $\cos \alpha$ B	$s^2$ $\sin^2 \alpha$ C	5.6847 9.6350 1.4016		
1st term	$''$ —16.9903	$h$	1.230200			6.7213		
2d term	+ 0.0005							
$-\Delta\phi$	—16.9898							
	$s$ $\sin \alpha$ $\Lambda'$ $\sec \phi'$	2.842358 9.817494 8.508994 0.149357	$\Delta\lambda$ $\sin \frac{1}{2}(\phi+\phi')$	1.318203 3.848303				
		1.318203		1.166506				
	$\Delta\lambda$	$''$ —20.8067	$-\Delta\alpha$	$''$ —14.67				

*Preliminary position computation,*

## STATION TELEGRAPH

$\alpha$	Cranberry Point to Lubec Channel Lighthouse				$^{\circ}$	$'$	$''$
Second angle	Lubec Channel Lighthouse and Telegraph				41	04	02.7
					+ 77	54	04.2
$\alpha$	Cranberry Point to Telegraph				118	58	06.9
$\phi\alpha$					—		37.4
$\alpha'$	Telegraph to Cranberry Point				180	00	00.00
	First angle of triangle				298	57	29.5
					29	52	34.9
$\phi$	$^{\circ}$	$'$	$''$	Cranberry Point	$\lambda$		
$\Delta\phi$	44	50	48.642		66	58	17.492
	+		20.860		+		52.980
$\phi'$	44	51	09.502	Telegraph	$\lambda'$	66	59
							10.472
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$	$'$	$''$	$s$	$s^2$		
	44	50	59	$\cos \alpha$	3.123766	6.2475	
				B	9.685141	9.8839	
					8.510480	1.4016	
1st term	$''$			h	1.319387	7.5330	
2d term	—20.8635						
	+ 0.0034						
$-\Delta\phi$	—20.8601						
	$s$			$\Delta\lambda$	$\sin \frac{1}{2}(\phi+\phi')$		
	$\sin \alpha$	3.123766				1.724112	
	$A'$	9.941951				9.848343	
	$\sec \phi'$	8.508994					
		0.149401					
		1.724112				1.572455	
		$''$				$''$	
	$\Delta\lambda$	+52.9800		$-\Delta\alpha$		+37.36	

secondary triangulation—Continued.

## STATION TELEGRAPH

$\alpha$	Lubec Channel Lighthouse to Cranberry Point					°	'	''
Third angle	Telegraph and Cranberry Point					221	03	48.0
						— 72	13	20.9
$\alpha$	Lubec Channel Lighthouse to Telegraph					148	50	27.1
$\Delta\alpha$						—		22.7
$\alpha'$	Telegraph to Lubec Channel Lighthouse					180	00	00.00
						328	50	04.4
$\phi$	°	'	''	Lubec Channel Lighthouse	$\lambda$			
$\Delta\phi$	44	50	31.652		$\Delta\lambda$	66	58	38.299
	+		37.850			+		32.173
$\phi'$	44	51	09.502	Telegraph	$\lambda'$	66	59	10.472
$\frac{1}{2}(\phi+\phi')$	°	'	''	$\cos \alpha$	$\sin^2 \alpha$			
	44	50	50	B	C			
1st term	—37.8511			3.135260	6.2705			
2d term	+ 0.0012			9.932338	9.4277			
				8.510480	1.4016			
$-\Delta\phi$	—37.8499			1.578078	7.0998			
	$\sin \alpha$	3.135260			$\Delta\lambda$	1.507496		
	A'	9.713841			$\sin \frac{1}{2}(\phi+\phi')$	9.848324		
	$\sec \phi'$	8.508994						
		0.149401						
		1.507496				1.355820		
	$\Delta\lambda$	+32.1733			$-\Delta\alpha$	+22.69		

## FORMATION OF OBSERVATION EQUATIONS

In the first column of the following table is given the assumed azimuths of the various lines. Under station Gunner the azimuth to Duck,  $299^{\circ} 38' 04''.3$ , comes from the position computation on page 110, and the other azimuths are obtained by adding to this the corrected angles from the preliminary computation of triangles. Thus in the first triangle in that list, Gunner-Duck-Larrabee, the angle at Gunner is  $45^{\circ} 43' 34''.2$ , which, added to the azimuth of Duck, gives  $345^{\circ} 21' 38''.5$ , as shown in the table. For any one station the assumed and observed azimuths of some one station may be taken as identical. At Gunner they are identical on station Duck. The observed azimuths in the second column of the table have their symbolic corrections affixed. These azimuths are obtained by adding the observed angles as derived from the list of observed directions on page 106 to the azimuth of the line Gunner-Duck.

At the fixed stations given in the lower part of the table the method of computing the assumed and observed azimuths is somewhat different. The assumed azimuths of the fixed lines come from the table of fixed positions on page 106. The assumed azimuths of the new lines are found by adding to one of these fixed azimuths the appropriate corrected angle from the computation of triangles on pages 107-109. In the second column of the table the observed azimuth of one fixed line used as an initial line is taken identical with its assumed azimuth, and the other observed azimuths, whether of fixed lines or of new ones, are found by adding to this azimuth the observed angles between the initial line and each of the others as derived from the list of directions.

The coefficients of the  $\delta\phi$ 's and  $\delta\lambda$ 's are found from the formulas on page 93.

## GUNNER

Assumed azimuth	Observed azimuth	Equation	Station observed
° ' "	° ' "		
299 38 04.3	299 38 04.3 - z <sub>1</sub> + v <sub>1</sub>	$v_1 = z_1 + 8669\phi_1 - 3509\lambda_1 + 0.0$	Duck
313 23 50.1	313 23 52.3 - z <sub>1</sub> + v <sub>2</sub>	$v_2 = z_1 + 2538\phi_1 - 1709\lambda_1 - 2.2$	Indian Point
345 21 38.5	345 21 33.7 - z <sub>1</sub> + v <sub>3</sub>	$v_3 = z_1 + 541\phi_1 - 1473\lambda_1 + 4.8$	Larabee
48 20 01.8	48 19 58.2 - z <sub>1</sub> + v <sub>4</sub>	$v_4 = z_1 - 2191\phi_1 - 1383\lambda_1 + 3.6$	Mam
55 12 54.3	55 12 47.2 - z <sub>1</sub> + v <sub>5</sub>	$v_5 = z_1 - 7756\phi_1 - 3833\lambda_1 + 7.1$	Lubec Channel L. H.
121 53 01.0	121 52 51.3 - z <sub>1</sub> + v <sub>6</sub>	$v_6 = z_1 - 3843\phi_1 + 1612\lambda_1 + 3643\phi_3 - 1612\lambda_3 + 9.7$	Telegraph
139 54 47.6	139 54 22.1 - z <sub>1</sub> + v <sub>7</sub>	$v_7 = z_1 - 1870\phi_1 + 1580\lambda_1 + 25.5$	Lubec church spire
145 18 53.0	145 18 45.9 - z <sub>1</sub> + v <sub>8</sub>	$v_8 = z_1 - 21313\phi_1 + 21909\lambda_1 + 21313\phi_3 - 21909\lambda_3 + 7.1$	Cranberry Point

## CRANBERRY POINT

325 18 49.9	325 18 49.9 - z <sub>2</sub> + v <sub>9</sub>	$v_9 = z_2 - 21313\phi_1 + 21909\lambda_1 + 21313\phi_3 - 21909\lambda_3 + 0.0$	Gunner
41 04 02.7	41 04 02.7 - z <sub>2</sub> + v <sub>10</sub>	$v_{10} = z_2 - 6013\phi_3 - 4910\lambda_3 + 0.0$	Lubec Channel L. H.
43 55 42.7	43 55 35.8 - z <sub>2</sub> + v <sub>11</sub>	$v_{11} = z_2 - 2010\phi_3 - 1455\lambda_3 - 13.1$	Mam
118 58 06.9	118 58 09.6 - z <sub>2</sub> + v <sub>12</sub>	$v_{12} = z_2 - 4189\phi_3 + 1650\lambda_3 + 4189\phi_3 - 1650\lambda_3 - 2.7$	Telegraph
139 27 33.7	139 27 22.2 - z <sub>2</sub> + v <sub>13</sub>	$v_{13} = z_2 - 2045\phi_3 + 1701\lambda_3 + 11.5$	Lubec church spire
157 11 31.9	157 11 25.1 - z <sub>2</sub> + v <sub>14</sub>	$v_{14} = z_2 - 637\phi_3 + 1078\lambda_3 + 6.8$	Treat <sub>2</sub>

## TELEGRAPH

170 19 22.2	170 19 22.2 - z <sub>3</sub> + v <sub>15</sub>	$v_{15} = z_3 - 1131\phi_3 + 4922\lambda_3 + 0.0$	Lubec church spire
298 57 29.5	298 58 22.1 - z <sub>3</sub> + v <sub>16</sub>	$v_{16} = z_3 - 4189\phi_3 + 1650\lambda_3 + 4189\phi_3 - 1650\lambda_3 - 52.6$	Cranberry Point
301 52 20.5	301 53 15.3 - z <sub>3</sub> + v <sub>17</sub>	$v_{17} = z_3 - 3643\phi_3 + 1612\lambda_3 + 3643\phi_3 - 1612\lambda_3 - 54.8$	Gunner
328 50 04.4	328 51 06.6 - z <sub>3</sub> + v <sub>18</sub>	$v_{18} = z_3 + 2413\phi_3 - 2830\lambda_3 - 62.2$	Lubec Channel L. H.

TREAT<sub>3</sub>

337 10 43.7	337 <sup>7</sup> 10 39.4 - z <sub>4</sub> + v <sub>19</sub>	$v_{19} = z_4 - 637\phi_3 + 1078\lambda_3 + 4.3$	Cranberry Point
354 45 17.5	354 45 17.5 - z <sub>4</sub> + v <sub>19</sub>	$v_{19} = z_4 + 0.0$	Lubec church spire

## MAM

Assumed azimuth	Observed azimuth	Equation	Station observed
° ' "	° ' "		
223 54 53.7	223 54 42.7	$v_{20} = z_3 - 2010\phi_1 - 1485\lambda_1 + 11.0 (+6.4)$	Cranberry Point Lubec Channel L. H. Gunner Duck Indian Point Larrabee
223 14 13.0	223 14 13.3	$v_{20}' = z_3 + 5.7 (+1.1)$	
228 19 09.7	228 19 04.0	$v_{21} = z_3 - 2191\phi_1 - 1388\lambda_1 + 5.7 (+1.1)$	
242 36 16.3	242 36 11.3	$v_{20}'' = z_3 + 5.0 (+0.4)$	
266 17 19.2	266 17 19.2	$v_{20}''' = z_3 + 0.0 (-4.6)$	
301 10 10.5	301 10 02.8	$v_{20}'''' = z_3 + 7.7 (+3.1)$	

## LARRABEE

121 11 26.7	121 11 24.1	$v_{23}' = z_3 + 2.6 (-0.2)$	Mam Lubec Channel L. H. Lubec church spire Gunner Duck Indian Point
152 22 33.8	152 22 34.2	$v_{23}'' = z_3 - 0.4 (-3.2)$	
154 36 03.7	154 35 54.7	$v_{23}''' = z_3 + 9.0 (+6.2)$	
165 22 02.6	165 22 00.6	$v_{23}'''' = z_3 + 541\phi_1 - 1473\lambda_1 + 2.0 (-0.8)$	
175 36 29.4	175 36 26.7	$v_{23}'''' = z_3 + 2.7 (-0.1)$	
199 23 35.7	199 23 35.7	$v_{23}'''' = z_3 + 0.0 (-2.8)$	

## INDIAN POINT

19 23 54.1	19 23 54.1	$v_{23}' = z_1 + 0.0 (-3.8)$	Larrabee Mam Lubec Channel L. H. Gunner Lubec church spire
46 18 53.8	46 18 55.1	$v_{23}'' = z_1 - 1.3 (-5.0)$	
114 48 33.8	114 48 32.5	$v_{23}''' = z_1 + 1.3 (-2.4)$	
133 24 32.6	133 24 31.6	$v_{23}'''' = z_1 + 2538\phi_1 - 1709\lambda_1 + 1.0 (-2.8)$	
136 58 04.6	136 57 49.6	$v_{23}'''' = z_1 + 15.0 (+11.2)$	

## DUCK

355 36 23.1	355 36 22.8	$v_{24}' = z_3 + 0.3 (+0.2)$	Larrabee Mam Lubec Channel L. H. Gunner.
62 37 26.2	Not observed	$v_{24}'' = z_3 + 0.0 (-0.1)$	
86 26 42.1	86 26 42.1	$v_{24}''' = z_3 + 8689\phi_1 - 3509\lambda_1 - 1.8 (-1.9)$	
119 38 22.1	119 38 23.9		

Figure 6 taken with the following two tables shows that  $z_1$  is for directions taken at Gunner,  $z_2$  for directions at Cranberry Point,  $z_3$  for directions at Telegraph, and  $z_5$  for directions at Mam. The scheme for eliminating these  $z$ 's by the use of the sum equations, as fictitious additional observation equations with negative weights, is used here in the same manner as in the previous example. Each weight is the negative reciprocal of the total number of observed lines in the adjustment that radiates from the point in question. The weights are, respectively,  $-1/8$ ,  $-1/6$ ,  $-1/4$ , and  $-1/6$ , as shown in Table 1.

At Treat<sub>2</sub>, Larrabee, Indian, and Duck where only one new line is to be determined the same process might be used, but the following method is identical in results and slightly shorter. Use is made of the fact that the directions taken at a point may each be changed by the same amount, a change equivalent to using merely a different zero point. Correct each of the directions by the averages of all the corrections necessary to reduce the observed results to the accepted results on the lines that have already been fixed. Then drop the  $z$  from the observation equation of the new line and assign the equation a positive weight equal to  $\frac{s}{s+1}$ , where  $s$  is the number of lines already fixed and therefore  $s+1$  is the total number of lines. Thus at Larrabee the constant terms of the observation equations representing pointings on lines already fixed are  $+2.6$ ,  $-0.4$ ,  $+9.0$ ,  $-0.1$ , and  $0.0$ , the mean of which is  $+2.8$ . Subtracting this from each of the preceding numbers we have  $-0.2$ ,  $-3.2$ ,  $+6.2$ ,  $-0.1$ , and  $-2.8$  as the new constant terms, also  $-0.8$  instead of  $+2.0$  on Gunner, the new point. These new values are inclosed in parentheses and are used in forming the normal equations. There are five fixed lines, so the weight of the new equation without  $z$  that is used to replace the six equations containing  $z$  is  $5/6$  and the equation itself is

$$v_{22} = +541\delta\phi_1 - 1473\delta\lambda_1 - 0.8$$

which in Table 2 corresponds to the line No. 22,

$$+0.49\delta\phi_1 - 1.34\delta\lambda_1 - 0.07.$$

The  $z$ 's are computed from the sum equations as in the previous example, the result of substitution in the right-hand side being divided by  $-1/r$ ,  $r$  being the total number of lines through the point to which  $z$  applies. For fixed points where only one new line occurs, substitute  $\delta\phi$  and  $\delta\lambda$  in the right-hand side of the observation equation on the new line omitting the  $z$ , and divide the result by  $-1/r$ . Thus at Duck (see p. 116),

$$z_8 = (8669\delta\phi_1 - 3509\delta\lambda_1 - 1.9) \div (-3)$$

as shown in the computation below.

When the  $z$ 's are known the  $v$ 's or corrections are computed from the equations on pages 115 and 116. The details are shown in the table on page 121.

For convenience of solution in the normals it is best to divide the constant terms by 10 and the coefficients by 1000. The solution will then give  $100\delta\phi_n$  and  $100\delta\lambda_n$ .

*Table for formation of normals No. 1*

		1	2	3	4	5	6			
		$\delta\phi_1$	$\delta\lambda_1$	$\delta\phi_2$	$\delta\lambda_2$	$\delta\phi_3$	$\delta\lambda_3$	$\eta$	$p$	$\sqrt{p}$
1	$z_1$									
2	1	+ 8669	- 3509					+ 0.0	1	1
3	1	+ 2538	- 1709					- 2.2	1	1
4	1	+ 541	- 1473					+ 4.8	1	1
5	1	- 2191	- 1388					+ 3.6	1	1
6	1	- 7756	- 3833					+ 7.1	1	1
7	1	- 3643	+ 1612			+3643	-1612	+ 9.7	1	1
8	1	- 1870	+ 1580					+ 25.5	1	1
Sum	8	-21313	+21909	+21313	-21909	+3643	-1612	+ 7.1	1	1
		-25025	+13189	+21313	-21909	+3643	-1612	+ 55.6	$-\frac{1}{2}$	0.353554
9	$z_2$									
10	1	-21313	+21909	+21313	-21909			+ 0.0	1	1
11	1			- 6013	- 4910			+ 0.0	1	1
12	1			- 2010	- 1485			- 13.1	1	1
13	1			- 4189	+ 1650	+4189	-1650	- 2.7	1	1
14	1			- 2045	+ 1701			+ 11.5	1	1
Sum	6	-21313	+21909	+ 6419	-23875	+4189	-1650	+ 6.8	1	1
								+ 2.5	$-\frac{1}{2}$	0.40834
15	$z_3$									
16	1					-1181	+4922	+ 0.0	1	1
17	1			- 4189	+ 1650	+4189	-1650	- 52.6	1	1
18	1	- 3643	+ 1612			+3643	-1612	- 54.8	1	1
19	1			- 4189	+ 1650	+2413	-2839	- 62.2	1	1
Sum	4	- 3643	+ 1612	- 4189	+ 1650	+9064	-1179	-169.6	$-\frac{1}{2}$	0.54
				- 637	+ 1078			+ 4.3	$\frac{1}{2}$	0.707
20	$z_5$									
21	1			- 2010	- 1485			+ 6.4	1	1
Sum	6	- 2191	- 1388	- 2010	- 1485			+ 1.1	1	1
22	1	- 2191	- 1388					+ 7.5	$-\frac{1}{2}$	0.40834
23	1	+ 541	- 1473					- 0.8	$\frac{1}{2}$	0.9129
24	1	+ 2538	- 1709					- 2.8	$\frac{1}{2}$	0.8944
		+ 8669	- 3509					- 1.9	$\frac{1}{2}$	0.8165

Table for formation of normals No. 2

	$\delta\phi_1$	$\delta\lambda_1$	$\delta\phi_2$	$\delta\lambda_2$	$\delta\phi_3$	$\delta\lambda_3$	$\eta$	$\Sigma$
1	+ 8.67	- 3.51					+0.00	+ 5.16
2	+ 2.54	- 1.71					-0.22	+ 0.61
3	+ 0.54	- 1.47					+0.48	- 0.45
4	- 2.19	- 1.39					+0.36	- 3.22
5	- 7.76	- 3.83					+0.71	-10.88
6	- 3.64	+ 1.61			+3.64	-1.61	+0.97	+ 0.97
7	- 1.87	+ 1.58					+2.55	+ 2.26
8	-21.31	+21.91	+21.31	-21.91			+0.71	+ 0.71
$z_1$	- 8.85 <i>i</i>	+ 4.66 <i>i</i>	+ 7.54 <i>i</i>	- 7.75 <i>i</i>	+1.29 <i>i</i>	-0.57 <i>i</i>	+1.97 <i>i</i>	- 1.71 <i>i</i>
9	-21.31	+21.91	+21.31	-21.91			+0.00	+ 0.00
10			- 6.01	- 4.91			+0.00	-10.92
11			- 2.01	- 1.48			-1.31	- 4.80
12			- 4.19	+ 1.65	+4.19	-1.65	-0.27	- 0.27
13			- 2.04	+ 1.70			+1.15	+ 0.81
14			- 0.64	+ 1.08			+0.68	+ 1.12
$z_2$	- 8.70 <i>i</i>	+ 8.95 <i>i</i>	+ 2.62 <i>i</i>	- 9.75 <i>i</i>	+1.71 <i>i</i>	-0.67 <i>i</i>	+0.10 <i>i</i>	- 5.74 <i>i</i>
15					-1.18	+4.92	+0.00	+ 3.74
16			- 4.19	+ 1.65	+4.19	-1.65	-5.26	- 5.26
17	- 3.64	+ 1.61			+3.64	-1.61	-5.48	- 5.48
18					+2.41	-2.84	-6.22	- 6.65
$z_3$	- 1.82 <i>i</i>	+ 0.81 <i>i</i>	- 2.09 <i>i</i>	+ 0.82 <i>i</i>	+4.53 <i>i</i>	-0.59 <i>i</i>	-8.48 <i>i</i>	- 6.82 <i>i</i>
19			- 0.45	+ 0.76			+0.30	+ 0.61
20			- 2.01	- 1.48			+0.64	- 2.85
21	- 2.19	- 1.39					+0.11	- 3.47
$z_5$	- 0.89 <i>i</i>	- 0.57 <i>i</i>	- 0.82 <i>i</i>	- 0.61 <i>i</i>			+0.31 <i>i</i>	- 2.58 <i>i</i>
22	+ 0.49	- 1.34					-0.07	- 0.92
23	+ 2.27	- 1.53					-0.25	+ 0.49
24	+ 7.08	- 2.86					-0.16	+ 4.06

## Normal equations

1	2	3	4	5	6	$\eta$	$\Sigma$
+987.3526	-852.5451 +913.2320	-823.2428 +876.4443 +923.5619	+781.3412 -837.7306 -831.4801 +842.4946	+ 8.0389 -13.2644 -39.8513 +36.7824 +43.7028	- 0.2285 + 3.9464 +18.6471 -15.9112 -33.6441 +41.7793	- 8.9085 + 6.5262 + 4.1465 + 2.6136 -18.8752 +30.2371	+ 91.8098 + 96.6088 +128.2256 - 21.8901 - 17.1109 + 44.8281



*Back solution*

$\delta\lambda_3$	$\delta\phi_3$	$\delta\lambda_2$	$\delta\phi_2$	$\delta\lambda_1$	$\delta\phi_1$
-0.86892	+0.69872	-0.12121	+0.02662	+0.00658	+0.00902
	-0.75924	-0.09758	+0.15787	+0.01840	-0.00020
-0.86892		+0.01446	-0.02003	-0.00216	+0.00049
	-0.06052		-0.06832	-0.18815	+0.16170
		-0.20433	+0.09614	-0.08990	+0.08016
				-0.25523	-0.22038
					+0.03079

*Computation of corrections*

1	2	3	4	5	6	7	8
+ 2.669 + 8.956 -11.412	+ 0.781 + 4.362 -11.412 - 2.2	+ 0.167 + 3.760 -11.412 + 4.8	- 0.675 + 3.543 -11.412 + 3.6	- 2.388 + 9.783 -11.412 + 7.1	- 1.122 - 4.114 - 2.205 +14.007 -11.412 + 9.7	- 0.576 - 4.033 -11.412 +25.5 + 9.479 + 9.5	- 6.562 -55.918 +20.490 +44.767 +11.412 + 7.1
+ 0.213 + 0.2	- 8.469 - 8.5	- 2.685 - 2.7	- 4.944 - 4.9	+ 3.083 + 3.1	+ 4.854 + 4.9		- 1.535 - 1.5
$z_1$	9	10	11	12	13	14	$z_2$
- 7.705 -33.662 +20.490 +44.767 - 2.205 +14.007 +55.6	- 6.562 -55.918 +20.490 +44.767 - 1.129 + 1.648 + 1.7	- 5.781 +10.033 - 1.129 + 3.123 + 3.1	- 1.932 + 3.035 - 1.129 -13.1 -13.126 -13.1	- 4.027 - 3.371 - 2.535 +14.337 - 1.129 - 2.7 + 0.575 + 0.6	- 1.966 - 3.476 - 1.129 +11.5 + 4.929 + 5.0	- 0.612 - 2.203 - 1.129 + 6.8 + 2.856 + 2.9	- 6.562 -55.918 + 6.171 +48.783 - 2.535 +14.337 + 2.5 + 6.776 - 1.129
+91.292 -11.412							
15	16	17	18	$z_3$	19	$z_4$	19'
+ 0.715 -42.768 +44.369	- 4.027 - 3.371 - 2.535 +14.337 +44.369 -52.6	- 1.122 - 4.114 - 2.205 +14.007 +44.369 -54.8	- 1.460 +24.669 +44.369 -62.2 + 5.378 + 5.4	- 1.122 - 4.114 - 4.027 - 3.371 - 5.486 +10.245 -109.6 -177.475 + 44.369	- 0.612 - 2.203 - 0.742 + 4.3 + 0.743 + 0.8	- 0.612 - 2.203 + 4.3 + 1.485 - 0.742	- 0.742 - 0.742 - 0.7 - 0.7
+ 2.316 + 2.3	- 3.827 - 3.8	- 3.865 - 3.8					
20	21	$z_5$	20'	20''	20'''	20''''	22
- 1.932 + 3.035 - 1.912 + 6.4	- 0.675 + 3.543 - 1.912 + 1.1	- 0.675 + 3.543 - 1.932 + 3.035 + 7.5	- 1.912 + 1.1 - 0.812 - 0.8	- 1.912 + 0.4 - 1.512 - 1.5	- 1.912 - 4.6 - 6.512 - 6.5	- 1.912 + 3.1 + 1.188 + 1.2	+ 0.167 + 3.760 - 0.521 - 0.8 + 2.606 + 2.6
+ 5.591 + 5.6	+ 2.056 + 2.1	+11.471 - 1.912					
$z_6$	22'	22''	22'''	22''''	22''	23	$z_7$
+ 0.167 + 3.760 - 0.8	- 0.521 - 0.2	- 0.521 - 3.2	- 0.521 + 6.2	- 0.521 - 0.1	- 0.521 - 2.8	+ 0.781 + 4.362 - 0.469 - 2.8	+ 0.781 + 4.362 - 2.8
+ 3.127 - 0.521	- 0.721 - 0.7	- 3.721 - 3.7	+ 5.679 + 5.7	- 0.621 - 0.6	- 3.321 - 3.3	+ 2.343 + 1.874 + 1.9	+ 2.343 - 0.469
23'	23''	23'''	23''''	24	$z_8$	24'	24''
- 0.469 - 3.8	- 0.469 - 5.0	- 0.469 - 2.4	- 0.469 +11.2	+ 2.669 + 8.956 - 3.242 - 1.9 + 6.483 + 6.5	+ 2.669 + 8.956 - 1.9 + 9.725 - 3.242	- 3.242 + 0.2 - 3.042 - 3.0	- 3.242 - 0.1 - 3.342 - 3.3

## Final computation of triangles

Symbol	Station	Observed angle ° ' "	Correc- tion "	Spheri- cal angle "	Spher- ical excess "	Plane angle ° ' "	Loga- rithm
-1+3 -24'+24 -22+22'''	Duck-Larrabee Gunner Duck Larrabee	45 43 29.4 124 02 01.1 10 14 26.1	- 2.9 + 9.5 - 3.2	26.5 10.6 22.9		° ' "	3.410111 0.145098 9.918389 9.249850
	Gunner-Larrabee Gunner-Duck		+ 3.4				3.473596 2.805057
-1+4 -21+20''	Duck-Mam Gunner Duck Mam Gunner-Mam Gunner-Duck	108 41 53.9 58.8 14 17 07.3	- 5.1 - 3.6	48.8 87.5 03.7		57 00 67.5	3.389282 0.023546 9.923684 9.332230 3.336512 2.805058-1
	Duck-Lubec Chan- nel Lighthouse						3.045619
-1+5 -24''+24	Gunner Duck Lubec Channel Lighthouse Gunner-Lubec Channel Light- house Gunner-Duck	115 34 42.9 33 11 41.8 35.3	+ 2.9 + 9.8	45.8 51.6 22.6		31 13 22.6	0.044799 9.738407 9.714639 2.828825+2 2.805057
	Indian Point-Lar- abee						3.238668
-2+3 -23'+23 -22+22''	Gunner Indian Point Larrabee	31 57 41.4 114 00 37.5 34 01 35.1	+ 5.8 + 6.1 - 5.9	47.2 43.6 29.2			0.276238 9.960689 9.747840
	Gunner-Larrabee Gunner-Indian Point		+ 6.0				3.473595+1 3.280746
-2+4 -23'+23 -21+20'''	Indian Point-Mam Gunner Indian Point Mam	94 56 05.9 47 05 36.5 37 58 15.2	+ 3.6 + 7.4 - 8.6	03.5 43.9 06.6			3.470097 0.001614 9.864802 9.789036
	Gunner-Mam Gunner-Indian Point		+ 2.4				3.336513-1 3.260747-1
	Indian Point-Lu- bec Channel Lighthouse						3.315792
-2+5 -23''+23	Gunner Indian Point Lubec Channel Lighthouse Gunner-Lubec Channel Light- house Gunner-Indian Point	161 48 54.9 18 35 59.1 66.0	+11.6 + 4.7	66.5 63.8 49.7		59 34 49.7	0.009305 9.503759 9.935679 2.828826+1 3.260746
-3+4 -22'+22 -21+20'''	Larrabee-Mam Gunner Larrabee Mam	62 58 24.5 44 10 36.5 72 50 58.8	- 2.2 + 3.3 - 0.9	22.3 39.8 57.9			3.443128 0.050224 9.843162 9.980246
	Gunner-Mam Gunner-Larrabee		+ 0.2				3.336512 3.473596
	Larrabee-Lubec Channel Light- house						3.449573
-3+5 -22''+22	Gunner Larrabee Lubec Channel Lighthouse Gunner-Lubec Channel Light- house Gunner-Larrabee	69 51 13.5 12 59 26.4 20.1	+ 5.8 + 6.3	19.3 32.7 08.0		97 09 08.0	0.027415 9.351839 9.996608 2.828827 3.473596

*Final computation of triangles—Continued*

Symbol	Station	Observed angle	Correc- tion	Spheri- cal angle	Spheri- cal excess	Plane angle	Loga- rithm
		° ' "	"	"		° ' "	
	Larabee-Lubec church spire						<b>3.702872</b>
-3+7	Gunner	154 32 48.4	+12.2	60.6			0.366814
-22''+22	Larabee	10 46 05.9	-3.1	02.8			9.271431
	Lubec church spire	65.7		56.6		14 40 56.6	9.403910
	Gunner-Lubec church spire						3.341117
	Gunner-Larabee						3.473596
	Mam-Lubec Channel Lighthouse						<b>3.176968</b>
-4+5	Gunner	6 52 49.0	+8.0	57.0			0.921421
-20'+21	Mam	3 04 50.7	+2.9	53.6		170 02 09.4	8.730438
	Lubec Channel Lighthouse	20.3		09.4			9.238122
	Gunner-Lubec Channel Light-house						2.828827
	Gunner-Mam						3.336511 <sup>+1</sup>
	Lubec church spire-Indian Point						<b>3.603122</b>
+2-7	Gunner	173 29 30.2	-18.0	12.2			0.945259
	Lubec church spire	11.8		21.0		2 57 21.0	8.712365
-23+23'''	Indian Point	3 33 18.0	+8.8	26.8			8.792736
	Gunner-Indian Point						3.260746
	Gunner-Lubec church spire						3.341117
	Gunner-Lubec Channel Light-house						2.828827
-9+10	Cranberry Point	75 45 12.8	+1.4	14.2			0.013565
-5+8	Gunner	90 05 58.7	-4.6	54.1		14 08 51.7	9.999999
	Lubec Channel Lighthouse	48.5		51.7			9.388141
	Cranberry Point-Lubec Channel Lighthouse						2.842391
	Cranberry Point-Gunner						2.230533 <sup>+1</sup>
	Gunner-Mam						3.336512
-9+11	Cranberry Point	78 36 65.9	-14.8	51.1			0.008632
-4+8	Gunner	96 58 47.7	+3.4	51.1			9.996769
-20+21	Mam	4 24 21.3	-3.5	17.8			8.885390
			-14.9				
	Cranberry Point-Mam						3.341913
	Cranberry Point-Gunner						2.230534
	Gunner-Lubec church spire						3.341117
-9+13	Cranberry Point	174 08 32.3	+3.28	35.58			0.991220
-7+8	Gunner	5 24 23.8	-11.02	12.78			8.973913
	Lubec church spire	03.9		11.64		0 27 11.64	7.898195
	Cranberry Point-Lubec church spire						3.306250
	Cranberry Point-Gunner						2.230532 <sup>+2</sup>
	Lubec Channel Lighthouse-Mam						<b>3.176968</b>
-10+11	Cranberry Point	2 51 53.1	-16.2	36.9		175 48 58.9	1.301899
	Lubec Channel Lighthouse	36.3		58.9			8.863046
-20+20'	Mam	1 19 30.6	-6.4	24.2			8.363526
	Cranberry Point-Mam						3.341913
	Cranberry Point-Lubec Channel Lighthouse						2.842339 <sup>-2</sup>

*Final computation of triangles—Continued*

Symbol	Station	Observed angle	Correc- tion	Spheri- cal angle	Spheri- cal excess	Plane angle	Loga- rithm
		° ' "	"	"		° ' "	
	Lubec church spire-Treat <sub>2</sub>						3.309932
-13+14	Cranberry Point	17 44 02.9	- 2.1	00.8			0.516283
	Lubec church spire	19.0		22.6		144 41 22.6	9.710102
-19+19'	Treat <sub>2</sub>	17 34 38.1	- 1.5	36.6			9.479985
	Cranberry Point-Treat <sub>2</sub>						3.588197
	Cranberry Point-Lubec church spire						3.306250
	Lubec church spire-Cranberry Point						3.306250
-15+16	Telegraph	128 38 50.9	- 6.1	53.8			0.107352
	Lubec church spire	47.5		49.2		30 51 49.2	9.710115
-12+13	Cranberry Point	20 29 12.6	+ 4.4	17.0			9.544083
	Telegraph-Cranberry Point						3.123717
	Telegraph-Lubec church spire						2.957685 <sup>-1</sup>
	Lubec church spire-Gunner						3.341117
-15+17	Telegraph	131 33 53.1	- 6.1	47.0			0.125967
	Lubec church spire	36.1		37.6		30 24 37.6	9.704315
- 6+ 7	Gunner	18 01 30.8	+ 4.6	35.4			9.490600
	Telegraph-Gunner						3.171399
	Telegraph-Lubec church spire						2.957684
	Cranberry Point-Gunner						2.230534
-16+17	Telegraph	2 54 53.2	0.0	53.2			1.293705
- 9+12	Cranberry Point	153 39 19.7	- 1.1	18.6			9.647160
- 6+ 8	Gunner	23 25 54.6	- 6.4	48.2			9.599478
			- 7.5				
	Telegraph-Gunner						3.171399
	Telegraph-Cranberry Point						3.123717
	Cranberry Point-Lubec Channel Lighthouse						2.842391
-16+18	Telegraph	29 52 44.5	+ 9.3	53.8			0.302588
-10+12	Cranberry Point	77 54 06.9	- 2.5	04.4			9.990245
	Lubec Channel Lighthouse	08.6		01.8		72 13 01.8	9.978738
	Telegraph-Lubec Channel Lighthouse						3.135224
	Telegraph-Cranberry Point						3.123717
	Gunner-Lubec Channel Lighthouse						2.828827
-17+18	Telegraph	26 57 51.3	+ 9.2	60.5			0.343447
- 5+ 6	Gunner	66 40 04.1	+ 1.8	05.9			9.962950
	Lubec Channel Lighthouse	64.6		53.6		86 21 53.6	9.999125
	Telegraph-Lubec Channel Lighthouse						3.135224
	Telegraph-Gunner						3.171399

## SECOND METHOD.

The only difference between the second method of adjustment and the first is in the treatment of the directions taken at the fixed points. At these points observation equations are written for the directions of new points only and the  $z$ 's are omitted. The observations taken over the fixed lines are not used, but the observed directions of the new lines are taken in connection with the adjusted direction of a fixed line, all directions being referred to a common initial line.

The equations with  $z$ 's omitted are the same as if the angle method of adjustment were used. (See p. 196.) In this treatment these equations are given unit weight. Jordan (*Vermessungskunde*, vol. 1, p. 179, of the third edition) suggests that on some accounts it would be better to assign the equations for observations at fixed points only half weight.

The observation equations for directions taken at Gunner, Cranberry Point, and Telegraph are the same as for the first method given on page 115 and are not repeated here. Below are given the observation equations for the remaining points, formed according to the second method. The assumed azimuths are identical with those used in the first method. As an example, to illustrate the computation of the observed azimuths, take the line Mam-Gunner. Use the observed direction for the new line and the adjusted one for the fixed line.

	°	'	''
Fixed azimuth Mam to Indian Point, page 106, =	266	17	19.2
Angle Indian Point to Gunner, page 106 ( $359^{\circ}$ $59' 55''.4$ to $322^{\circ} 01' 44''.8$ ), =	322	01	49.4
Observed azimuth Mam to Gunner, =	228	19	08.6

or by reckoning from any other fixed line through Mam the same result is reached, thus,

	°	'	''
Fixed azimuth Mam to Lubec Channel Light- house, =	225	14	19.0
Angle Lubec Channel Lighthouse to Gunner, page 106 ( $318^{\circ} 56' 55''.2$ to $322^{\circ} 01' 44''.8$ ), =	3	04	49.6
Observed azimuth Mam to Gunner, =	228	19	08.6

Note that the coefficients of the  $\delta\phi$ 's and  $\delta\lambda$ 's are exactly the same as for the first method and that the  $z$ 's are omitted.

*Formation of observation equations*TREAT<sub>2</sub>

Assumed azimuth ° ' "	Observed azimuth ° ' "	Equation	Station observed
337 10 43.7	337 10 39.4 + $v_9$	$v_9 = -637\delta\phi_2 + 1078\delta\lambda_2 + 4.3$	Cranberry Point

## MAM

223 44 53.7	223 44 47.3 + $v_{20}$	$v_{20} = -2010\delta\phi_2 - 1485\delta\lambda_2 + 6.4$	Cranberry Point Gunner
223 19 09.7	223 19 08.6 + $v_{21}$	$v_{21} = -2191\delta\phi_1 - 1383\delta\lambda_1 + 1.1$	

## LARRABEE

165 22 02.6	165 22 03.7 + $v_{22}$	$v_{22} = +541\delta\phi_1 - 1473\delta\lambda_1 - 1.1$	Gunner
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## INDIAN POINT

133 24 32.6	133 24 30.1 + $v_{23}$	$v_{23} = +2538\delta\phi_1 - 1709\delta\lambda_1 + 2.5$	Gunner
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## DUCK

119 38 22.1	119 38 24.0 + $v_{24}$	$v_{24} = +8669\delta\phi_1 - 3509\delta\lambda_1 - 1.9$	Gunner
-------------	------------------------	--	--------

The first part of each of tables 1 and 2 for the formation of normals according to the first method, pages 118 and 119, down to the line for  $v_{19}$  will serve for the second method also and is not repeated here. The remainder of the tables according to the second method is given below.

In forming the normal equations the four  $z$ 's that occur are eliminated by the device of the sum equation serving as a fictitious observation equation with negative weight. The other observations that do not contain  $z$ 's enter into the formation of the normal equations in the usual way. After the normal equations have been solved the four  $z$ 's are found from the sum equations in the way previously explained and enter into the computation of the  $v$ 's, or corrections, from 1 to 18 and 20 and 21, but not into the others.

Table for formation of normals No. 1

	1	2	3	4	5	6			
	$\partial\phi_1$	$\partial\lambda_1$	$\partial\phi_2$	$\partial\lambda_2$	$\partial\phi_3$	$\partial\lambda_3$	$\eta$	$p$	$\sqrt{p}$
19			- 637	+ 1078			+ 4.3	1	1
20			- 2010	- 1485			+ 6.4	1	1
21	- 2191	- 1388					+ 1.1	1	1
22	+ 541	- 1473					- 1.1	1	1
23	+ 2538	- 1709					+ 2.5	1	1
24	+ 8669	- 3509					- 1.9	1	1

Table for formation of normals No. 2

	1	2	3	4	5	6		
	$\partial\phi_1$	$\partial\lambda_1$	$\partial\phi_2$	$\partial\lambda_2$	$\partial\phi_3$	$\partial\lambda_3$	$\eta$	$\Sigma$
19			- 0.64	+ 1.08			+ 0.43	+ 0.87
20			- 2.01	- 1.48			+ 0.64	- 2.85
21	- 2.19	- 1.39					+ 0.11	- 3.47
22	+ 0.54	- 1.47					- 0.11	- 1.04
23	+ 2.54	- 1.71					+ 0.25	+ 1.08
24	+ 8.67	- 3.51					- 0.19	+ 4.97

Normal equations

1	2	3	4	5	6	$\eta$	$\Sigma$
+1014.5374	-863.2282 +918.6459	-822.5130 +876.9117 +924.4414	+781.8841 -837.3829 -831.3291 +843.4555	+ 8.0389 -13.2644 -39.8513 +36.7824 +43.7028	- 0.2265 + 3.9464 +18.6471 -15.9112 -33.6441 +41.7793	- 8.5215 + 5.8167 + 3.7521 + 2.6609 -18.8752 +30.2371	+109.9712 + 91.4452 +130.0589 - 19.8403 - 17.1109 + 44.8281

## Solution of normals

1	2	3	4	5	6	$\eta$	$\zeta$
+1014.5374 $\partial\phi_1$	-863.2282 + 0.800589	-822.5130 + 0.8107271	+781.8841 - 0.7708904	+ 8.0389 - 0.0079237	- 0.2265 + 0.0002233	- 8.5215 + 0.0083994	+109.9712 - 0.1083954
1	+918.6459 -734.4554	+876.9117 -699.8425	+837.3529 +665.2730	-13.2644 + 6.8400	+ 3.9464 - 0.1927	+ 5.8167 - 7.2506	+ 91.4452 + 98.5700
	+184.1605 $\partial h$	+177.0692 - 0.961494	-172.1099 + 0.934565	- 6.4244 + 0.034885	+ 3.7537 - 0.020383	- 1.4339 + 0.007786	+185.0152 - 1.004641
	1 2	+924.4414 -686.8336 -170.2510	+831.3291 +633.8946 +165.4826	-39.8513 + 6.5174 + 6.1770	+18.6471 - 0.1336 - 3.6092	+ 3.7521 - 6.9086 + 1.3787	+130.0589 + 89.1566 -177.8910
		+ 87.3568 $\partial\phi_2$	- 31.9519 + 0.365763	-27.1569 + 0.310873	+14.8543 - 0.170042	- 1.7778 + 0.020351	+ 41.3245 - 0.473054
		1 2 3	+843.4555 -602.5828 -100.8479 - 11.6868	+36.7824 - 6.1954 - 6.0040 - 9.9830	-15.9112 + 0.1746 + 2.5081 + 3.4332	+ 2.6609 + 6.5674 - 1.3401 - 0.6503	- 19.8403 - 84.7528 +172.9087 + 13.1150
			+ 68.3380 $\partial\lambda_3$	+14.6500 - 0.214376	- 6.7953 + 0.099437	+ 7.2379 - 0.105913	+ 83.4306 - 1.220852
		1 2 3 4		+43.7028 - 0.0637 - 0.2241 - 8.4423 - 3.1406	-33.6441 + 0.0018 + 0.1309 + 4.6178 + 1.4567	-18.8752 + 0.0675 - 0.0500 - 0.5527 - 1.5516	17.1109 - 0.8714 + 6.4543 + 12.8407 - 17.8855
				+31.8321 $\partial\phi_3$	-27.4369 + 0.861926	-20.9620 + 0.658518	- 16.5668 - 0.520443
				1 2 3 4 5	+41.7793 - 0.0001 - 0.0765 - 2.5259 - 0.6757 -23.6486	+30.2371 - 0.0019 + 0.0282 + 0.3023 + 0.7197 -13.0677	44.8281 + 0.0246 - 3.7712 - 7.0289 8.2861 - 14.2794
					+14.8525 $\partial\lambda_3$	+13.2187 - 0.89900	+ 28.0712 - 1.89000

*Back solution*

$\delta\lambda_3$	$\delta\phi_3$	$\delta\lambda_2$	$\delta\phi_2$	$\delta\lambda_1$	$\delta\phi_1$
-0.89000	+0.65852	-0.10591	+0.02035	+0.00779	+0.00840
	-0.76711	-0.08850	+0.15131	+0.01814	-0.00020
-0.89000		+0.02328	-0.03376	-0.00379	+0.00086
	-0.10859		-0.15993	-0.07244	+0.13189
		-0.17113	+0.07534	-0.21023	+0.06108
					-0.17888
					+0.02315

*Computation of corrections*

1	2	3	4	5	6	7	8
+ 2.007	+ 0.588	+ 0.125	- 0.507	- 1.796	- 0.843	- 0.433	- 4.934
+ 7.377	+ 3.593	+ 3.097	+ 2.918	+ 8.058	- 3.389	- 3.322	-46.059
-10.753	-10.753	-10.753	-10.753	-10.753	- 3.956	-10.753	+16.057
	- 2.2	+ 4.8	+ 3.6	+ 7.1	+14.347	+25.5	+37.493
- 1.369					-10.753		-10.753
- 1.4	- 8.772	- 2.731	- 4.742	+ 2.609	+ 9.7	+10.992	+ 7.1
	- 8.8	- 2.8	- 4.8	+ 2.6		+11.0	
					+ 5.106		- 1.096
					+ 5.1		- 1.1
$z_1$	9	10	11	12	13	14	$z_2$
- 5.793	- 4.934	- 4.530	- 1.514	- 3.156	- 1.541	- 0.480	- 4.934
-27.727	-46.059	+ 8.402	+ 2.541	- 2.824	- 2.911	- 1.845	-46.059
+16.057	+16.057	- 1.223	- 1.223	- 4.549	- 1.223	- 1.223	+ 4.836
+37.493	+37.493		-13.1	+14.685	+11.5	+ 6.8	+40.857
- 3.956	- 1.223	+ 2.649		- 1.223			- 4.549
+14.347		+ 2.7	-13.296	- 2.7	+ 5.825	+ 3.252	+14.685
+55.6	+ 1.334		-13.3		+ 5.8	+ 3.2	+ 2.5
	+ 1.3			+ 0.233			
+86.021				+ 0.2			+ 7.336
-10.753							- 1.223
15	16	17	18	$z_3$	19	20	21
+ 1.282	- 3.156	- 0.843	- 2.620	- 0.843	- 0.480	- 1.514	- 0.507
-43.806	- 2.824	- 3.389	+25.267	- 3.889	- 1.845	+ 2.541	+ 2.918
+44.790	- 4.549	- 3.956	+44.790	- 3.156	+ 4.3	+ 6.4	+ 1.1
	+14.685	+14.347	-62.2	- 2.824			
+ 2.266	+44.790	+44.790		- 9.843	+ 1.975	+ 7.427	+ 3.511
+ 2.2	-52.6	-54.8	+ 5.237	+10.493	+ 2.0	+ 7.5	+ 3.5
			+ 5.3	-169.6			
	- 3.654	- 3.851					
	- 3.7	- 3.9		-179.162			
				+ 44.790			
22	23	24					
+ 0.125	+ 0.588	+ 2.007					
+ 3.097	+ 3.593	+ 7.377					
- 1.1	+ 2.5	- 1.9					
+ 2.122	+ 6.681	+ 7.484					
+ 2.1	+ 6.7	+ 7.5					

*Final computation of triangles*

Symbol	Station	Observed angle	Correc- tion	Spher- ical angle	Spher- ical excess	Plane angle	Loga- rithm
		° ' "	"	"		° ' "	
-1+ 3	Duck-Larrahee						<b>3.410111</b>
+24	Gunner	45 43 29.4	- 1.4	28.0			0.145093
-22	Duck	124 02 00.9	+ 7.5	08.4			9.918392
	Larrahee	10 14 25.7	- 2.1	23.6			9.249858
			+ 4.0				
	Gunner-Larrahee						3.473596-1
	Gunner-Duck						2.805062
-1+ 4	Duck-Mam						<b>3.389282</b>
+24	Gunner	108 41 53.9	- 3.4	50.5			0.023547
-21	Duck	57 00 57.8	+ 7.5	65.3			9.923681
	Mam	14 17 07.7	- 3.5	04.2			9.392234
			+ 0.6				
	Gunner-Mam						3.336510
	Gunner-Duck						2.805063-1
-1+ 5	Duck-Lubec Channel Light- house						<b>3.045619</b>
+24	Gunner	115 34 42.9	+ 4.0	46.9			0.044800
	Duck	33 11 41.9	+ 7.5	49.4			9.738400
	Lubec Channel Lighthouse	35.2		23.7		31 13 23.7	9.714643
	Gunner-Lubec Channel Lighthouse						2.828819
	Gunner-Duck						2.805062
-2+ 3	Indian Point-Larrahee						<b>3.236668</b>
+23	Gunner	31 57 41.4	+ 6.0	47.4			0.276237
-22	Indian Point	114 00 36.0	+ 6.7	42.7			9.960690
	Larrahee	34 01 32.0	- 2.1	29.9			9.747842
			+10.6				
	Gunner-Larrahee						3.473595
	Gunner-Indian Point						3.260747
-2+ 4	Indian Point-Mam						<b>3.470097</b>
+23	Gunner	94 56 05.9	+ 4.0	09.9			0.001614
-21	Indian Point	47 05 36.3	+ 6.7	43.0			9.864800
	Mam	37 58 10.6	- 3.5	07.1			9.789037
			+ 7.2				
	Gunner-Mam						3.336511-1
	Gunner-Indian Point						3.260748-1
-2+ 5	Indian Point-Lubec Channel Lighthouse						<b>3.315762</b>
+23	Gunner	101 48 54.9	+11.4	66.3			0.009305
	Indian Point	18 35 56.3	+ 6.7	63.0			9.503754
	Lubec Channel Lighthouse	68.8		50.7		59 34 50.7	9.935680
	Gunner-Lubec Channel Lighthouse						2.828821-1
	Gunner-Indian Point						3.260747

*Final computation of triangles—Continued*

Symbol	Station	Observed angle	Correc- tion	Spher- ical angle	Spher- ical excess	Plane angle	Loga- rithm
		° ' "	"	"		° ' "	
-3+ 4 +22 -21	Larrabee-Mam						3.443126
	Gunner	62 58 24.5	- 2.0	22.5			0.050224
	Larrahee	44 10 37.0	+ 2.1	39.1			9.843160
	Mam	72 50 61.9	- 3.5	58.4			9.980246
			- 3.4				
	Gunner-Mam						3.336510
	Gunner-Larrabee						3.473596 <sup>-1</sup>
	Larrabee-Lubec Channel Lighthouse						3.449573
-3+ 5 +22	Gunner	69 51 13.5	+ 5.4	18.9			0.027415
	Larrabee	12 59 29.9	+ 2.1	32.0			9.351833
	Lubec Channel Lighthouse	16.6		09.1		97 09 09.1	9.996607
	Gunner-Lubec Channel Lighthouse						2.828821 <sup>-1</sup>
	Gunner-Larrabee						3.473595
	Larrabee-Lubec church spire						3.702872
-3+ 7 +22	Gunner	154 32 48.4	+13.8	62.2			0.366819
	Larrabee	10 46 00.0	+ 2.1	02.1		14 40 55.7	9.271423
	Lubec church spire	71.6		55.7			9.408903
	Gunner-Lubec church spire						3.341114 <sup>+1</sup>
	Gunner-Larrabee						3.473594 <sup>+1</sup>
	Mam-Lubec Channel Light- house						3.176968
-4+ 5 +21	Gunner	6 52 49.0	+ 7.4	56.4			0.921432
	Mam	3 04 49.6	+ 3.5	53.1		170 02 10.5	8.730418
	Lubec Channel Lighthouse	21.4		10.5			9.238109
	Gunner-Lubec Channel Light- house						2.828818 <sup>+2</sup>
	Gunner-Mam						3.336509 <sup>+1</sup>
	Lubec church spire-Indian Point						3.603122
+2- 7 -23	Gunner	173 29 30.2	-19.8	10.4		2 57 21.9	0.945226
	Lubec church spire	56 55.4		81.9			8.712401
	Indian Point	3 33 34.4	- 6.7	27.7			8.792767
	Gunner-Indian Point						3.260749 <sup>-2</sup>
	Gunner-Lubec church spire						3.341115
	Gunner-Lubec Channel Light- house						2.828820
-9+10 -5+ 8	Cranberry Point	75 45 12.8	+ 1.4	14.2			0.013565
	Gunner	90 05 58.7	- 3.7	55.0		14 08 50.8	9.999999
	Lubec Channel Lighthouse	48.5		50.8			9.388133
	Cranberry Point-Lubec Chan- nel Lighthouse						2.842384
	Cranberry Point-Gunner						2.230518

## Final computation of triangles—Continued

Symbol	Station	Observed angle	Correc- tion	Spher- ical angle	Spher- ical excess	Plane angle	Loga- rithm
		° ' "	"	"		° ' "	
-9+11	Gunner-Mam						3.336510
-4+ 8	Cranberry Point	78 36 65.9	-14.6	51.3			0.008632
-20+21	Gunner	96 58 47.7	+ 3.7	51.4			9.996768
	Mam	4 24 21.3	- 4.0	17.3			8.885376
			-14.9				
	Cranberry Point-Mam						3.341910
	Cranberry Point-Gunner						2.230518
-9+13	Gunner-Lubec church spire						3.341115
-7+ 8	Cranberry Point	174 08 32.3	+ 4.5	36.8			0.991245
	Gunner	5 24 23.8	-12.1	11.7			8.973889
	Lubec church spire	03.9		11.5		0 27 11.5	7.898156
	Cranberry Point-Lubec church spire						3.306249
	Cranberry Point-Gunner						3.230516 <sup>+2</sup>
	Lubec Channel Lighthouse-Mam						3.176968
-10+11	Cranberry Point	2 51 53.1	-16.0	37.1			1.301891
-20	Lubec Channel Lighthouse	35.2		58.7		175 48 58.7	8.863051
	Mam	1 19 31.7	- 7.5	24.2			8.363526
	Cranberry Point-Mam						3.241910
	Cranberry Point-Lubec Channel Lighthouse						2.842385 <sup>-1</sup>
-13+14	Lubec church spire-Treat <sub>2</sub>						3.309982
	Cranberry Point	17 44 02.9	- 2.6	00.3			0.516286
	Lubec church spire	19.0		23.6		144 41 23.6	9.761929
-19	Treat <sub>2</sub>	17 34 38.1	- 2.0	36.1			9.479981
	Cranberry Point-Treat <sub>2</sub>						3.588197
	Cranberry Point-Lubec church spire						3.306249
	Lubec church spire-Cranberry Point						3.306249
-15+16	Telegraph	128 38 59.9	- 5.9	54.0			0.107352
	Lubec church spire	47.5		47.8		30 51 47.8	9.710110
-12+13	Cranberry Point	20 29 12.6	+ 5.6	18.2			9.544090
	Telegraph-Cranberry Point						3.123711
	Telegraph-Lubec church spire						2.957691
-15+17	Lubec church spire-Gunner						3.341115
	Telegraph	131 33 53.1	- 6.1	47.0			0.125967
	Lubec church spire	36.1		36.3		30 24 36.3	9.704310
-6+ 7	Gunner	18 01 30.8	+ 5.9	36.7			9.490609
	Telegraph-Gunner						3.171382
	Telegraph-Lubec church spire						2.957691
-16+17	Cranberry Point-Gunner						2.230518
-9+12	Telegraph	2 54 53.2	- 0.2	53.0			1.293713
-6+ 8	Cranberry Point	153 39 19.7	- 1.1	18.6			9.647161
	Gunner	23 25 54.6	- 6.2	48.4			9.599479
			- 7.5				
	Telegraph-Gunner						3.171382
	Telegraph-Cranberry Point						3.123710 <sup>+1</sup>

*Final computation of triangles—Continued*

Symbol	Station.	Observed angle	Correc- tion	Spher- ical angle	Spher- ical excess	Plane angle	Loga- rithm
		° ' "	"	"		° ' "	
	Cranberry Point-Lubec Chan- nel Lighthouse						2.842384
-16+18	Telegraph	29 52 44.5	+ 9.0	53.5			0.302589
-10+12	Cranberry Point	77 54 06.9	- 2.5	04.4			9.990245
	Lubec Channel Lighthouse	08.6		02.1		72 13 02.1	9.978738
	Telegraph-Lubec Channel Lighthouse						3.135218
	Telegraph-Cranberry Point						3.123711
	Gunner-Lubec Channel Lighthouse						2.828820
-17+18	Telegraph	26 57 51.3	+ 9.2	60.5			0.343447
- 5+ 6	Gunner	66 40 04.1	+ 2.5	06.6			9.962951
	Lubec Channel Lighthouse	04.6		52.9		86 21 52.9	9.999125
	Telegraph-Lubec Channel Lighthouse						3.135218
	Telegraph-Gunner						3.171392

*Final position computation,*

STATION GUNNER

$\alpha$ Second angle $\alpha$ $\Delta\alpha$ $\alpha'$	Duck to Lubec Channel Lighthouse Lubec Channel Lighthouse and Gunner			$^{\circ}$ 86 +33	$'$ 26 11	$''$ 42.1 49.4	
	Duck to Gunner			119 —	38	31.5 17.8	
	Gunner to Duck			180 299 115	38 34	13.7 46.9	
	First angle of triangle						
$\phi$ $\Delta\phi$ $\phi'$	$^{\circ}$ 44 +	$'$ 50	$''$ 33.886 10.227	Duck	$\lambda$ $\Delta\lambda$ +	66 57 47.822 25.263	
	44	50	44.113	Gunner	$\lambda'$	66 58 13.085	
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 44	$'$ 50	$''$ 39	$s$ $\cos \alpha$ B	2.805062 9.694237 8.510480	$s^2$ $\sin^2 \alpha$ C	5.6101 9.8782 1.4016
1st term	$''$ —10.2277		h	1.009779	6.8899		
2d term	+ 0.0008						
$-\Delta\phi$	—10.2269						
	$s$ $\sin \alpha$ $A'$ $\sec \phi'$	2.805062 9.939086 8.508994 0.149348	$\Delta\lambda$ $\sin \frac{1}{2}(\phi+\phi')$	1.402490 9.848301	1.250791		
	1.402490						
	$\Delta\lambda$	$''$ +25.2633	$-\Delta\alpha$	$''$ +17.82			

## STATION CRANBERRY POINT

$\alpha$ Second angle	Gunner to Lubec Channel Lighthouse Lubec Channel Lighthouse and Cranberry Point			$^{\circ}$ 55 + 90	$'$ 13 05	$''$ 00.6 55.0		
	Gunner to Cranberry Point			145 —	18	55.6 03.1		
$\alpha$ $\Delta\alpha$	Cranberry Point to Gunner			180 325 75	18 45	52.5 14.2		
$\alpha'$	First angle of triangle							
$\phi$ $\Delta\phi$	$^{\circ}$ 44 +	$'$ 50	$''$ 44.113 04.529	Gunner	$\lambda$ $\Delta\lambda$	66 +	58	13.085 04.406
$\phi'$	44	50	48.642	Cranberry Point	$\lambda'$	66	58	17.491
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 44	$'$ 50	$''$ 46	$s$ $\cos \alpha$ B	2.230518 9.915029 8.510480	$s^2$ $\sin^2 \alpha$ C	4.4610 9.5103 1.4016	
1st term	$''$ —4.5293		h	0.656027		5.3729		
2d term	+0.0000							
$-\Delta\phi$	—4.5293							
	$s$ $\sin \alpha$ $A'$ $\sec \phi'$	2.230518 9.755156 8.508994 0.149357	$\sin \frac{\Delta\lambda}{2(\phi+\phi')}$	0.644025 9.848315	0.492340			
	0.644025							
	$\Delta\lambda$	+4.4058	$-\Delta\alpha$	+3.1				

secondary triangulation

## STATION GUNNER

$\alpha$ Third angle	Lubec Channel Lighthouse to Duck Gunner and Duck					$^{\circ}$ 266 — 31	$'$ 26 13	$''$ 06.5 23.7
$\alpha$ $\Delta\alpha$	Lubec Channel Lighthouse to Gunner					235 +	12	42.8 17.8
$\alpha'$	Gunner to Lubec Channel Lighthouse					180 55	13	00.6
$\phi$	$^{\circ}$ 44	$'$ 50	$''$ 31.652	Lubec Channel Lighthouse	$\lambda$	66	58	38.299
$\Delta\phi$	+		12.461		$\Delta\lambda$	—		25.214
$\phi'$	44	50	44.113	Gunner	$\lambda'$	66	58	13.085
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 44	$'$ 50	$''$ 36	$s$ $\cos \alpha$ B	2.828819 9.756288 8.510480	$s^2$ $\sin^2 \alpha$ C	5.6576 9.8290 1.4016	
1st term	$''$ —12.4620			h	1.095587		6.8882	
2d term	+ 0.0008							
$-\Delta\phi$	—12.4612							
	$s$ $\sin \alpha$ A'	2.828819 9.914485 8.508994		$\Delta\lambda$ $\sin \frac{1}{2}(\phi+\phi')$	1.401646 9.848294			
	$\sec \phi'$	0.149348						
		1.401646			1.249940			
		$''$			$''$			
	$\Delta\lambda$	—25.2142		$-\Delta\alpha$	—17.78			

## STATION CRANBERRY POINT

$\alpha$ Third angle	Lubec Channel Lighthouse to Gunner Cranberry Point and Gunner					$^{\circ}$ 235 — 14	$'$ 12 08	$''$ 42.8 50.8
$\alpha$ $\Delta\alpha$	Lubec Channel Lighthouse to Cranberry Point					221 +	03	52.0 14.7
$\alpha'$	Cranberry Point to Lubec Channel Lighthouse					180 41	04	06.7
$\phi$	$^{\circ}$ 44	$'$ 50	$''$ 31.652	Lubec Channel Lighthouse	$\lambda$	66	58	38.299
$\Delta\phi$	+		16.990		$\Delta\lambda$	—		20.808
$\phi'$	44	50	48.642	Cranberry Point	$\lambda'$	66	58	17.491
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 44	$'$ 50	$''$ 40	$s$ $\cos \alpha$ B	2.842384 9.877355 8.510480	$s^2$ $\sin^2 \alpha$ C	5.6848 9.6350 1.4016	
1st term	$''$ —16.9910			h	1.230219		6.7214	
2d term	+ 0.0005							
$-\Delta\phi$	—16.9905							
	$s$ $\sin \alpha$ A'	2.842384 9.817504 8.508994		$\Delta\lambda$ $\sin \frac{1}{2}(\phi+\phi')$	1.318239 9.848303			
	$\sec \phi'$	0.149357						
		1.318239			1.166542			
		$''$			$''$			
	$\Delta\lambda$	—20.8084		$-\Delta\alpha$	—14.67			

*Final position computation,*

## STATION TELEGRAPH

$\alpha$	Cranberry Point to Lubec Channel Lighthouse					$^{\circ}$	$'$	$''$	
Second angle	Lubec Channel Lighthouse and Telegraph					41	04	06.7	
						+ 77	54	04.4	
$\alpha$	Cranberry Point to Telegraph					118	58	11.1	
$\Delta\alpha$						—		37.4	
$\alpha'$	Telegraph to Cranberry Point					180			
	First angle of triangle					298	57	33.7	
						29	52	53.4	
$\phi$	$^{\circ}$	$'$	$''$	Cranberry Point	$\lambda$	66	58	17.491	
$\Delta\phi$	44	50	48.642						
	+		20.858		$\Delta\lambda$	+		52.973	
$\phi'$	44	51	09.500	Telegraph	$\lambda'$	66	59	10.464	
			+1						
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$	$'$	$''$	$s$	3.123711	$s^2$	6.2474		
	44	50	59	$\cos \alpha$	9.685157	$\sin^2 \alpha$	9.8839		
				B	8.510480	C	1.4016		
1st term	$''$			h	1.319348		7.5329		
2d term	-20.8617								
	+ 0.0034								
$-\Delta\phi$	-20.8583								
	$s$	3.123711	$\sin \frac{1}{2}(\phi+\phi')$						
	$\sin \alpha$	9.941946				1.724052			
	$\Delta'$	8.508994				9.848343			
	$\sec \phi'$	0.149401				1.572395			
		1.724052							
		$''$							
$\Delta\lambda$		+52.9728		$-\Delta\alpha$		+37.36			



*Computation of probable errors*

	Adopted $v$ 's first solution	$v^2$		Adopted $v$ 's second solution	$v^2$
1	+0.2	0.04	1	-1.4	1.96
2	-8.5	72.25	2	-8.8	77.44
3	-2.7	7.29	3	-2.8	7.84
4	-4.9	24.01	4	-4.8	23.04
5	+3.1	9.61	5	+2.6	6.76
6	+4.9	24.01	6	+5.1	26.01
7	+9.5	90.25	7	+11.0	121.00
8	-1.5	2.25	8	-1.1	1.21
9	+1.7	2.89	9	+1.3	1.69
10	+3.1	9.61	10	+2.7	7.29
11	-13.1	171.61	11	-13.3	176.89
12	+0.6	0.36	12	+0.2	0.04
13	+5.0	25.00	13	+5.8	33.64
14	+2.9	8.41	14	+3.2	10.24
15	+2.3	5.29	15	+2.2	4.84
16	-3.8	14.44	16	-3.7	13.69
17	-3.8	14.44	17	-3.9	15.21
18	+5.4	29.16	18	+5.3	28.09
19	+0.8	0.64	19	+2.0	4.00
19'	-0.7	0.49	20	+7.5	56.25
20	+5.6	31.36	21	+3.5	12.25
21	+2.1	4.41	22	+2.1	4.41
20'	-0.8	0.64	23	+6.7	44.89
20''	-1.5	2.25	24	+7.5	56.25
20'''	-6.5	42.25			
20''''	+1.2	1.44			734.93
22	+2.6	6.76			
22'	-0.7	0.49			
22''	-3.7	13.69			
22'''	+5.7	32.49			
22''''	-0.6	0.36			
22''	-3.3	10.89			
23	+1.9	3.61			
23'	-4.2	17.64			
23''	-5.5	30.25			
23'''	-2.8	7.84			
23''''	+10.7	114.49			
24	+6.5	42.25			
24'	-3.0	9.00			
24''	-3.4	11.56			
		895.72			

In the first method of adjustment, there are 40 equations to determine 14 unknown quantities, namely,  $6\delta\phi$ 's and  $\delta\lambda$ 's and  $8z$ 's. The probable error of an observed direction is therefore,

$$0.6745\sqrt{\frac{895.72}{40-14}} = \pm 4.0''$$

In the second method there are 24 equations to determine 9 unknown quantities, namely,  $6\delta\phi$ 's and  $\delta\lambda$ 's and  $3z$ 's at new points. The probable error of an observed direction is therefore,

$$0.6745\sqrt{\frac{734.93}{15}} = \pm 4.7''$$

## ADJUSTMENT OF A FIGURE CONTAINING LATITUDE, LONGITUDE, AZIMUTH, AND LENGTH CONDITIONS BY THE METHOD OF VARIATION OF GEOGRAPHIC COORDINATES

This example illustrates the fitting of a chain of triangulation in between fixed lines at the ends. The necessary preliminary computations of the assumed positions and directions could have been carried out in much the same way as in the preceding examples. A preliminary figure adjustment was, however, available and the results of it were used in the preliminary computations of the triangles and the geographic positions, pages 140–157. The fixed lines at the ends of the chain are shown in the following list of fixed positions. The list of observed directions is not given. The necessary data may be derived from the observed angles of the triangles, pages 140–143, taken in connection with figure 7, page 157.

*Table of fixed positions*

Station	Latitude and longitude	Azimuth	Back azimuth	To station	Loga- rithm of distance.
	° ' "	° ' "	° ' "		
Fort Morgan.	30 13 42.242 88 01 23.228				
Dauphin Island east base.	30 14 56.379 88 08 14.288	281 42 17.9	101 45 44.9	Fort Morgan.	4.050203
Dauphin Island west base.	30 14 21.492 88 14 51.034	264 11 22.1	84 14 41.9	Dauphin Island east base	4.027832
Biloxi Lighthouse.	30 23 39.419 88 54 03.820				
Ship Island Lighthouse.	30 12 45.341 88 57 57.464	197 12 19.7	17 14 17.6	Biloxi Lighthouse.	4.323998

*Preliminary computation of triangles*

Station	Observed angle	Correc- tion	Spher- ical angle	Spher- ical excess	Plane angle	Loga- rithm
Fort Morgan-Dauphin Island east base Cedar	43 54 21.8	+ 0.0	21.8	0.1	21.7	4. 050203 0. 158968
Fort Morgan	42 33 37.7	+ 0.3	38.0	0.1	37.9	9. 830183
Dauphin Island east base	93 31 59.6	+ 0.9	60.5	0.1	32 00.4	9. 999174
		+ 1.2		0.3		
Cedar-Dauphin Island east base Cedar-Fort Morgan						4. 039354 4. 208345
Dauphin Island east base-Dauphin Is- land west base						4. 027832
Cedar	37 26 33.0	- 0.9	32.1	0.1	32.0	0. 216124
Dauphin Island east base	103 55 36.4	- 0.9	35.5	0.1	35.4	9. 987043
Dauphin Island west base	38 37 52.5	+ 0.2	52.7	0.1	52.6	9. 795398
		- 1.6		0.3		
Cedar-Dauphin Island west base Cedar-Dauphin Island east base						4. 230999 4. 039354
Cedar-Dauphin Island east base Cat	69 30 32.7	+ 0.8	33.5	0.0	33.5	4. 039354 0. 028386
Cedar	60 11 14.2	+ 1.4	15.6	0.1	15.5	9. 938349
Dauphin Island east base	50 18 11.4	- 0.3	11.1	0.1	11.0	9. 886171
		+ 1.9		0.2		
Cat-Dauphin Island east base Cat-Cedar						4. 006089 3. 953911
Cedar-Dauphin Island west base Cat	135 31 54.9	+ 2.7	57.6	0.0	57.6	4. 230999 0. 154580
Cedar	22 44 41.2	+ 2.3	43.5	0.1	43.4	9. 587303
Dauphin Island west base	21 43 18.7	+ 0.4	19.1	0.1	19.0	9. 568322
		+ 5.4		0.2		
Cat-Dauphin Island west base Cat-Cedar						3. 972892 3. 953911
Dauphin Island east base-Dauphin Is- land west base						4. 027832
Cat	66 01 22.2	+ 1.9	24.1	0.1	24.0	0. 039191
Dauphin Island east base	53 37 25.0	- 0.6	24.4	0.1	24.3	9. 905869
Dauphin Island west base	60 21 11.2	+ 0.6	11.8	0.1	11.7	9. 939066
		+ 1.9		0.3		
Cat-Dauphin Island west base Cat-Dauphin Island east base						3. 972892 4. 006089
Cedar-Cat Pins	23 22 40.3	- 1.8	38.5	0.1	38.4	3. 953911 0. 401445
Cedar	30 54 61.5	- 3.5	58.0	0.1	57.9	9. 710779
Cat	18.4		23.7	0.0	125 42 23.7	9. 909565
				0.2		
Pins-Cat Pins-Cedar						4. 066135-1 4. 264921
Cedar-Dauphin Island west base Pins	58 45 26.0	- 2.8	23.2	0.2	23.0	4. 230999 0. 068049
Cedar	53 39 42.7	- 1.2	41.5	0.2	41.3	9. 906082
Dauphin Island west base	67 34 57.9	- 2.0	55.9	0.2	55.7	9. 965873
		- 6.0		0.6		
Pins-Dauphin Island west base Pins-Cedar						4. 205130 4. 264921
Cat-Dauphin Island west base Pins	35 22 45.7	- 1.0	44.7	0.1	44.6	3. 972892 0. 237334
Cat	35.3		38.7	0.0	98 45 38.7	9. 994903
Dauphin Island west base	45 51 39.2	- 2.4	36.8	0.1	36.7	9. 855908
				0.2		
Pins-Dauphin Island west base Pins-Cat						4. 205129-1 4. 066134

*Preliminary computation of triangles—Continued*

Station	Observed angle	Correc- tion	Spher- ical angle	Spher- ical excess	Plane angle	Loga- rithm
	" ' "	"	"	"	" ' "	
Pins-Dauphin Island west base						4.205130
Grand	54 52 01.6	+ 1.4	00.2	0.2	00.0	0.087345
Pins	85 13 07.0	+ 1.3	08.3	0.1	08.2	9.998486
Dauphin Island west base	39 54 50.9	+ 1.1	52.0	0.2	51.8	9.807293
		+ 1.0		0.5		
Grand-Dauphin Island west base						4.290961
Grand-Pins						4.099768
Grand-Pins						4.099768
Petit	33 09 08.7	- 0.3	08.4	0.1	08.3	0.262119
Grand	114 03 53.6	- 0.6	53.0	0.1	52.9	9.960511
Pins	32 46 57.5	+ 1.4	58.9	0.1	58.8	9.733565
		+ 0.5		0.3		
Petit-Pins						4.322398
Petit-Grand						4.095452
Grand-Dauphin Island west base						4.290961
Petit	81 41 28.2	+ 0.1	28.3	0.1	28.2	0.004583
Grand	59 11 52.0	+ 0.8	52.8	0.2	52.6	9.933964
Dauphin Island west base	39 06 39.1	+ 0.3	39.4	0.2	39.2	9.799908
		+ 1.2		0.5		
Petit-Dauphin Island west base						4.229508
Petit-Grand						4.095452
Pins-Dauphin Island west base						4.205130
Petit	48 32 19.5	+ 0.4	19.9	0.2	19.7	0.125284
Pins	52 26 09.5	- 0.1	09.4	0.2	09.2	9.890093
Dauphin Island west base	79 01 30.0	+ 1.4	31.4	0.3	31.1	9.991984
		+ 1.7		0.7		
Petit-Dauphin Island west base						4.229507 <sup>+1</sup>
Petit-Pins						4.322398
Grand-Petit						4.095452
Pascagoula	37 39 20.6	+ 3.0	23.6	0.1	23.5	0.214011
Grand	104 18 56.1	+ 1.5	57.6	0.2	57.4	9.986300
Petit	38 01 38.6	+ 0.6	39.2	0.1	39.1	9.789603
		+ 5.1		0.4		
Pascagoula-Petit						4.295763
Pascagoula-Grand						4.099072
Pascagoula-Grand						4.099072
Horn	38 49 39.0	+ 3.8	42.8	0.1	42.7	0.202738
Pascagoula	97 55 54.0	+ 3.5	57.5	0.2	57.3	9.995824
Grand	43 14 18.9	+ 1.2	20.1	0.1	20.0	9.835717
		+ 8.5		0.4		
Horn-Grand						4.297634 <sup>+1</sup>
Horn-Pascagoula						4.137527
Pascagoula-Petit						4.295763
Horn	77 06 13.2	+ 2.9	16.1	0.2	15.9	0.011094
Pascagoula	60 16 33.4	+ 0.5	33.9	0.2	33.7	9.938732
Petit	42 37 10.3	+ 0.3	10.6	0.2	10.4	9.830670
		+ 3.7		0.6		
Horn-Petit						4.245589
Horn-Pascagoula						4.137527
Grand-Petit						4.095452
Horn	38 16 34.2	- 0.9	33.3	0.2	33.1	0.207995
Grand	61 04 37.2	+ 0.3	37.5	0.2	37.3	9.942142
Petit	80 38 48.9	+ 0.9	49.8	0.2	49.6	9.994188
		+ 0.3		0.6		
Horn-Petit						4.245589
Horn-Grand						4.297635

*Preliminary computation of triangles—Continued*

Station	Observed angle	Correc- tion	Spher- ical angle	Spher- ical excess	Plane angle	Loga- rithm
	° ' "	"	"	"	° ' "	
Pascagoula-Horn						4.137527
Belle	48 58 49.8	- 1.8	48.0	0.2	47.8	0.123352
Pascagoula	69 43 28.0	- 3.4	24.6	0.1	24.5	9.972217
Horn	61 17 53.5	- 5.6	47.9	0.2	47.7	9.943058
		-10.8		0.5		
Belle-Horn						4.232096
Belle-Pascagoula						4.202937
Belle-Pascagoula						4.202937
Club	58 40 43.3	+ 1.1	44.4	0.1	44.3	0.068406
Belle	89 28 55.3	- 0.6	54.7	0.2	54.5	9.999982
Pascagoula	31 50 23.4	- 2.1	21.3	0.1	21.2	9.722253
		- 1.6		0.4		
Club-Pascagoula						4.271325
Club-Belle						3.993596
Belle-Horn						4.232096
Club	105 43 56.9	- 0.4	56.5	0.1	56.4	0.016582
Belle	40 30 05.5	+ 1.2	06.7	0.1	06.6	9.812561
Horn	33 45 53.7	+ 3.4	57.1	0.1	57.0	9.744918
		+ 4.2		0.3		
Club-Horn						4.061239
Club-Belle						3.993596
Pascagoula-Horn						4.137527
Club	47 03 13.6	- 1.5	12.1	0.1	12.0	0.135426
Pascagoula	37 53 04.6	- 1.3	03.3	0.1	03.2	9.788216
Horn	95 03 47.2	- 2.2	45.0	0.2	44.8	9.998302
		- 5.0		0.4		
Club-Horn						4.061239
Club-Pascagoula						4.271325
Belle-Club						3.993596
Deer	41 02 10.7	+ 0.7	11.4	0.1	11.3	0.182739
Belle	102 35 18.5	+ 1.5	20.0	0.0	20.0	9.989432
Club	36 22 26.8	+ 2.0	28.8	0.1	28.7	9.773101
		+ 4.2		0.2		
Deer-Club						4.165767
Deer-Belle						3.949436
Deer-Belle						3.949436
Ship	33 10 60.2	- 1.8	58.4	0.1	58.3	0.261704
Deer	97 49 37.9	- 1.8	36.1	0.1	36.0	9.995035
Belle	48 59 22.9	+ 2.9	25.8	0.1	25.7	9.877717
		- 0.7		0.3		
Ship-Belle						4.207136 <sup>+</sup>
Ship-Deer						4.088917
Deer-Club						4.165767
Ship	70 52 35.0	- 0.5	34.5	0.2	34.3	0.024054
Deer	56 47 27.2	- 2.5	24.7	0.1	24.6	9.922554
Club	52 20 03.5	- 2.3	01.2	0.1	01.1	9.898496
		- 5.3		0.4		
Ship-Club						4.112075
Ship-Deer						4.088917
Belle-Club						3.993596
Ship	37 41 34.8	+ 1.3	36.1	0.1	36.0	0.213650
Belle	53 35 55.6	- 1.4	54.2	0.1	54.1	9.905729
Club	88 42 30.3	- 0.3	30.0	0.1	29.9	9.999890
		- 0.4		0.3		
Ship-Club						4.112075
Ship-Belle						4.207136

*Preliminary computation of triangles—Continued*

Station	Observed angle	Correc- tion	Spher- ical angle	Spher- ical excess	Plane angle	Loga- rithm
Deer-Ship Biloxi Lighthouse	° ' "	"	"	"	° ' "	4.088917
Deer	48 11 17.4	+ 3.6	21.0	0.1	20.9	0.127640
Ship	96 30 31.2	+ 2.3	33.5	0.1	33.4	9.997191
	35 18 04.4	+ 1.4	05.8	0.1	05.7	9.761838
		+ 7.3		0.3		
Biloxi Lighthouse-Ship Biloxi Lighthouse-Deer						4.213748 3.973395
Biloxi Lighthouse-Deer Ship Island Lighthouse	25 30 02.1	+ 1.4	03.5	0.2	03.3	3.973395 0.366001
Biloxi Lighthouse Deer	81 55 36.0	- 1.7	34.3	0.1	34.2	9.995674
	72 34 26.4	- 3.7	22.7	0.2	22.5	9.979593
		- 4.0		0.5		
Ship Island Lighthouse-Deer Ship Island Lighthouse-Biloxi Light- house						4.340070 4.323989
Biloxi Lighthouse-Ship Ship Island Lighthouse	50 31 41.2	- 0.9	40.3	0.2	40.1	4.213748 0.112420
Biloxi Lighthouse Ship	33 44 18.6	- 5.3	13.3	0.2	13.1	9.744591
	95 44 07.0	- 0.1	06.9	0.1	06.8	9.997821
		- 6.3		0.5		
Ship Island Lighthouse-Ship Ship Island Lighthouse-Biloxi Light- house						4.070759 4.323989
Deer-Ship Ship Island Lighthouse	25 01 29.1	- 2.3	36.8	0.1	36.7	4.088917 0.373615
Deer	23 56 04.8	+ 6.0	10.8	0.1	10.7	9.608227
Ship	131 02 11.4	+ 1.3	12.7	0.1	12.6	9.887537
		+ 5.0		0.3		
Ship Island Lighthouse-Ship Ship Island Lighthouse-Deer						4.070759 4.340069 <sup>+1</sup>

*Preliminary position computation,*

## STATION CAT

$\alpha$ Second angle	East base to west base West base and Cat					$^{\circ}$ 84 +53	$'$ 14 37	$''$ 41.9 24.4
$\alpha$ $\Delta\alpha$	East base to Cat					137 —	52 2	06.3 08.4
$\alpha'$	Cat to east base					180 317 66	00 49 01	00.00 57.9 24.1
	First angle of triangle							
$\phi$	$^{\circ}$ 30	$'$ 14	$''$ 56.379	Dauphin Island east base	$\lambda$	88	08	14.288
$\Delta\phi$	+	4	04.171		$\Delta\lambda$	+	4	14.637
$\phi'$	30	19	00.550	Cat	$\lambda'$	88	12	28.925
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30	$'$ 16	$''$ 58	$\cos \alpha$ B	$\frac{s^2}{\sin^2 \alpha}$ C	$\frac{s^2}{\sin^2 \alpha}$ D	$\frac{-h}{s^2 \sin^2 \alpha}$ E	$\frac{-h}{s^2 \sin^2 \alpha}$ F
1st term 2d, 3d, and 4th terms } $-\Delta\phi$	—244.2407			h	2.387818	8.8366	7.108	5.970
	+ 0.0700					+0.0686	+0.0013	+0.0001
	—244.1707							
	$\frac{s}{\sin \alpha}$ A'	$\frac{s}{\sin \alpha}$ 4.006089 9.826616 8.509352	$\frac{s}{\sin \alpha}$ B	$\frac{s}{\sin \alpha}$ 9.870173 8.511556	$\frac{s^2}{\sin^2 \alpha}$ C	$\frac{s^2}{\sin^2 \alpha}$ D	$\frac{-h}{s^2 \sin^2 \alpha}$ E	$\frac{-h}{s^2 \sin^2 \alpha}$ F
	$\sec \phi'$	0.063865	$\sin \frac{1}{2}(\phi+\phi')$	2.405922	9.702663	2.108585		
		2.405922		2.108585				
		"		"				
	$\Delta\lambda$	+254.6373	$-\Delta\alpha$	+128.40				

## STATION CEDAR

$\alpha$ Second angle	East base to west base West base and Cedar				$^{\circ}$ 84 +103	$'$ 14 55	$''$ 41.9 35.5		
$\alpha$ $\Delta\alpha$	East base to Cedar				188 +	10	17.4 29.4		
$\alpha'$	Cedar to east base				180 8 37	00 10 26	00.00 46.8 32.1		
	First angle of triangle								
$\phi$	$^{\circ}$ 30	$'$ 14	$''$ 56.379	Dauphin Island east base	$\lambda$	88	08 14.288		
$\Delta\phi$	+	5	51.941		$\Delta\lambda$	—	58.265		
$\phi'$	30	20	48.320 +1	Cedar	$\lambda'$	88	07 16.023		
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30	$'$ 17	$''$ 52	$\cos \alpha$ B	$\frac{s}{\sin \alpha}$ 4.039354 9.995568 8.511556	$\sin^2 \alpha$ C	$\frac{s^2}{\sin^2 \alpha}$ 8.0787 8.3054 1.1712	$\frac{-h}{s^2 \sin^2 \alpha}$ D	$\frac{-h}{s^2 \sin^2 \alpha}$ E
1st term	—351.9476		h	2.546478	7.5553	7.425	+0.0027		
2d and 3d terms	+ 0.0063								
$-\Delta\phi$	—351.9413								
	$\frac{s}{\sin \alpha}$ A'	$\frac{s}{\sin \alpha}$ 4.039354 9.152708 8.509351	$\sin \frac{1}{2}(\phi+\phi')$	$\frac{s}{\sin \alpha}$ 9.995568 8.511556	$\frac{s}{\sin \alpha}$ 4.039354 9.152708 8.509351	$\frac{s}{\sin \alpha}$ 4.039354 9.152708 8.509351	$\frac{s}{\sin \alpha}$ 4.039354 9.152708 8.509351		
	$\sec \phi'$	0.063997							
		1.765408			1.765408	9.702857			
		"			1.468265				
$\Delta\lambda$		—58.2650	$-\Delta\alpha$		—29.39				

## secondary triangulation

## STATION CAT

$\alpha$ Third angle	West base to east base Cat and east base						°	'	''
							264	11	22.1
							— 60	21	11.8
$\alpha$ $\Delta\alpha$	West base to Cat						203	50	10.3
							+	1	11.7
$\alpha'$	Cat to west base						180	00	00.00
							23	51	22.0
$\phi$	°	'	''	Dauphin Island west base	$\lambda$		88	14	51.034
	30	14	21.492						
$\Delta\phi$	+	4	39.058	Cat	$\Delta\lambda$		—	2	22.109
	30	19	00.550						
$\phi'$	°	'	''	Cat	$\lambda'$		88	12	28.925
	30	16	41						
$\frac{1}{2}(\phi+\phi')$	°	'	''	$s$	$\sin^2 \alpha$	$h^2$	$s^2 \sin^2 \alpha$	$-\frac{h}{E}$	
	30	16	41	$\cos \alpha$	C	D	E		
				B					
1st term	—279.0808	h	2.445730		8.3299		7.222		2.445
2d, 3d, and	+ 0.0231				+0.0214		+0.0017		7.159
4th terms									5.917
$-\Delta\phi$	—279.0577								5.521
	$s$	3.972892	$\sin \frac{1}{2}(\phi+\phi')$	$\Delta\lambda$	2.152623				
	$\sin \alpha$	9.606514							
	$\Lambda'$	8.509352			9.702600				
	$\sec \phi'$	0.063865							
		2.152623			1.855223				
		''			''				
$\Delta\lambda$	—142.1095		$-\Delta\alpha$		—71.65				

## STATION CEDAR

$\alpha$ Third angle	West base to east base Cedar and east base						°	'	''
							264	11	22.1
							— 38	37	52.7
$\alpha$ $\Delta\alpha$	West base to Cedar						225	33	29.4
							+	3	49.5
$\alpha'$	Cedar to west base						180	00	00.00
							45	37	18.9
$\phi$	°	'	''	Dauphin Island west base	$\lambda$		88	14	51.034
	30	14	21.492						
$\Delta\phi$	+	6	26.829	Cedar	$\Delta\lambda$		—	7	35.011
	30	20	48.321						
$\phi'$	°	'	''	Cedar	$\lambda'$		88	07	16.023
	30	17	35						
$\frac{1}{2}(\phi+\phi')$	°	'	''	$s$	$\sin^2 \alpha$	$h^2$	$s^2 \sin^2 \alpha$	$-\frac{h}{E}$	
	30	17	35	$\cos \alpha$	C	D	E		
				B					
1st term	—387.0517	h	2.587769		9.3404		7.507		2.588
2d, 3d, and	+ 0.2227				+0.2190		+0.0032		8.169
4th terms									5.917
$-\Delta\phi$	—386.8290								6.674
	$s$	4.230999	$\sin \frac{1}{2}(\phi+\phi')$	$\Delta\lambda$	2.658022				
	$\sin \alpha$	9.853675							
	$\Lambda'$	8.509351			9.702795				
	$\sec \phi'$	0.063997							
		2.658022			2.360817				
		''			''				
$\Delta\lambda$	—455.0111		$-\Delta\alpha$		—229.52				



secondary triangulation—Continued

## STATION PINS

$\alpha$ Third angle	West base to Cedar Pins and Cedar					$^{\circ}$ 225 — 67	$'$ 33 34	$''$ 29. 4 55. 9			
$\alpha$ $\Delta\alpha$	West base to Pins					157 —	58 1	33. 5 53. 7			
$\alpha'$	Pins to west base					180 337	00 56	00. 00 39. 8			
$\phi$	$^{\circ}$ 30	$'$ 14	$''$ 21. 492	Dauphin Island west base	$\lambda$	88	14	51. 034			
$\Delta\phi$	+	8	02. 754		$\Delta\lambda$	+	3	45. 228			
$\phi'$	30	22	24. 246 +1	Pins	$\lambda'$	88	18	36. 262			
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30	$'$ 18	$''$ 23	$s$ $\cos \alpha$ B	4. 205130 9. 967092 8. 511557	$s^2$ $\sin^2 \alpha$ C	8. 4103 9. 1480 1. 1711	$h^2$ D	5. 368 2. 331	$-\frac{h}{s^2 \sin^2 \alpha}$ E	2. 684 7. 558 5. 917
1st term 2d, 3d, and 4th terms } $-\Delta\phi$	—482. 8131		$h$	2. 683779	8. 7294	7. 699	6. 159				
	+ 0. 0587				+0. 0534	+0. 0050	+0. 0001				
	—482. 7544										
	$s$ $\sin \alpha$ $A'$ $\sec \phi'$	4. 205130 9. 574026 8. 509350 0. 064116	$\sin \frac{\Delta\lambda}{2}(\phi+\phi')$	2. 352622 9. 702967	2. 055589						
		2. 352622		2. 055589							
		"		"							
	$\Delta\lambda$	+225. 2278	$-\Delta\alpha$	+113. 66							

## STATION GRAND

$\alpha$ Third angle	West base to Pins Grand and Pins					$^{\circ}$ 157 — 39	$'$ 58 54	$''$ 33.5 52.0	
$\alpha$ $\Delta\alpha$	West base to Grand					118 —	03 5	41.5 25.5	
$\alpha'$	Grand to west base					180 297	00 58	00.00 16.0	
$\phi$	$^{\circ}$ 30	$'$ 14	$''$ 21.492	Dauphin Island west base	$\lambda$	88	14	51.034	
$\Delta\phi$	+	4	58.097			$\Delta\lambda$	+	10	45.488
$\phi'$	30	19	19.589	Grand	$\lambda'$	88	25	36.522	
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30	$'$ 16	$''$ 50	$s$ $\cos \alpha$ B	$s^2$ $\sin^2 \alpha$ C	8.5819 9.8914 1.1711	$h^2$ D	$-\frac{h}{s^2 \sin^2 \alpha}$ E	2.475 8.473 5.917
1st term 2d, 3d, and 4th terms } $-\Delta\phi$	—298.5403		h	2.475003	9.6444	7.281	+0.0019	6.865 +0.0007	
	+ 0.4436								
	—298.0967								
	$s$ $\sin \alpha$ $\Delta'$ $\sec \phi'$	4.290861 9.945687 8.509352 0.063888	$\sin \frac{\Delta\lambda}{2}(\phi+\phi')$	2.809888 9.702633	2.512521	"	"	"	
		2.809888							
		"							
	$\Delta\lambda$	+645.4877	$-\Delta\alpha$	+325.48					

*Preliminary position computation,*

## STATION PETIT

$\alpha$ Second angle	Grand to west base West base and Petit			$^{\circ}$ 297 + 59	$'$ 58 11	$''$ 16.0 52.8			
$\alpha$ $\Delta\alpha$	Grand to Petit			357 +	10	08.8 11.6			
$\alpha'$	Petit to Grand			180 177 81	00 10 41	00.00 20.4 28.3			
				First angle of triangle					
$\phi$ $\Delta\phi$	$^{\circ}$ 30 —	$'$ 19 6	$''$ 19.589 44.089	Grand	$\lambda$ $\Delta\lambda$	88 — 25	36.522 23.005		
$\phi'$	30	12	35.500	Petit	$\lambda'$	88	25 13.517		
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30 15 58	$'$ 15	$''$ 58	$s$ cos $\alpha$ B	4.095452 9.999470 8.511551	$s^2$ C 1.1725	8.1909 7.3872 1.1725	$h^2$ D	5.213 2.332
1st term 2d and 3d terms $-\Delta\phi$	$''$ +404.0853		h	2.606473	6.7506 +0.0006	7.545 0.0035			
	+ 0.0041								
	+404.0894								
	$s$ $\sin \alpha$ $A'$ $\sec \phi'$	4.095452 8.693623 8.509354 0.063392	$\sin \frac{1}{2}(\phi+\phi')$	1.361821 9.702445	1.064266				
	$\Delta\lambda$	$''$ -23.0049	$-\Delta\alpha$	$''$ -11.59					

## STATION HORN

$\alpha$ Second angle	Grand to Petit Petit and Horn				$^{\circ}$ 357 + 61	$'$ 10 04	$''$ 08.8 37.5		
	Grand to Horn				58 —	14 5	46.3 18.1		
	Horn to Grand				180 238	00 09	00.00 28.2 —, 1 33.3		
	First angle of triangle				38	16			
$\phi$ $\Delta\phi$	$^{\circ}$ 30 —	$'$ 19 5	$''$ 19.589 39.578	Grand	$\lambda$ $\Delta\lambda$	88 +	25 10	36.522 31.017	
$\phi'$	30	13	40.011	Horn	$\lambda'$	88	36	07.539	
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30 16 30	$''$ cos $\alpha$ B	$s$ 4.297635 9.721209 8.511551	$\sin^2 \alpha$ C	$s^2$ 8.5953 9.8592 1.1725	$h^2$ D	5.061 2.332	$-\frac{h}{s^2 \sin^2 \alpha}$ E	2.530 8.454 5.918
	1st term	+339.1525 + 0.4253 +339.5778	h	2.530395	9.6270	7.393	6.902		
	2d, 3d, and 4th terms }				+0.4236	+0.0025		-0.0008	
	$-\Delta\phi$								
	$s$ $\sin \alpha$ $A'$ $\sec \phi'$	4.297635 9.929581 8.509354 0.063471	$\sin \frac{1}{2}(\phi+\phi')$	2.800041 9.702560					
		2.800041		2.502601					
	$\Delta\lambda$	$''$ +631.0170	$-\Delta\alpha$	$''$ +318.13					

## secondary triangulation—Continued

## STATION PETIT

$\alpha$ Third angle	West base to Grand Petit and Grand					$^{\circ}$ 118 — 39	$'$ 06	$''$ 41.5 39.4			
$\alpha$ $\Delta\alpha$	West base to Petit					78 —	57 5	02.1 13.4			
$\alpha'$	Petit to west base					180 258	00 51	00.00 48.7			
$\phi$ $\Delta\phi$	$^{\circ}$ 30 —	$'$ 14 1	$''$ 21.492 45.992	Dauphin Island west base	$\lambda$ $\Delta\lambda$	88 +	14 10	51.034 22.484			
$\phi'$	30	12	35.500	Petit	$\lambda'$	88	25	13.518 —1			
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30 13 28	$'$ 13 28	$''$ 4.229508 9.282521 8.511557	$\cos \alpha$ B	$\sin^2 \alpha$ C	8.4590 9.9837 1.1711	$h^2$ D	4.047 2.331	$-\frac{h}{s^2} \sin^2 \alpha$ E	2.024 8.443 5.917	
1st term	+105.5810				h	2.023586		9.6138		6.378	6.384
2d, 3d, and 4th terms	+ 0.4109							+0.4109		+0.0002	—0.0002
$-\Delta\phi$	+105.9919										
	$s$ $\sin \alpha$ $\Delta'$ $\sec \phi'$	4.229508 9.991874 8.509354 0.063392	$\sin \frac{1}{2}(\phi+\phi')$	$\Delta\lambda$	2.794128 9.701905	2.496033					
		2.794128				2.496033					
		$''$				$''$					
	$\Delta\lambda$	+622.4837	$-\Delta\alpha$			+313.35					

## STATION HORN

$\alpha$ Third angle	Petit to Grand Grand and Horn					$^{\circ}$ 177 — 80	$'$ 10 38	$''$ 20.4 49.8		
	Petit to Horn					96 —	31 5	30.6 29.2		
	Horn to Petit					180 276	00 26	00.00 01.4		
$\phi$ $\Delta\phi$	$^{\circ}$ 30 +	$'$ 12 1	$''$ 35.500 04.511	Petit	$\lambda$ $\Delta\lambda$	88 +	25 10	13.517 54.021		
$\phi'$	30	13	40.011	Horn	$\lambda'$	88	36	07.538 —1		
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30 13 08	$'$ 13 08	$''$ 4.245589 9.055530 8.511559	$\cos \alpha$ B	$s^2$ C	8.4912 9.9943 1.1706	$h^2$ D	3.625 2.331	$-\frac{h}{s^2} \sin^2 \alpha$ E	1.813 8.485 5.916
1st term 2d, 3d, and 4th terms — $\Delta\phi$	—64.9648		h	1.812678		9.6561 +0.4530	5.956 +0.0001			6.214 +0.0002
	+ 0.4533									
	—64.5115									
	$s$ $\sin \alpha$ $\Delta'$ $\sec \phi'$	4.245589 9.997178 8.509354 0.063471	$\sin \frac{1}{2}(\phi+\phi')$	2.815592 9.701830		2.517422				
	2.815592									
	$''$ $\Delta\lambda$	+654.0214								

*Preliminary position computation,*

## STATION PASCAGOULA

$\alpha$	Grand to Horn						$^{\circ}$	$'$	$''$
Second angle	Horn and Pascagoula						58	14	46.3
$\Delta\alpha$	Grand to Pascagoula						+ 43	14	20.1
$\alpha'$	Pascagoula to Grand						101	29	06.4
	First angle of triangle						—	3	52.8
$\phi$	$^{\circ}$	$'$	$''$	Grand		$\lambda$	180	00	00.00
$\Delta\phi$	30	19	19.589			$\Delta\lambda$	281	25	13.6
	+	1	21.006				97	55	57.5
$\phi'$	30	20	40.595	Pascagoula		$\lambda'$	88	25	36.522
							+	7	40.922
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$	$'$	$''$	$\cos \alpha$	$\sin^2 \alpha$	$h^2$	$-\frac{h}{s^2} \sin^2 \alpha$		
	30	20	00	B	C	D	E		
1st term									
2d, 3d, and 4th terms	—81.2312			h					
$-\Delta\phi$	+ 0.2256								
	—81.0056								
	$\frac{s}{\sin \alpha}$			$\sin \frac{\Delta\lambda}{2(\phi+\phi')}$					
	A'								
	sec $\phi'$								
	$\Delta\lambda$			$-\Delta\alpha$					

## STATION BELLE

$\alpha$	Pascagoula to Horn						$^{\circ}$	$'$	$''$
Second angle	Horn and Belle						19	21	11.1
$\Delta\alpha$	Pascagoula to Belle						+ 69	43	24.6
$\alpha'$	Belle to Pascagoula						89	04	35.7
	First angle of triangle						—	5	01.8
$\phi$	$^{\circ}$	$'$	$''$	Pascagoula		$\lambda$	180	00	00.00
$\Delta\phi$	30	20	40.595			$\Delta\lambda$	268	59	33.9
	—		8.730				48	58	48.0
$\phi'$	30	20	31.865	Belle		$\lambda'$	88	33	17.444
			—1				+	9	57.323
							88	43	14.767
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$	$'$	$''$	$\cos \alpha$	$\sin^2 \alpha$	$h^2$	$-\frac{h}{s^2} \sin^2 \alpha$		
	30	20	36	B	C				
1st term									
2d term	+8.3511			h					
$-\Delta\phi$	+0.3790								
	+8.7301								
	$\frac{s}{\sin \alpha}$			$\sin \frac{\Delta\lambda}{2(\phi+\phi')}$					
	A'								
	sec $\phi'$								
	$\Delta\lambda$			$-\Delta\alpha$					

secondary triangulation—Continued

## STATION PASCAGOULA

$\alpha$ Third angle	Horn to Grand Pascagoula and Grand				$^{\circ}$ 238 — 38	$'$ 09 49	$''$ 28.1 42.8				
$\alpha$ $\Delta\alpha$	Horn to Pascagoula				199 +	19 1	45.3 25.8				
$\alpha'$	Pascagoula to Horn				180 19	00 21	00.00 11.1				
$\phi$ $\Delta\phi$	$^{\circ}$ 30 +	$'$ 13 7	$''$ 40.011 00.584	Horn	$\lambda$ $\Delta\lambda$	88 —	36 2	07.539 50.094			
$\phi'$	30	20	40.595	Pascagoula	$\lambda'$	88	33	17.445 —1			
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30 17 10	$'$ 10	$''$ 10	$s$ $\cos \alpha$ B	4.137527 9.974803 8.511558	$s^2$ $\sin^2 \alpha$ C	8.2750 9.0396 1.1709	$h^2$ D	5.248 2.331	$-\frac{h}{s^2 \sin^2 \alpha}$ E	2.624 7.315 5.917
1st term	—420.6182			h	2.623888	8.4855	7.579	5.856			
2d, 3d, and 4th terms	+ 0.0345					+0.0306	+0.0038	+0.0001			
$-\Delta\phi$	—420.5837										
	$s$ $\sin \alpha$ $\Lambda'$ $\sec \phi'$	4.137527 9.519824 8.509351 0.063988	$\Delta\lambda$ $\sin \frac{1}{2}(\phi+\phi')$	2.230690 9.702706							
		2.230690		1.933396							
	$\Delta\lambda$	—170.0944	$-\Delta\alpha$	—85.78							

## STATION BELLE

$\alpha$ Third angle	Horn to Pascagoula Belle and Pascagoula				$^{\circ}$ 199 — 61	$'$ 17	$''$ 45.3 47.9				
$\alpha$ $\Delta\alpha$	Horn to Belle				138 —	01 3	57.4 35.5				
$\alpha'$	Belle to Horn				180 317	00 58	00.00 21.9				
$\phi$ $\Delta\phi$	$^{\circ}$ 30 +	$'$ 13 6	$''$ 40.011 51.853	Horn	$\lambda$ $\Delta\lambda$	88 +	36 7 07.539 07.228				
$\phi'$	30	20	31.864	Belle	$\lambda'$	88	43 14.767				
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30 17 06	$'$ 06	$''$ 06	$s$ $\cos \alpha$ B	4.232096 9.871296 8.511558	$s^2$ $\sin^2 \alpha$ C	8.4642 9.6505 1.1709	$h^2$ D	5.230 2.331	$-\frac{h}{s^2 \sin^2 \alpha}$ E	2.615 8.115 5.917
1st term 2d, 3d, and 4th terms	—412.0501 + 0.1976	h	2.614950		9.2856 +0.1936		7.561 +0.0036		6.647 +0.0004		
$-\Delta\phi$	—411.8525										
	$s$ $\sin \alpha$ $\Lambda'$ $\sec \phi'$	4.232096 9.825236 8.509351 0.063977	$\Delta\lambda$ $\sin \frac{1}{2}(\phi+\phi')$	2.630660 9.702690							
		2.630660		2.333350							
	$\Delta\lambda$	—427.2283	$-\Delta\alpha$	+215.45							

*Preliminary position computation,*

## STATION CLUB

$\alpha$ Second angle	Belle to Horn Horn and Club						$^{\circ}$ 317 + 40	$'$ 58 30	$''$ 21.9 06.7
	Belle to Club						358 +	28	28.6 05.0
	Club to Belle			First angle of triangle			180 178 105	00 28 43	00.00 33.6 56.5
$\phi$ $\Delta\phi$	$^{\circ}$ 30 —	$'$ 20 5	$''$ 31.864 19.886	Belle	$\lambda$ $\Delta\lambda$		88 —	43	14.767 09.812
$\phi'$	30	15	11.978	Club	$\lambda'$		88	43	04.955 +1
$\frac{1}{2}(\phi+\phi')$  1st term 2d and 3d terms }  $-\Delta\phi$	$^{\circ}$ 30	$'$ 17	$''$ 52	$s$ $\cos \alpha$ B	3.993596 9.999846 8.511550	$s^2$ $\sin^2 \alpha$ C	7.9872 6.8504 1.1729	$h^2$ D	5.010 2.332
	+319.8836			h	2.504992		6.0105 +0.0001		7.342 +0.0022
	+ 0.0023								
	+319.8859								
	$s$ $\sin \alpha$ $\Lambda'$ $\sec \phi'$	3.993596 8.425207 8.509353 0.063584	$\sin \frac{1}{2}(\phi+\phi')$	0.991740 9.702856					
		0.991740		0.694596					
	$\Delta\lambda$	$''$ -9.8116	$-\Delta\alpha$	$''$ -4.95					

## STATION DEER

$\alpha$ Second angle	Belle to Club Club and Deer						$^{\circ}$ 358 +102	$'$ 28 35	$''$ 28.6 20.0
$\alpha$ $\Delta\alpha$	Belle to Deer						101 —	03 2	48.6 45.3
$\alpha'$	Deer to Belle						180 281	00 01	00.00 03.3 +1 11.4
				First angle of triangle			41	02	
$\phi$ $\Delta\phi$	$^{\circ}$ 30 +	$'$ 20	$''$ 31.864 55.356	Belle	$\lambda$ $\Delta\lambda$		88 +	43 5	14.767 27.103
$\phi'$	30	21	27.220	Deer	$\lambda'$		88	48	41.870
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30	$'$ 21	$''$ 00	$s$ $\cos \alpha$ B	3.949436 9.283068 8.511550	$s^2$ $\sin^2 \alpha$ C	7.8989 9.9837 1.1729	$h^2$ D	3.488 2.332
1st term 2d and 3d terms } $-\Delta\phi$	—55.4695		h	1.744054			9.0555 +0.1136		5.820 +0.0001
	+ 0.1137								
	—55.3558								
	$s$ $\sin \alpha$ $\Lambda'$ $\sec \phi'$	3.949436 9.991853 8.509351 0.064045	$\sin \frac{1}{2}(\phi+\phi')$	2.514685 9.703531					
		2.514685		2.218216					
	$\Delta\lambda$	$''$ +327.1033	$-\Delta\alpha$	$''$ +165.28					

secondary triangulation—Continued

## STATION CLUB

$\alpha$ Third angle	Horn to Belle Club and Belle					$^{\circ}$ 138 — 33	$'$ 01 45	$''$ 57.4 57.1			
$\alpha$ $\Delta\alpha$	Horn to Club					104 —	16 3	00.3 30.2			
$\alpha'$	Club to Horn					180 284	00 12	00.00 30.1			
$\phi$ $\Delta\phi$	$^{\circ}$ 30 +	$'$ 13 1	$''$ 40.011 31.967	Horn	$\lambda$ $\Delta\lambda$	88 +	36 6	07.539 57.418			
$\phi'$	30	15	11.978	Club	$\lambda'$	88	43	04.957 —1			
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30 14 26	$'$ 13	$''$ 40.011 31.967	$s$ $\cos \alpha$ B	$4.061239$ 9.391705 8.511558	$s^2$ $\sin^2 \alpha$ C	$8.1225$ 9.9728 1.1709	$h^2$ D	$3.929$ 2.331	$-\frac{h}{s^2} \sin^2 \alpha$ E	1.964 8.095 5.917
1st term 2d, 3d, and 4th terms } $-\Delta\phi$	—92.1514 + 0.1848		h	1.964502		9.2662 +0.1845	6.260 +0.0002	5.976 +0.0001			
	—91.9666										
	$s$ $\sin \alpha$ A'	4.061239 9.986395 8.509353	$\Delta\lambda$ $\sin \frac{1}{2}(\phi+\phi')$	2.620571 9.702113							
	$\sec \phi'$	0.063584		2.322684							
		2.620571									
	$\Delta\lambda$	+	417.4179	$-\Delta\alpha$	+	210.22					

## STATION DEER

$\alpha$ Third angle	Club to Belle Deer and Belle			$^{\circ}$ 178 — 36	$'$ 28 22	$''$ 33.6 28.8					
$\alpha$ $\Delta\alpha$	Club to Deer			142 —	06 2	04.8 50.0					
$\alpha'$	Deer to Club			180 322	00 03	00.00 14.8					
$\phi$ $\Delta\phi$	$^{\circ}$ 30 +	$'$ 15 6	$''$ 11.978 15.242	Club	$\lambda$ $\Delta\lambda$	88 +	43 5	04.956 36.915			
$\phi'$	30	21	27.220	Deer	$\lambda'$	88	48	41.871 —1			
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30 18 20	$'$ 15	$''$ 11.978 15.242	$s$ $\cos \alpha$ B	$4.165767$ 9.897131 8.511556	$s^2$ $\sin^2 \alpha$ C	$8.3315$ 9.5767 1.1713	$h^2$ D	$5.149$ 2.331	$-\frac{h}{s^2} \sin^2 \alpha$ E	$2.574$ 7.908 5.918
1st term 2d, 3d, and 4th terms }	—375.3652 + 0.1233		h	2.574454	9.0795 +0.1200	7.480 +0.0030	6.400 +0.0003				
$-\Delta\phi$	—375.2419										
	$s$ $\sin \alpha$ A'	$4.165767$ 9.788357 8.509351	$\Delta\lambda$ $\sin \frac{1}{2}(\phi+\phi')$	$2.527520$ 9.702957							
	$\sec \phi'$	0.064045		2.230477							
		2.527520									
	$\Delta\lambda$	+	336.9147	$-\Delta\alpha$	+	170.01					

*Preliminary position computation,*

## STATION SHIP

$\alpha$ Second angle	Deer to Club Club and Ship						$^{\circ}$ 322 + 56	$'$ 03 47	$''$ 14.8 24.7
$\alpha$ $\Delta\alpha$	Deer to Ship						18	50 1	39.5 14.8
$\alpha'$	Ship to Deer			First angle of triangle			180 198 70	00 49 52	00.00 24.7 34.5
$\phi$ $\Delta\phi$	30 —	21 6	27.220 17.200	Deer	$\lambda$ $\Delta\lambda$		88 +	48 2	41.870 28.269
$\phi'$	30	15	10.020	Ship	$\lambda'$		88	51	10.139
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30	$'$ 18	$''$ 19	$s$ cos $\alpha$ B	4.088917 9.976075 8.511549	$s^2$ sin <sup>2</sup> $\alpha$ C	8.1778 9.0184 1.1731	$h^2$ D	5.153 2.332
1st term 2d and 3d terms — $\Delta\phi$	"			h	2.576541		8.3693 +0.0234		7.485 +0.0031
	+377.1734 + 0.0265								
	+377.1999								
	$s$ sin $\alpha$ $\Lambda'$ sec $\phi'$	4.088917 9.509200 8.509353 0.063581		sin $\frac{1}{2}(\phi+\phi')$	2.171051 9.702952				
		2.171051			1.874003				
	$\Delta\lambda$	" +148.2692		— $\Delta\alpha$	" +74.82				

## STATION BILOXI LIGHTHOUSE

$\alpha$ Second angle	Deer to Ship Ship and Biloxi Lighthouse						$^{\circ}$ 18 + 96	$'$ 50 30	$''$ 39.5 33.5
$\alpha$ $\Delta\alpha$	Deer to Biloxi Lighthouse						115 —	21 2	13.0 42.9
$\alpha'$	Biloxi Lighthouse to Deer			First angle of triangle			180 295 48	00 18 11	00.00 30.1 21.0
$\phi$ $\Delta\phi$	30 +	21 2	27.220 12.200	Deer	$\lambda$ $\Delta\lambda$		88 +	48 5	41.870 22.085
$\phi'$	30	23	39.420 —1	Biloxi Lighthouse	$\lambda'$		88	54	03.955 —1
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30	$'$ 22	$''$ 33	$s$ cos $\alpha$ B	3.978395 9.631650 8.511549	$s^2$ sin <sup>2</sup> $\alpha$ C	7.9568 9.9120 1.1731	$h^2$ D	4.243 2.332
1st term 2d, 3d, and 4th terms — $\Delta\phi$	"			h	2.121594		9.0419 +0.1102		—h $s^2$ sin <sup>2</sup> $\alpha$ E 7.869 5.919
	—132.3104 + 0.1107								
	—132.1997								
	$s$ sin $\alpha$ $\Lambda'$ sec $\phi'$	3.978395 9.956016 8.509350 0.064209		sin $\frac{1}{2}(\phi+\phi')$	2.507970 9.703868				
		2.507970			2.211838				
	$\Delta\lambda$	" +322.0846		— $\Delta\alpha$	" +162.87				

## secondary triangulation—Continued

## STATION SHIP

$\alpha$ Third angle	Club to Deer Ship and Deer					$^{\circ}$ 142 — 52	$'$ 06 20	$''$ 04.8 01.2
$\alpha$ $\Delta\alpha$	Club to Ship					89 —	46 4	03.6 04.4
$\alpha'$	Ship to Club					180 269	00 41	00.00 59.2
$\phi$ $\Delta\phi$	$^{\circ}$ 30 —	$'$ 15	$''$ 11.978 01.958	Club	$\lambda$ $\Delta\lambda$	88 +	43 8	04.956 05.182
$\phi'$	30	15	10.020	Ship	$\lambda'$	88	51	10.138 +1
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30	$'$ 15	$''$ 11	$s$ $\cos \alpha$ B	4.112975 7.607988 8.511556	$s^2$ $\sin^2 \alpha$ C	8.2260 0.0000 1.1713	
1st term	+1.7081		h	0.232519	9.3973			
2d term	+0.2496				+0.2496			
$-\Delta\phi$	+1.9577							
	$s$ $\sin \alpha$ $\Delta'$ $\sec \phi'$	4.112975 9.999996 8.509353 0.063581	$\sin \frac{1}{2}(\phi+\phi')$		2.685905 9.702275			
		2.685905			2.388180			
	$\Delta\lambda$	+	485.1824	$-\Delta\alpha$	+	244.44		

## STATION BILOXI LIGHTHOUSE

$\alpha$ Third angle	Ship to Deer Biloxi Lighthouse and Deer					$^{\circ}$ 198 — 35	$'$ 49 18	$''$ 24.7 05.8			
$\alpha$ $\Delta\alpha$	Ship to Biloxi Lighthouse					163 —	31 1	18.9 27.8			
$\alpha'$	Biloxi Lighthouse to Ship					180 343	00 29	00.00 51.1			
$\phi$ $\Delta\phi$	$^{\circ}$ 30 +	$'$ 15 8	$''$ 10.020 29.399	Ship	$\lambda$ $\Delta\lambda$	88 +	51 2	10.139 53.815			
$\phi'$	30 Fixed value	23	39.419 39.419	Biloxi Lighthouse	$\lambda'$	88	54	03.954 03.820			
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30	$'$ 19	$''$ 25	$s$ $\cos \alpha$ B	4.213748 9.981786 8.511556	$s^2$ $\sin^2 \alpha$ C	8.4275 8.9056 1.1713	$h^2$ D	5.414 2.331	$-\frac{h}{E}$ $s^2 \sin^2 \alpha$	2.707 7.333 5.918
1st term	—509.4364		h	2.707090	8.5044		7.745	5.958			
2d, 3d, and 4th terms }	+ 0.0376				+0.0319		+0.0056	+0.0001			
$-\Delta\phi$	—509.3988										
	$s$ $\sin \alpha$ $\Delta'$ $\sec \phi'$	4.213748 9.452781 8.509350 0.064209	$\sin \frac{1}{2}(\phi+\phi')$	2.240088 9.703190							
		2.240088		1.943278							
		$''$		$''$							
	$\Delta\lambda$	+173.8153	$-\Delta\alpha$	+87.76							

Preliminary position computation,

STATION SHIP ISLAND LIGHTHOUSE

$\alpha$	Biloxi Lighthouse to Ship					$^{\circ}$	$'$	$''$			
Second angle	Ship and Ship Island Lighthouse					343	29	51.1			
$\alpha$	Biloxi Lighthouse to Ship Island Lighthouse					+ 33	44	13.3			
$\Delta\alpha$						17	14	04.4			
						—	1	57.9			
$\alpha'$	Ship Island Lighthouse to Biloxi Lighthouse					180	00	00.00			
	First angle of triangle					197	12	06.5			
						50	31	40.3			
$\phi$	$^{\circ}$	$'$	$''$	Biloxi Lighthouse	$\lambda$	$\Delta\lambda$					
$\Delta\alpha$	30	23	39.419						88	54	03.954
	—	10	54.077		+	3	53.590				
$\phi'$	30	12	45.342	Ship Island Light-house	$\lambda'$	88	57	57.544			
			+1								
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$	$'$	$''$	$s$	$4.323989$	$s^2$	$8.6480$	$h^2$	$5.631$	$-\frac{h}{E}$	$2.816$
	30	18	12	$\cos \alpha$	$9.980049$	$\sin^2 \alpha$	$8.9434$	D	$2.332$	$s^2 \sin^2 \alpha$	$7.592$
				B	$8.511546$	C	$1.1738$				$5.919$
1st term	+654.0094			h	2.815584		8.7652		7.963		6.327
2d, 3d, and 4th terms	+ 0.0673						+0.0583		+0.0092		-0.0002
$-\Delta\phi$	+654.0767										
	$s$	4.323989		$\sin \frac{1}{2}(\phi+\phi')$	$\Delta\lambda$	2.368455	2.071385				
	$\sin \alpha'$	9.471708									
	$\Delta'$	8.509354									
	$\sec \phi'$	0.063404									
		2.368455									
		"					"				
	$\Delta\lambda$	+233.5904		$-\Delta\alpha$		+117.87					

Fixed  $\alpha$  Biloxi Lighthouse to Ship Island Lighthouse,  $17^\circ 14' 17.6''$ .



## FORMATION OF OBSERVATION EQUATIONS

The position computation was carried westward from the fixed lines at the eastern end of the scheme and the observation equations were formed in the same order. The treatment of fixed lines is the same as in the first adjustment of figure 6, pages 105 et seq. No new detail arises until the points Deer and Ship are reached, which have lines connecting them with the fixed points Biloxi Lighthouse and Ship Island Lighthouse. Suppose for the moment that Biloxi Lighthouse were not fixed but that its latitude and longitude were to receive corrections of  $\delta\phi_{12}$  and  $\delta\lambda_{12}$  respectively. The observation equation for  $v_{57}$  would then read,

$$v_{57} = z_{13} - 603\delta\phi_{11} + 248\delta\lambda_{11} + 603\delta\phi_{12} - 248\delta\lambda_{12} + 0.5$$

The latitude and longitude of Biloxi Lighthouse as developed by the preliminary position computation are  $30^\circ 23' 39.''419$  and  $88^\circ 54' 03.''954$ , while the fixed values are  $30^\circ 23' 39.''419$  and  $88^\circ 54' 03.''820$ , so that to reduce the preliminary to the fixed values on which the adjustment is built corrections of  $\delta\phi_{12} = 0.000^*$  and  $\delta\lambda_{12} = -0.134$  are necessary. By substituting these values in the equation for  $v_{57}$  there results,

$$v_{57} = z_{13} - 603\delta\phi_{11} + 248\delta\lambda_{11} + 603 (0.000) - 248 (-0.134) + 0.5$$

or,

$$v_{57} = z_{13} - 603\delta\phi_{11} + 248\delta\lambda_{11} + 33.7$$

as given in the table page 16.

The equation for the reverse direction,  $v_{63}$ , if Biloxi Lighthouse were not fixed, would be,

$$v_{63} = z_{15} - 603\delta\phi_{11} + 248\delta\lambda_{11} + 603\delta\phi_{12} - 248\delta\lambda_{12} + 0.0$$

which with the use of the above values of  $\delta\phi_{12}$  and  $\delta\lambda_{12}$  becomes,

$$v_{63} = z_{15} - 603\delta\phi_{11} + 248\delta\lambda_{11} + 33.2$$

Similar computations must be made for  $v_{56}$ ,  $v_{58}$ ,  $v_{59}$ ,  $v_{64}$ ,  $v_{67}$ , and  $v_{68}$ .

The known terms in the expressions for  $v_{65}$ , and  $v_{66}$  may be found in a similar way by allowing for the fixity of both ends of the line. If the corrections needed to the preliminary position of Ship Island Lighthouse in order to reduce it to the fixed position are called  $\delta\phi_{13}$  and  $\delta\lambda_{13}$  then for the line Biloxi Lighthouse to Ship Island Lighthouse the formula gives,

$$d\alpha = 89(\delta\phi_{13} - \delta\phi_{12}) + 250 (\delta\lambda_{13} - \delta\lambda_{12})$$

If the line were free to be turned in azimuth, then by the adjustment,

$$v_{65} = z_{15} + 89(\delta\phi_{13} - \delta\phi_{12}) + 250(\delta\lambda_{13} - \delta\lambda_{12}) - 1.7$$

---

\* This zero discrepancy is merely accidental.

But to reduce the positions of the preliminary computation to the fixed positions,  $\delta\phi_{13} = -0.002$ ,  $\delta\phi_{12} = 0.000$ ,  $\delta\lambda_{13} = -0.080$ , and  $\delta\lambda_{12} = -0.134$  (see pp. 155, 157). Substituting these values gives,

$$v_{65} = z_{15} + 11.6$$

as given in the table.  $v_{66}$  is found by a similar process.

The effect of using for the preliminary computation values from a previous adjustment for the figure but not for the positions appears in the constant term of the normal equations until the effect of closure in positions comes in. These constant terms should be zero except for the effect of accumulated errors in the last place of the two computations and a slight difference in the treatment of directions (3) and (3a). They are in fact almost negligible until the effect of closure appears in the equation for  $\delta\phi_8$ , from which point they become quite large.

Fort Morgan-Dauphin Island east base:

$$\begin{array}{lll} \text{Assumed azimuth, } 101 & 45 & 44.9 \\ \text{Observed azimuth, } 101 & 45 & 44.9 - z_1 + v_1 \\ & & 0 = 0.0 + z_1 - v_1 \\ & & v_1 = z_1 + 0.0 \end{array}$$

Fort Morgan-Cedar:

$$\begin{array}{lll} \text{Assumed azimuth, } 144 & 19 & 22.9 + d\alpha \\ \text{Observed azimuth, } 144 & 19 & 22.6 - z_1 + v_2 \\ & & 0 = +0.3 + z_1 - v_2 + 229\delta\phi_1 - 277\delta\lambda_1 \\ & & v_2 = z_1 + 229\delta\phi_1 - 277\delta\lambda_1 + 0.3 \end{array}$$

#### DAUPHIN ISLAND EAST BASE

Assumed azimuth	Observed azimuth	Equation	Station observed
° / "	° / "		
84 14 41.9	84 14 41.9 - $z_3 + v_3$	$v_3 = z_3 + 0.0$	West base Cat Cedar Fort Morgan
137 52 06.3	137 52 06.9 - $z_2 + v_4$	$v_4 = +z_2 + 420\delta\phi_2 - 403\delta\lambda_2 - 0.6$	
188 10 17.4	188 10 18.3 - $z_2 + v_5$	$v_5 = +z_2 - 82\delta\phi_1 - 498\delta\lambda_1 - 0.9$	
281 42 17.9	281 42 17.9 - $z_2 + v_{3a}$	$v_{3a} = z_2 + 0.0$	

#### (1) CEDAR

324 16 25.0	324 16 25.0 - $z_3 + v_{4a}$	$v_{4a} = z_3 + 229\delta\phi_1 - 277\delta\lambda_1 + 0.0$	Fort Morgan East base West base Cat Pins
8 10 46.8	8 10 46.8 - $z_3 + v_{5a}$	$v_{5a} = z_3 - 82\delta\phi_1 - 498\delta\lambda_1 + 0.0$	
45 37 18.9	45 37 19.8 - $z_3 + v_6$	$v_6 = z_3 - 266\delta\phi_1 - 226\delta\lambda_1 - 0.9$	
68 22 02.4	68 22 01.0 - $z_3 + v_7$	$v_7 = z_3 - 657\delta\phi_1 - 226\delta\lambda_1 + 657\delta\phi_2 + 226\delta\lambda_2 + 1.4$	
99 17 00.4	99 17 02.5 - $z_3 + v_8$	$v_8 = z_3 - 341\delta\phi_1 + 48\delta\lambda_1 + 341\delta\phi_2 - 48\delta\lambda_2 - 2.1$	

#### (2) CAT

23 51 22.0	23 51 22.0 - $z_4 + v_{17}$	$v_{17} = z_4 - 273\delta\phi_2 - 537\delta\lambda_2 + 0.0$	West base Pins Cedar East base
122 37 00.7			
248 19 24.4	248 19 27.1 - $z_4 + v_{15}$	$v_{15} = z_4 - 657\delta\phi_1 - 226\delta\lambda_1 + 657\delta\phi_2 + 226\delta\lambda_2 - 2.7$	
317 49 57.9	317 49 59.8 - $z_4 + v_{16}$	$v_{16} = z_4 + 420\delta\phi_2 - 403\delta\lambda_2 - 1.9$	

#### DAUPHIN ISLAND WEST BASE

264 11 22.1	264 11.22.1 - $z_5 + v_{14}$	$v_{14} = z_5 + 0.0$	East base Petit Grand Pins Cat Cedar
78 57 02.1	78 57 01.7 - $z_5 + v_9$	$v_9 = z_5 + 367\delta\phi_4 + 62\delta\lambda_4 + 0.4$	
118 03 41.5	118 03 40.8 - $z_5 + v_{10}$	$v_{10} = z_5 + 287\delta\phi_5 - 133\delta\lambda_5 + 0.7$	
157 58 33.5	157 58 31.7 - $z_5 + v_{11}$	$v_{11} = z_5 + 148\delta\phi_3 - 318\delta\lambda_3 + 1.8$	
203 50 10.3	203 50 10.9 - $z_5 + v_{12}$	$v_{12} = z_5 - 273\delta\phi_2 - 537\delta\lambda_2 - 0.6$	
225 33 29.4	225 33 29.6 - $z_5 + v_{13}$	$v_{13} = z_5 - 266\delta\phi_1 - 226\delta\lambda_1 - 0.2$	

## (3) PINS

Assumed azimuth	Observed azimuth	Equation	Station observed
° ' "	° ' "		
279 11 16.6	279 11 16.6— $z_8+v_{18}$	$v_{18}=z_8-341\delta\phi_1+48\delta\lambda_1+341\delta\phi_3-48\delta\lambda_3+0.0$	Cedar
302 33 55.1	302 33 56.9— $z_6+v_{19}$	$v_{19}=z_8-459\delta\phi_3+255\delta\lambda_3+459\delta\phi_5-255\delta\lambda_5-1.8$	Cat
337 56 39.8	337 56 42.6— $z_6+v_{20}$	$v_{20}=z_6+148\delta\phi_3-318\delta\lambda_3-2.8$	West base
30 22 49.2	30 22 52.1— $z_6+v_{21}$	$v_{21}=z_6-153\delta\phi_3-228\delta\lambda_3+153\delta\phi_4+228\delta\lambda_4-2.9$	Petit
63 09 48.1	63 09 49.6— $z_6+v_{22}$	$v_{22}=z_8-450\delta\phi_3-198\delta\lambda_3+450\delta\phi_5+198\delta\lambda_5-1.5$	Grand

## (4) PETIT

96 31 30.6	96 31 30.6— $z_7+v_{28}$	$v_{28}=z_7-358\delta\phi_4+36\delta\lambda_4+358\delta\phi_6-36\delta\lambda_6+0.0$	Horn
139 08 41.2	139 08 40.9— $z_7+v_{29}$	$v_{29}=z_7-209\delta\phi_4+211\delta\lambda_4+209\delta\phi_7-211\delta\lambda_7+0.3$	Pascagoula
177 10 20.4	177 10 19.5— $z_7+v_{30}$	$v_{30}=z_7-25\delta\phi_4+442\delta\lambda_4+25\delta\phi_5-442\delta\lambda_5+0.9$	Grand
210 19 28.8	210 19 28.2— $z_7+v_{31}$	$v_{31}=z_7-153\delta\phi_3-228\delta\lambda_3+153\delta\phi_4+228\delta\lambda_4+0.6$	Pins
258 51 48.7	258 51 47.7— $z_7+v_{32}$	$v_{32}=z_7+367\delta\phi_4+62\delta\lambda_4+1.0$	West base

## (5) GRAND

243 06 15.8	243 06 15.8— $z_8+v_{23}$	$v_{23}=z_8-450\delta\phi_3-198\delta\lambda_3+450\delta\phi_5+198\delta\lambda_5+0.0$	Pins
297 58 16.0	297 58 17.4— $z_8+v_{24}$	$v_{24}=z_8+287\delta\phi_4-133\delta\lambda_4-1.4$	West base
357 10 08.8	357 10 09.4— $z_8+v_{25}$	$v_{25}=z_8-25\delta\phi_4+442\delta\lambda_4+25\delta\phi_5-442\delta\lambda_5-0.6$	Petit
58 14 46.3	58 14 46.6— $z_8+v_{26}$	$v_{26}=z_8-272\delta\phi_5-146\delta\lambda_5+272\delta\phi_6+146\delta\lambda_6-0.3$	Horn
101 29 06.4	101 29 05.5— $z_8+v_{27}$	$v_{27}=z_8-495\delta\phi_5+87\delta\lambda_5+495\delta\phi_7-87\delta\lambda_7+0.9$	Pascagoula

## (6) HORN

104 16 00.3	104 16 00.3— $z_8+v_{38}$	$v_{38}=z_8-535\delta\phi_6+118\delta\lambda_6+535\delta\phi_8-118\delta\lambda_8+0.0$	Club
138 01 57.4	138 01 54.0— $z_8+v_{39}$	$v_{39}=z_8-249\delta\phi_6+240\delta\lambda_6+249\delta\phi_9-240\delta\lambda_9+3.4$	Belle
199 19 45.3	199 19 47.5— $z_8+v_{40}$	$v_{40}=z_8+153\delta\phi_6+379\delta\lambda_6-153\delta\phi_7-379\delta\lambda_7-2.2$	Pascagoula
238 09 28.1	238 09 26.5— $z_8+v_{41}$	$v_{41}=z_8-272\delta\phi_6-146\delta\lambda_6+272\delta\phi_8+146\delta\lambda_8+1.6$	Grand
276 26 01.4	276 26 00.7— $z_8+v_{42}$	$v_{42}=z_8-358\delta\phi_4+36\delta\lambda_4+358\delta\phi_6-36\delta\lambda_6+0.7$	Petit

## (7) PASCAGOULA

281 25 13.6	281 25 13.6— $z_{10}+v_{33}$	$v_{33}=z_{10}-495\delta\phi_6+87\delta\lambda_6+495\delta\phi_7-87\delta\lambda_7+0.0$	Grand
319 04 37.2	319 04 34.2— $z_{10}+v_{34}$	$v_{34}=z_{10}-209\delta\phi_4+211\delta\lambda_4+209\delta\phi_7-211\delta\lambda_7+3.0$	Petit
19 21 11.1	19 21 07.6— $z_{10}+v_{35}$	$v_{35}=z_{10}+153\delta\phi_6+379\delta\lambda_6-153\delta\phi_7-379\delta\lambda_7+3.5$	Horn
57 14 14.4	57 14 12.2— $z_{10}+v_{36}$	$v_{36}=z_{10}-286\delta\phi_7-160\delta\lambda_7+286\delta\phi_8+160\delta\lambda_8+2.2$	Club
89 04 35.7	89 04 35.6— $z_{10}+v_{37}$	$v_{37}=z_{10}-398\delta\phi_7-6\delta\lambda_7+398\delta\phi_9+6\delta\lambda_9+0.1$	Belle

## (8) BELLE

268 59 33.9	268 59 33.9— $z_{11}+v_{43}$	$v_{43}=z_{11}-398\delta\phi_7-6\delta\lambda_7+398\delta\phi_9+6\delta\lambda_9+0.0$	Pascagoula
317 58 21.9	317 58 23.7— $z_{11}+v_{44}$	$v_{44}=z_{11}-249\delta\phi_6+240\delta\lambda_6+249\delta\phi_9-240\delta\lambda_9-1.8$	Horn
358 28 28.6	358 28 29.2— $z_{11}+v_{45}$	$v_{45}=z_{11}-17\delta\phi_6+559\delta\lambda_6+17\delta\phi_9-559\delta\lambda_9-0.6$	Club
52 04 22.8	52 04 24.8— $z_{11}+v_{46}$	$v_{46}=z_{11}-311\delta\phi_9-211\delta\lambda_9+311\delta\phi_{10}+211\delta\lambda_{10}-2.0$	Ship
101 03 48.6	101 03 47.7— $z_{11}+v_{47}$	$v_{47}=z_{11}-700\delta\phi_9+119\delta\lambda_9+700\delta\phi_{11}-119\delta\lambda_{11}+0.9$	Deer

## (8) CLUB

89 46 03.6	89 46 03.6— $z_{12}+v_{48}$	$v_{48}=z_{12}-490\delta\phi_8-2\delta\lambda_8+490\delta\phi_{12}+2\delta\lambda_{12}+0.0$	Ship
142 06 04.8	142 06 07.1— $z_{12}+v_{49}$	$v_{49}=z_{12}-266\delta\phi_8+297\delta\lambda_8+266\delta\phi_{11}-297\delta\lambda_{11}-2.3$	Deer
178 28 33.6	178 28 33.9— $z_{12}+v_{50}$	$v_{50}=z_{12}-17\delta\phi_8+559\delta\lambda_8+17\delta\phi_9-559\delta\lambda_9-0.3$	Belle
237 09 18.0	237 09 17.2— $z_{12}+v_{51}$	$v_{51}=z_{12}-286\delta\phi_7-160\delta\lambda_7+286\delta\phi_8+160\delta\lambda_8+0.8$	Pascagoula
284 12 30.1	284 12 30.8— $z_{12}+v_{52}$	$v_{52}=z_{12}-535\delta\phi_6+118\delta\lambda_6+535\delta\phi_8-118\delta\lambda_8-0.7$	Horn

## (11) DEER

Assumed azimuth	Observed azimuth	Equation	Station observed
° ' "	° ' "		
281 01 03.4	281 01 03.4— $z_{13}+v_{53}$	$v_{53}=z_{13}-700\delta\phi_9+119\delta\lambda_9+700\delta\phi_{11}-119\delta\lambda_{11}+0.0$	Belle Club Ship Ship Island Lighthouse. Biloxi Lighthouse
322 03 14.8	322 03 14.1— $z_{13}+v_{54}$	$v_{54}=z_{13}-266\delta\phi_8+297\delta\lambda_8+266\delta\phi_{11}-297\delta\lambda_{11}+0.7$	
18 50 39.5	18 50 41.3— $z_{13}+v_{55}$	$v_{55}=z_{13}+167\delta\phi_{10}+425\delta\lambda_{10}-167\delta\phi_{11}-425\delta\lambda_{11}-1.8$	
42 46 50.3	42 46 46.1— $z_{13}+v_{56}$	$v_{56}=z_{13}-197\delta\phi_{11}-185\delta\lambda_{11}-11.0$	
115 21 13.0	115 21 12.5— $z_{13}+v_{57}$	$v_{57}=z_{13}-603\delta\phi_{11}+248\delta\lambda_{11}+33.7$	

## (10) SHIP

67 47 12.0	67 47 12.0— $z_{14}+v_{58}$	$v_{58}=z_{14}-500\delta\phi_{10}-177\delta\lambda_{10}-15.2$	Ship Island Lighthouse Biloxi Lighthouse Deer Belle Cluh
163 31 18.9	163 31 19.0— $z_{14}+v_{59}$	$v_{59}=z_{14}-110\delta\phi_{10}+323\delta\lambda_{10}+43.2$	
198 49 24.7	198 49 23.4— $z_{14}+v_{60}$	$v_{60}=z_{14}+167\delta\phi_{10}+425\delta\lambda_{10}-167\delta\phi_{11}-425\delta\lambda_{11}+1.3$	
232 00 23.1	232 00 23.6— $z_{14}+v_{61}$	$v_{61}=z_{14}-311\delta\phi_9-211\delta\lambda_9+311\delta\phi_{10}+211\delta\lambda_{10}-0.5$	
269 41 59.2	269 41 58.4— $z_{14}+v_{62}$	$v_{62}=z_{14}-490\delta\phi_8-2\delta\lambda_8+490\delta\phi_{10}+2\delta\lambda_{10}+0.8$	

## BILOXI LIGHTHOUSE

295 18 30.1	295 18 30.1— $z_{15}+v_{63}$	$v_{63}=z_{15}-603\delta\phi_{11}+248\delta\lambda_{11}+33.2$	Deer Ship Ship Island Lighthouse
343 29 51.1	343 29 47.5— $z_{15}+v_{64}$	$v_{64}=z_{15}-110\delta\phi_{10}+323\delta\lambda_{10}+46.9$	
17 14 04.4	17 14 06.1— $z_{15}+v_{65}$	$v_{65}=z_{15}+11.6$	

## SHIP ISLAND LIGHTHOUSE

197 12 06.5	197 12 06.5— $z_{16}+v_{66}$	$v_{66}=z_{16}+13.3$	Biloxi Lighthouse Deer Ship
222 42 10.0	222 42 08.6— $z_{16}+v_{67}$	$v_{67}=z_{16}-197\delta\phi_{11}-185\delta\lambda_{11}-13.8$	
247 43 46.8	247 43 47.7— $z_{16}+v_{68}$	$v_{68}=z_{16}-500\delta\phi_{10}-177\delta\lambda_{10}-16.1$	

In order to get the quantities on a better relative basis, it is best to adopt  $100\delta\phi_1$ ,  $100\delta\lambda_1$ , etc., as unknowns in the equations. The coefficients throughout will then be divided by 100, and from the solution we shall determine one hundred times the corrections in seconds to the various latitudes and longitudes.

Table for formation of normals, No. 1

	$\partial\phi_1$	$\partial\lambda_1$	$\partial\phi_2$	$\partial\lambda_2$	$\partial\phi_3$	$\partial\lambda_3$	$\partial\phi_4$	$\partial\lambda_4$	$\partial\phi_5$	$\partial\lambda_5$	$\partial\phi_6$	$\partial\lambda_6$	$\partial\phi_7$	$\partial\lambda_7$	$\partial\phi_8$	$\partial\lambda_8$	$\eta$	$\mathcal{I}$	$V/p$
$z_1$	1																0.0	1	1
1	229	-277															+0.3	1	1
2	229	-277															+0.3	-1	0.7071
$z_2$	1																0.0	1	1
1	82	-498	+420	-403													-0.6	1	1
1	82	-498	+420	-403													-0.9	1	1
1	82	-498	+420	-403													0.0	1	1
4	82	-498	+420	-403													-1.5	-1	0.51
$z_3$	1																0.0	1	1
1	229	-277															0.0	1	1
1	82	-498															-0.9	1	1
1	266	-226															+1.4	1	1
1	657	-48	+657	+226													-2.1	1	1
1	341	+1179	+657	+226	+341	-48											-1.6	-1	0.4471
5	-1117				+341	-48													
$z_4$	1																0.0	1	1
1	657	-226	-273	-537													-2.7	1	1
1	657	-226	+657	+226													-1.9	1	1
1	657	-226	+420	-403													-4.0	-1	0.5771
3	657	-226	+804	-714															
$z_5$	1																0.0	1	1
1	226	-226	-273	-537	+148	-318	+367	+62	+287	-133							+0.4	1	1
1	226	-226	-273	-537	+148	-318	+367	+62	+287	-133							+0.7	1	1
1	226	-226	-273	-537	+148	-318	+367	+62	+287	-133							+1.8	1	1
1	226	-226	-273	-537	+148	-318	+367	+62	+287	-133							-0.6	1	1
6	266	-226	-273	-537	+148	-318	+367	+62	+287	-133							-0.2	1	1
$z_6$	1																+2.1	-1	0.4084
1	341	+48	-459	+255	+341	-48											0.0	1	1
1	341	+48	-459	+255	+459	-255											-1.8	1	1
1	341	+48	-459	+255	+148	-318											-2.8	1	1
1	341	+48	-459	+255	+153	-228	+153	+228	+450	+198							-2.9	1	1
1	341	+48	-459	+255	+153	-228	+153	+228	+450	+198							-1.5	1	1
5	341	+48	-459	+255	+345	-1047	+153	+228	+450	+198							-9.0	-1	0.4471
$z_7$	1																0.0	1	1
1	229	-277															+0.3	1	1
1	229	-277															+0.3	1	1
1	229	-277															+0.9	1	1
1	229	-277															+0.6	1	1
1	229	-277															+1.0	1	1
5	229	-277															+2.8	-1	0.4471







Table for formation of normals, No. 2 \*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	$\gamma$	$\tau$
$\partial\phi_1$	$\partial\lambda_1$	$\partial\phi_2$	$\partial\lambda_2$	$\partial\phi_3$	$\partial\lambda_3$	$\partial\phi_4$	$\partial\lambda_4$	$\partial\phi_5$	$\partial\lambda_5$	$\partial\phi_6$	$\partial\lambda_6$	$\partial\phi_7$	$\partial\lambda_7$		
+2.29 +1.62i	-2.77 -1.96i	+4.20	-4.03											+0.3 +0.21i	-0.18 -0.13i
-0.82 -0.41i	-4.98 -2.49i	+2.10i	-2.02i											-0.6 -0.9	-0.43 -6.70
+2.29 -0.82	-2.77 -4.98													-0.75i 0.0	-3.57i -0.48
-2.66 -2.26	-2.26 -3.41	+6.57	+2.26											0.0 -0.9	-5.80 -5.82
-6.57 -3.41	-2.26 +0.48			+3.41 +1.52i	-0.48 -0.21i									+1.4 -2.1	+1.40 -2.10
-4.98i	-5.27i	+2.94i	+1.01i											-0.72i 0.0	-5.72i -8.10
-2.73	-2.73	-2.73	-5.37											-2.7 -1.9	-2.70 -1.73
+6.57 +4.03	+6.57 +4.20	+6.57 +4.64i	+2.26 -3.12i											-2.65i +0.4	-7.22i +4.69
-3.79i	-1.30i					+3.67	+0.62	+2.87	-1.33					+0.7 +1.8	+2.24 -8.70
				+1.48	-3.18									-0.6 -0.2	+0.10 -5.12
-2.66 -1.06i -3.41	-2.26 -0.92i +0.48	-2.73 -1.11i -4.59	-5.37 -2.19i +2.55	+0.60i +3.41 +4.59	-1.30i -0.48 -2.55	+1.50i	+0.25i	+1.17i	-0.54i					+0.86i 0.0	-2.77i 0.0
						+1.53	+2.28	+4.50	+1.98					-1.8 -2.8	-1.80 -4.50
						+0.68i	+1.02i	+2.01i	+0.89i					-2.9 -1.5	-2.90 -1.50
-1.52i	+0.21i	-2.05i	+1.14i	+1.54i	-4.68i									-4.02i 0.0	-4.78i 0.0
				-1.53	-2.28			+0.25	-4.42	+3.58	-0.36	+2.09	-2.11	+0.3 +0.9	+0.30 +0.90
				-0.68i	-1.02i									+0.6 +1.0	+0.60 +5.29
				-4.50	-1.88	+3.67	+0.62							+1.25i 0.0	+3.17i 0.0
						-0.32i	+4.38i			+1.60i	-0.16i	+0.93i	-0.94i	-1.4 -0.6	+0.14 -0.60
						-0.25	+4.42			+2.72	+1.46			-0.3 +0.9	-0.30 +0.90
				-2.01i	-0.89i	-0.11i	+1.98i			+1.22i	+0.65i	+4.95 +2.21i	-0.87 -0.39i	+0.3 -0.63i	+0.30 +0.63i

\* All values in this table except those in the  $\gamma$  and  $\tau$  columns have been divided by 100.



## Normal equations

1	2	3	4	5	6	7	8	9	10	11	12
+90.0150	+ 4.1730 +45.4427	- 57.5385 - 3.5323 +117.5659	-41.7538 -17.5325 +33.7556 +78.7027	-12.6766 - 11.5126 -21.7139 + 9.7277 + 84.3424	- 6.3049 - 1.7307 + 1.2849 - 3.8022 + 6.2298 +19.9567	+ 2.6686 + 1.2372 3.0590 2.5098 7.0677 - 2.2637 + 59.8618	+ 1.8229 0.0158 2.3685 0.6153 - 1.7394 + 0.9316 - 0.3988 +34.2849	+ 4.3305 0.6543 5.4192 0.2709 -24.1325 44.2628 6.7978 7.2211 - 1.6977 +109.1066	+ 0.7642 - 0.6837 + 1.2251 - 2.1972 -24.1325 + 0.8894 - 8.1327 -27.5748 + 6.3958 +46.8472	+ 3.5402 2.7178 - 24.3542 7.4852 - 13.4456 2.6606 +103.3016	+ 1.1977 0.4153 + 10.2336 3.0404 - 0.4469 1.5031 6.3476 28.4613
13	14	15	16	17	18	19	20	21	22	$\eta$	$\Sigma$
+ 5.0745 + 2.9155 - 9.8322 + 1.0340 -51.1966 +14.5520 -13.4998 - 6.7278 +99.0967	- 1.4231 - 1.3059 + 2.2660 - 0.2006 - 1.6849 - 3.7637 - 8.8081 -15.4350 + 1.1318 +25.6494	+ 5.0144 - 1.5856 5.7446 + 1.0543 - 57.7023 + 1.3790 + 13.8524 + 0.1385 +122.2200	- 0.1784 - 0.5920 + 0.9446 - 0.6253 +24.4953 - 6.7932 + 0.4952 - 0.3391 -19.7152 +63.1452	+ 3.4314 1.8508 5.2880 0.0273 - 15.1399 - 6.3530 - 32.5324 8.1200 - 11.8987 8.9105 +142.9446	- 1.6841 + 0.1430 - 1.2391 + 0.7072 + 1.5610 - 1.9532 -11.4363 - 3.6212 + 3.1968 -43.8742 -21.3622 +61.5179	+ 6.7770 2.6480 5.2774 1.6185 43.9722 13.4344 - 12.7934 - 1.0383 +106.8200	+ 1.0673 + 1.0111 - 1.6860 + 0.0354 9.8031 4.8929 - 0.8560 - 2.8738 +27.9899 +50.9179	+ 6.3184 3.9798 7.0946 0.9507 15.7932 3.1960 - 94.2862 31.3248 + 16.8102 - 9.2141 +173.0271	- 3.7670 + 1.2720 - 2.6458 + 0.9735 7.7351 6.3741 - 6.4115 + 8.1976 - 7.0763 -28.1027 -54.7848	+ 0.5960 + 1.0971 0.4396 - 0.5294 0.2811 - 0.1405 + 0.0662 0.1210 0.8374 0.2900 0.0595 4.4519 0.2802 3.5317 2.1778 37.1447 28.6369 6.1221 30.0408 7.1924 34.6511 +167.6744 -115.8192 +163.8779	

## Solution of normals

1	2	3	4	5	6	7	8	9	10	11	12	13	14	$\eta$	$\Sigma$
+90.0150 $\partial\phi_1$	+4.1780 -0.046356	-57.5385 +0.639210	-41.7538 +0.463854	-12.0766 +0.140828	-6.3049 +0.070043	+2.6686 -0.029646	+1.8229 -0.020251	+4.3305 -0.048109	+0.7642 -0.008490					+0.5960 -0.006621	-13.9086 +0.154459
1	+45.4427 -0.1085	-3.5323 +2.6674	-17.5325 +1.9357	+11.5126 +0.5877	-1.7807 +0.2923	+1.2372 -0.1237	+0.0158 -0.0845	+0.6543 -0.2908	0.6837 -0.0354					+1.0071 -0.0276	+40.5135 +0.6446
	+45.2492 $\partial\lambda_1$	-0.8649 +0.019114	-15.5968 +0.346687	+12.1003 -0.267415	-1.4854 +0.032893	+1.1135 -0.024608	+0.0687 -0.001518	+0.4535 -0.010222	0.7191 -0.015892					+0.9795 -0.021647	+41.1581 +0.909587
	1	+117.5650 -86.7732	+33.7556 -26.6894	-21.7139 -8.1030	+1.2849 -4.0302	+3.0590 +1.7058	+2.3685 +1.1652	+5.4192 +2.7681	1.2251 +0.4885					-0.4396 +0.3310	+81.4539 +8.8873
	2	-0.0165	-0.2951	+0.2313	-0.0294	+0.0213	-0.0013	+0.0087	0.0187					+0.0187	+0.7867
		+80.7702 $\partial\phi_2$	+6.7681 -0.083795	-29.5836 +0.366294	-2.7737 +0.034341	+4.7961 -0.039256	+3.5324 -0.043734	+8.1900 -0.101473	1.6999 -0.021046					-0.0390 +0.000494	+73.3535 -0.908175
		1	+78.7027 -19.3677	+9.7277 -5.8901	-3.8022 -2.9246	+2.5098 +1.2378	-0.6153 +0.8456	+0.2709 +2.0087	-2.1972 0.3545					+0.5294 +0.2765	+59.5351 -6.4492
		2	-5.3760	+4.1708	-0.5130	+0.8338	-0.0237	+0.1563	-0.2479					+0.3376	+14.1867
		3	-0.5671	+2.4791	+0.2324	-0.4011	-0.2960	+0.6868	-0.1424					+0.0083	-6.1467
			+53.3919 $\partial\lambda_2$	+10.4975 -0.196612	-7.0074 +0.131245	+3.7303 -0.069866	-0.0894 +0.001674	+1.7491 -0.032760	-2.2330 +0.041823					+1.1468 -0.021479	+61.1858 -1.145975
		1	+84.3424 -1.7352	+6.2298 -0.8879	-7.0677 +0.3758	-1.7394 +0.2567	-44.2628 +0.6099	-24.1325 0.1076	+8.3278 -11.9580					-0.2811 +0.0839	+8.3278 -11.9580
		2	-3.2358	+0.3860	-0.1844	+0.1844	0.1213	+0.1923	+0.1923					-0.2619 -0.0146	-11.0063 +26.8680
		3	-10.8570	+1.0160	-0.1660	+1.7331	+1.2869	+3.0921	+0.6227					-0.2255	-12.0299
		4	-2.0639	+1.3777	+0.7334	+0.0176	+0.3439	+0.4380	+0.4380					-0.6892 +0.010527	+10.2025 -0.153605
			+66.4205 $\partial\phi_3$	+6.1016 +0.091863	-5.9700 +0.089852	-0.1528 +0.002300	-41.1160 -0.619026	-22.7709 +0.342829	+3.5402 -0.053300					-0.6892 +0.010527	+10.2025 -0.153605
		1	+19.9567 -0.4416	+19.9567 -0.4416	+0.9316 -0.1277	+0.9316 -0.1277	+0.9316 -0.1277	+0.9316 -0.1277	+0.9316 -0.1277					-0.1405 +0.0417	+3.9182 -0.9738
		2	-0.0490	-0.0490	-0.0366 -0.0223	-0.0366 -0.0223	-0.0366 -0.0223	-0.0366 -0.0223	-0.0366 -0.0223					+0.0322 -0.0014	+0.9738 -1.3538
		3	-0.0953	-0.0953	+0.1644 -0.1213	+0.1644 -0.1213	+0.1644 -0.1213	+0.1644 -0.1213	+0.1644 -0.1213					+0.0014 -0.0150	+2.5190 +8.0303
		4	-0.9197	-0.9197	+0.4896 -0.0117	+0.4896 -0.0117	+0.4896 -0.0117	+0.4896 -0.0117	+0.4896 -0.0117					+0.1505 +0.0642	+8.0303 -0.9372
		5	-0.5605	-0.5605	+0.5484 -0.0140	+0.5484 -0.0140	+0.5484 -0.0140	+0.5484 -0.0140	+0.5484 -0.0140					+0.1307 -0.4662	+0.9372 -13.9098
			+17.8906 $\partial\lambda_3$	+17.8906 -0.065990	-0.8428 +0.047109	-0.8428 +0.047109	-0.8428 +0.047109	-0.8428 +0.047109	-0.8428 +0.047109					+0.1467 -0.008200	+13.9098 -0.777492

### Solution of normals—Continued

	7	8	9	10	11	12	13	14	15	16	17	18	$\eta$	$\Sigma$
1	+59.8618	-0.3988	-7.2211	+0.8894	-24.3542	+10.2386	-9.8322	+2.2660	+5.0144	-0.1784	+3.4314	-1.6841	+0.0662	+38.2322
2	-0.0701	-0.0540	-0.1284	-0.0277									-0.0177	+0.4122
3	-0.0274	+0.0017	-0.0112	+0.1077									-0.0241	-1.0128
4	-0.2836	+0.0093	-0.4857	+0.1077									+0.0024	-4.3466
5	-0.2608	+0.0062	-0.1222	+0.1560									-0.0801	-4.2748
6	-0.5366	+0.0137	-3.6956	-2.0467	+0.3182	+0.1077	+0.4561	-0.1279					-0.0628	+0.9170
7	-0.0397	+0.0556	-0.1032	-0.2942	+0.1127	+0.0144	+0.1154	-0.0654					+0.0069	+0.0553
8	+58.6348	-0.6123	-11.7674	-1.4012	-23.9233	+10.3557	+9.2607	+2.0827	+5.0144	-0.1784	+3.4314	-1.6841	+0.1092	+30.5824
$\delta\phi$		+0.010443	+0.200690	+0.023897	+0.408005	-0.176614	+0.175739	-0.035520	-0.035519	+0.069043	-0.058522	+0.028722	+0.001862	+0.521574
1	+34.2849	+1.6977	-27.5748	-0.0155	-7.4852	-3.0404	+1.0340	-0.2006	-1.5856	-0.5920	-1.8508	+0.1430	-0.1210	-2.9055
2	-0.0369	-0.0877	-0.0011	-0.0011									-0.0121	+0.2816
3	-0.0001	-0.0077	-0.0011	-0.0011									+0.0015	+0.0925
4	-0.0001	-0.0029	-0.0037	-0.0037									+0.0017	-3.2080
5	-0.0004	-0.0046	-0.0524	-0.0037	+0.0081	+0.0028	+0.0117	-0.0083					+0.0019	+0.1024
6	-0.0779	+0.1446	+0.4122	+0.4122	-0.1579	-0.0201	+0.1616	+0.0776					-0.0016	+0.0235
7	-0.0064	-0.1229	-0.0146	-0.0146	+0.2498	+0.1081	-0.0497	+0.0217	+0.0524	-0.0019	+0.0358	-0.0176	-0.0097	+0.9179
8	+34.0086	-1.1823	-27.3242	-0.3450	-7.8848	-2.9496	+0.7874	-0.1046	-1.5382	-0.5939	-1.8150	+0.1254	-0.1404	-6.2420
$\delta\lambda$		+0.034765	+0.803452	+0.021847	+0.086731	-0.023153	+0.038076	-0.003076	+0.045083	+0.017463	-0.053369	+0.03687	+0.004128	+0.183542
1	+109.1066	+6.3958	-13.4456	+0.4469	-13.4456	+0.4469	-51.1966	-1.6849	+5.7446	+0.9446	+5.2880	-1.2391	+0.8374	+15.2886
2	-0.2083	-0.0368											-0.0287	+0.6689
3	-0.0045	+0.0072											-0.0098	-0.4125
4	-0.8317	-0.1725											+0.0040	-7.4434
5	-0.0673	-0.0732											-0.0376	-2.0044
6	-25.4519	-14.0958	+2.1915	+0.7414	+2.1915	+0.7414	+3.1412	-0.8809					-0.4328	+6.3156
7	-0.2684	-0.7651	+0.2931	+0.0374	+0.2931	+0.0374	+0.3000	-0.1440	+1.0063	-0.0858	+0.6886	-0.3380	+0.0180	+1.7089
8	-2.3616	-0.2812	+4.8012	+2.0783	+4.8012	+2.0783	-1.8585	+0.4180	+0.0636	+0.0206	+0.0631	-0.0044	+0.0219	+6.1376
$\delta\phi$		+0.0411	-0.2741	+0.9499	-0.2741	+0.1025	-0.0274	+0.0036	+6.8042	+0.9294	+6.0897	-1.5815	+0.0049	+20.4707
1	+79.8818	-7.9253	-15.4881	+3.4065	-15.4881	+3.4065	-49.6413	-2.2882	+6.8042	-0.0294	+6.0897	-1.5815	+0.3335	+31.2170
$\delta\phi$		+0.099213	+0.193888	-0.042644	+0.042644	-0.021484	+14.5520	-3.7637	+1.0543	-0.6253	+0.0273	-0.7072	-0.004175	-0.256292
1	+46.8472	-2.6606											+0.2900	+0.0595
2	-0.0065												-0.0051	+0.1180
3	-0.0114												+0.0150	+0.6541
4	-0.0358												+0.0008	-1.5438
5	-0.0834												+0.0080	+2.5590
6	-7.8065												+0.2317	+3.4977
7	-2.1905												+0.0512	+4.8561
8	-0.0635												-0.0026	-0.7308
9	-21.9536												-0.1128	-5.0151
$\delta\lambda$		+13.9997											+0.0631	+2.0310
$\delta\phi$		+0.94576											-0.0785	+7.9474
$\delta\lambda$		+0.906261											-0.005631	-0.570127

11	12	13	14	15	16	17	18	19	20	21	22	$\eta$	$\Sigma$
+108.3016	-	-13.4998	-	-57.7023	+24.4953	-15.1399	+1.5610	+6.7770	+1.0673	+6.3184	-3.7670	-0.5202	-4.4519
-	6.8476	-0.2705	-	-	-	-	-	-	-	-	-	+0.0378	-0.5438
-	0.0638	-0.0408	-	+2.0459	-0.0728	+1.4000	0.6871	-	-	-	-	+0.0196	-1.8602
-	0.3200	-3.7784	-	+0.3555	-0.1377	+0.4208	0.0291	-	-	-	-	-0.0440	+12.4778
-	9.7608	-0.1826	-	+0.3555	-0.1377	+0.4208	0.0291	-	-	-	-	-0.0326	-1.4472
-	1.8281	-0.6359	-	+1.3193	-0.1802	+1.1710	0.3066	-	-	-	-	+0.0647	-3.9690
-	0.0030	-0.6605	-	+0.4010	-0.6591	-0.4871	-0.5219	-	-	-	-	+0.0510	+5.1824
-	5.8818	-1.8028	-	+3.1979	-	-	-	-	-	-	-	-	+13.3061
-	4.5532	-19.1125	-11.3911	-54.2916	+23.8059	-13.4768	0.0745	+6.7770	+1.0673	+6.3184	-3.7670	-0.4640	+13.3061
+82.3192	+0.055312	+0.232175	+0.138377	+0.659525	-0.289190	+0.103714	+0.000905	-0.082326	-0.012965	-0.076755	+0.045761	+0.005637	-0.161640
+28.4613	-	-6.7278	-15.4350	+1.3790	-6.7932	+6.3530	-1.9532	-2.6480	-1.0111	-3.9798	+1.2720	-0.0948	-0.2802
-	0.0216	-0.0915	-0.0257	-	-	-	-	-	-	-	-	+0.0126	-0.1840
6	0.0052	-0.0418	+0.0201	-	-	-	-	-	-	-	-	-0.0025	-0.2374
7	1.8290	+1.6356	-0.3678	-0.8856	+0.0315	-0.6060	0.2974	-	-	-	-	+0.0193	-5.4013
8	0.2588	-0.0983	-0.0091	-0.1380	-0.0815	-0.1574	0.0109	-	-	-	-	-0.0122	-0.5414
9	0.453	+2.1169	-0.0970	-0.2902	-0.0896	-0.2576	0.1670	-	-	-	-	-0.0142	-0.8730
10	-0.5525	+2.5151	-0.1229	+0.1229	-0.2020	-0.1493	0.0604	+0.3748	+0.0590	+0.3495	-0.2084	+0.0156	-1.5823
11	-0.2518	+1.0652	-0.6301	+3.0030	+1.3168	-0.7454	-0.7354	+0.0041	-0.0521	-3.6303	+1.0636	-0.0287	-0.7360
+25.4001	-	-1.5824	-17.2788	-2.8099	-5.7380	+4.4373	1.7334	-2.2732	-0.9521	-3.6303	+1.0636	-0.1019	-5.1990
$\delta\lambda_0$	-	+0.062299	+0.680265	+0.110626	+0.225905	-0.174696	+0.068244	+0.089496	+0.037484	+0.142925	-0.041874	+0.004012	-0.204684
5	+99.0937	+1.1318	-	-13.5624	+0.4952	-32.5324	-11.4363	+5.2774	+1.6860	+7.0946	-2.6458	-0.2161	-3.5317
6	-0.3877	+0.1087	+0.1067	-	-	-	-	-	-	-	-	+0.0534	-0.7795
7	-0.3353	+0.1009	+0.1009	+0.7920	-0.0282	+0.5420	0.2680	-	-	-	-	-0.0201	-1.9043
8	-1.4626	+0.3289	+0.0024	+0.0355	+0.0138	+0.0420	-0.0029	-	-	-	-	-0.0172	-4.8302
9	-0.0182	+0.0024	+0.0024	+0.2284	+0.5776	+3.7533	0.9828	-	-	-	-	-0.0083	-0.1445
10	-30.8488	-1.4220	+4.2284	+0.5595	+0.9195	+0.6793	0.7282	+1.5734	+0.2478	+1.4670	-0.8746	-0.0711	-7.2024
11	-11.4488	-4.4616	-4.4616	-12.6052	+5.5271	-3.1290	0.0173	-	-	-	-	-0.1077	-3.0893
12	-4.4374	-2.6447	-2.6447	-12.6052	+5.5271	-3.1290	0.0173	+0.1416	+0.0593	+0.2262	+0.0663	-0.0063	-0.3239
13	-0.0986	-1.0765	-1.0765	-1.1751	-0.3575	-0.2764	0.1080	-	-	-	-	-0.0063	-0.7433
	+50.0593	+1.1011	+1.1011	-22.1363	7.1475	-30.3832	-12.0505	+6.7092	+1.8745	+8.3354	-3.4541	-0.1746	7.0433
$\delta\sigma$	-	+0.021995	+0.021995	+0.442202	-0.142781	+0.606645	-0.240724	-0.134025	-0.037446	-0.166511	+0.069000	+0.003488	-0.140699
5	+25.6494	-	+25.6494	-0.1385	-0.3391	+0.81200	-3.6212	+1.6185	+0.0854	+0.9507	-0.9785	-0.0496	2.1778
6	-0.0905	-	-0.0905	-	-	-	-	-	-	-	-	-0.0150	-0.2186
7	-0.0772	-	-0.0772	-	-	-	-	-	-	-	-	+0.0096	-0.9137
8	-0.0740	-	-0.0740	-	-	-	-	-	-	-	-	+0.0039	1.0863
9	-0.0003	-	-0.0003	-	-	-	-	-	-	-	-	-0.0004	-0.0192
10	-0.0655	-	-0.0655	-	-	-	-	-	-	-	-	+0.0096	-0.5864
11	-0.1949	-	-0.1949	-	-	-	-	-	-	-	-	-0.0277	-2.8068
12	-0.2180	-	-0.2180	-	-	-	-	-	-	-	-	-0.0642	-1.8413
13	-7.5127	-	-7.5127	-	-	-	-	-	-	-	-	-0.0642	-2.8068
14	-1.5763	-	-1.5763	-	-	-	-	-	-	-	-	-0.0693	-3.5367
15	-1.9115	-	-1.9115	-	-	-	-	-	-	-	-	-0.0693	-3.5367
16	-1.7542	-	-1.7542	-	-	-	-	-	-	-	-	+0.0038	-0.5849
17	-0.0242	-	-0.0242	-	-	-	-	-	-	-	-	-0.0038	-0.5849
18	+10.3085	-	+10.3085	-	-	-	-	-	-	-	-	-0.0447	-3.7470
$\delta\lambda$	-	-	-	-	-	-	-	-	-	-	-	+0.004336	-0.363486

## Solution of normals—Continued

	15	16	17	18	19	20	21	22	$\eta$	$\Sigma$
7	+122.2200	-19.7152	-11.8987	+3.1998	-43.9722	+9.8031	-15.7932	+7.7351	+37.1447	+28.6969
8	-0.4283	+0.0153	-0.2694	+0.1440					+0.0093	-2.6154
9	-0.0691	-0.0268	-0.0818	+0.0056					-0.2814	-0.0063
10	-0.5796	-0.0792	-0.5144	+0.1347					-0.0284	-1.7437
11	-0.0273	+0.0449	-0.0352	+0.0356					-0.0035	-0.3520
12	+35.8067	+15.7006	+8.8833	+0.0491	+4.4696	+0.7039	+4.1671	-2.4844	-0.3060	+8.7757
13	-0.3108	-0.6348	-0.4909	-0.1918	-0.2515	-0.1063	+0.4016	+0.1177	-0.0113	-0.5751
14	-9.7387	+3.1606	+13.4289	-5.3288	+2.9668	-0.8289	+3.6859	-1.5274	-0.0772	+3.1146
15	+7.5905	-1.2234	+8.3426	-4.1136	+0.7399	-0.4340	+0.7104	-0.5966	-0.0384	+3.2153
		-2.7640	-26.2388	+6.0654	-36.0474	+10.7966	-9.0522	+3.2444	+36.6829	+38.1746
		+0.040876	+0.388042	+0.089700	+0.533100	-0.159669	+0.133872	-0.047981	-0.542498	-0.564559
7	+63.1452	-43.8742	-43.8742	-13.4344		-4.8829	+3.1960	-6.3741	-10.5001	-6.1221
8	-0.0005	-0.0104	-0.0051	-0.0022					-0.0003	-0.0931
9	-0.0104	-0.0317	+0.0184	+0.0184					-0.0025	-0.1090
10	-0.0738	-0.0793	-0.0585	-0.0585					-0.0039	-0.2882
11	-6.8844	-0.0546	-0.0546	-0.0215					+0.0057	+0.5785
12	-1.2962	+1.0094	+3.8974	-0.3916	-1.9598	-0.3087	-1.8272	+1.0694	+0.1342	-3.8490
13	-1.0205	+4.3360	-0.7206	+0.3916	-0.5135	-0.2151	-0.8201	+0.2403	-0.0230	-1.1745
14	-0.1991	-1.3512	+0.6663	-0.6663	-0.9579	-0.2676	-1.1901	+0.4932	+0.0249	-1.0056
15	-0.1130	-1.0725	-0.2479	-0.2479	+0.1198	+0.0703	-0.1151	-0.0966	+0.0062	-0.5208
		+53.5365	+18.2788	-43.5239	-13.2193	-5.3033	-0.3700	+1.1265	+1.4995	+1.5604
		$\partial\phi_3$	+0.341427	+0.812976	+0.040315	+0.090660	+0.021042	-4.5152	-8.8717	-9.7446
7	+142.9446	-21.3622	+142.9446	-12.7934		-0.8560	-94.2862	+2.4115	+43.2741	+30.0408
8	-0.2008	+0.0986	-0.2008						+0.0064	-1.7897
9	-0.0969	+0.0067	-0.0969						-0.0075	-0.3331
10	-0.4566	+0.1196	-0.4566						-0.0252	-1.5477
11	-0.0403	-0.0432	-0.0403						+0.0042	+0.4375
12	-2.2033	+0.0122	-2.2033						+0.0760	+2.1784
13	-0.7752	+0.3028	-0.7752						+0.0178	+0.4082
14	-18.4227	-7.3104	-18.4227						+0.1089	-4.2728
15	-2.1892	+4.3212	-2.1892						+0.0422	-3.5339
16	-10.1813	-2.3536	-10.1813						+14.2345	+14.8133
	-6.2409	+14.8602	-6.2409						+3.0290	+3.3271
									+90.3936	+48.7633
									-0.634694	-0.512468

## Solution of normals—Continued

	18	19	20	21	22	$\eta$	$\Sigma$
7	+ 61.5179	— 1.0383	— 2.8738	+ 31.3248	— 8.1976	+ 7.4331	+ 7.1924
8	— 0.0484					— 0.0031	+ 0.8784
9	— 0.0005					+ 0.0005	+ 0.0230
10	— 0.0313					+ 0.0066	+ 0.4053
11	— 0.0463					+ 0.0045	+ 0.4581
12	— 0.0001	— 0.0061	— 0.0010	— 0.0057	+ 0.0034	+ 0.0004	+ 0.0120
13	— 0.1183	— 0.1551	— 0.0650	— 0.2477	+ 0.0726	— 0.0070	+ 0.3548
14	— 2.9008	+ 1.6151	+ 0.4512	+ 2.0065	— 0.8315	— 0.0420	+ 1.6955
15	— 2.2294	+ 0.4010	— 0.2352	— 0.3850	— 0.3233	+ 0.0208	+ 1.7425
16	— 0.5441	+ 3.2335	+ 0.9685	— 0.8120	+ 0.2910	+ 3.2905	+ 3.4243
17	— 35.3839	— 14.8119	+ 4.3115	— 0.9158	— 3.6707	— 7.2125	— 7.9221
18	— 1.3061	— 1.8508	+ 0.8318	— 10.5336	+ 0.3480	+ 7.0757	+ 5.7131
	+ 18.9087	— 19.0796	— 5.2350	+ 20.4315	— 12.3081	+ 10.5259	+ 13.2434
	$\partial\lambda_9$	+ 1.009038	+ 0.276857	— 1.080534	+ 0.650923	— 0.556670	— 0.700387
11	+106.8200	+27.9899	— 16.8102	— 7.0763	+ 34.6511	+ 85.3611	
12	— 0.5579	— 0.0879	— 0.5202	+ 0.3101	+ 0.0382	— 1.0954	
13	— 0.2034	— 0.0852	— 0.3249	+ 0.0852	— 0.0091	— 0.4653	
14	— 0.8992	— 0.2512	— 1.1172	+ 0.4629	+ 0.0234	— 0.9440	
15	— 0.0721	+ 0.0423	+ 0.0693	+ 0.0582	+ 0.0037	— 0.3134	
16	— 19.2169	+ 5.7557	— 4.8257	+ 1.7296	+ 19.5557	+ 20.3509	
17	— 6.2003	— 1.8048	— 0.3834	— 1.5366	— 3.0192	+ 3.3162	
18	— 2.6226	+ 1.1786	— 14.9264	+ 0.4931	+ 10.0264	+ 8.0956	
19	— 19.2520	— 5.2823	+ 20.6162	— 12.4193	+ 10.6210	+ 13.3631	
	+ 57.7956	+27.4551	— 18.2225	— 17.8831	+ 71.8912	+121.0363	
	$\partial\phi_{10}$	— 0.475038	+ 0.315292	+ 0.309420	— 1.243887	— 2.094213	
11	+50.9179	— 9.2141	— 26.1027	+167.6744	+214.2334		
12	— 0.0138	— 0.0819	+ 0.0488	+ 0.0060	— 0.1725		
13	— 0.0357	— 0.1361	+ 0.0399	— 0.0038	— 0.1949		
14	— 0.0702	— 0.3121	+ 0.1293	+ 0.0065	— 0.2637		
15	— 0.0248	— 0.0406	— 0.0341	— 0.0022	+ 0.1839		
16	— 1.7239	+ 1.4454	— 0.5180	— 5.8571	— 6.0953		
17	— 0.5253	— 0.1116	— 0.4473	— 0.8788	— 0.9653		
18	— 0.5297	+ 6.7081	— 0.2216	— 4.5060	— 3.6382		
19	— 1.4493	+ 5.6566	— 3.4076	+ 2.9142	+ 3.6665		
	—13.0422	+ 8.6564	+ 8.4952	+ 34.1511	— 57.4968		
	+33.5030	+ 12.5701	— 22.0181	+125.2021	+149.2571		
	$\partial\lambda_{10}$	— 0.375193	+ 0.657198	— 3.737041	— 4.455037		
11	+173.0271	— 35.3120	—162.3283	—115.8122			
12	— 0.4850	+ 0.2891	+ 0.0356	— 1.0213			
13	— 0.5189	+ 0.1520	— 0.0146	— 0.7431			
14	— 1.3879	+ 0.5751	+ 0.0291	— 1.1728			
15	— 0.0665	— 0.0558	— 0.0036	+ 0.3009			
16	— 1.2118	+ 0.4343	+ 4.9108	+ 5.1105			
17	— 0.0237	— 0.0950	+ 0.1867	+ 0.2050			
18	—84.9517	+ 2.8063	+ 57.0642	+ 46.0750			
19	—22.0769	+ 13.2993	+ 11.3736	+ 14.3099			
20	— 5.7454	— 5.6384	+ 22.6667	+ 38.1618			
21	— 4.7162	+ 8.2610	— 46.9750	— 56.0002			
	+ 51.8431	— 15.2841	—136.1754	— 99.6164			
	$\partial\phi_{11}$	+ 0.294815	+ 2.626683	+ 1.921498			
11	+ 54.7848	+188.1235	+163.8779				
12	— 0.1724	— 0.0212	+ 0.6089				
13	— 0.0445	+ 0.0043	+ 0.2177				
14	— 0.2383	— 0.0120	+ 0.4860				
15	— 0.0469	— 0.0030	+ 0.2527				
16	— 0.1557	— 1.7601	— 1.8317				
17	— 0.3808	— 0.7482	— 0.8218				
18	— 0.0927	— 1.8851	— 1.5220				
19	— 8.0116	+ 6.8516	+ 8.6204				
20	— 5.5334	+ 22.2446	+ 37.4511				
21	— 14.4703	+ 82.2826	+ 98.0915				
22	— 4.5060	— 40.1466	— 29.3684				
	+ 21.1322	+254.9304	+276.0626				
	$\partial\lambda_{11}$	— 12.06360	— 13.06360				

*Back solution*

22	21	20	19	18	17	16	15
-12.0636	+2.6267	- 3.7370	-1.2439	- 0.5567	-0.6347	+0.1657	-0.5425
	-3.5565	- 7.9282	-3.7327	- 7.8525	+0.3765	-1.0174	+0.5788
-12.0636		+ 0.3489	-0.2932	+ 1.0047	-0.8785	-0.0196	-0.1245
	-0.9298		+5.3757	- 3.1330	+0.8443	-1.1210	+1.8069
		-11.3163		+ 0.1069	+0.0176	+0.0360	+0.0565
			+0.1059		-1.2220	-8.4798	-0.9356
				-10.4306		+0.5110	-0.5808
					-1.4968		-0.4057
						-9.9251	
							-0.1469
14	13	12	11	10	9	8	7
+0.0043	+0.0035	+0.0040	+0.0056	-0.0056	-0.0042	+0.0041	+0.0019
-0.8137	-0.8324	+0.5052	-0.5520	-0.6012	-0.2065	+0.0385	-0.2996
-0.0747	+0.1548	-0.1329	+0.0714	-0.0805	+0.1132	-0.0799	+0.0876
-0.5552	+0.4238	-0.4242	+0.1467	-0.7224	+0.1155	-0.1733	-0.0302
-0.0089	-0.0142	+0.0095	-0.0087	+0.0065	+0.0125	-0.0066	+0.0126
-4.8507	-2.5109	-0.7118	+0.0094	-2.2577	-0.1831	-0.0197	+0.2271
+1.4117	-0.9080	+0.2615	-0.2450	+1.9853	-1.3614	+0.0507	-0.3460
-1.3794	+1.4171	-2.2421	+2.8702	-1.4399	+0.3084	-0.6273	+1.2773
-0.1261	-0.0650	-0.0163	-0.0969	+0.2647	+0.0790	+0.0945	+0.1663
	+0.1406	-4.3487	-0.8846		-0.2828	-2.2905	-0.0681
-6.3927		-0.1365	-0.5086	-2.8508		+0.0490	-0.2829
	-2.1907		-0.4000		-1.4094		-0.0309
		-7.2323				-2.9605	
			+0.4075				+0.7151
6	5	4	3	2	1		
-0.0082	+0.0105	-0.0215	+0.0005	-0.0216	-0.0066		
-0.4199	-0.1370	-0.1192	+0.0600	-0.0453	+0.0242		
+0.2999	+0.1674	+0.0462	+0.1430	+0.0141	+0.0678		
+0.1234	+0.1304	-0.0050	+0.1295	-0.0045	+0.0600		
-0.0545	-0.0217	-0.0500	-0.0424	-0.0176	-0.0212		
-0.9952	-0.9773	-0.1310	-0.0343	-0.0328	-0.0699		
-0.1726	-0.8725	+0.3049	-0.5681	+0.4148	-0.2184		
+0.1954	-0.0068		-0.0020	+0.0084	+0.0113		
+0.0337	+0.0643	+0.0244		-0.0060	-0.2006		
	+0.0917		-0.3138		-0.0143		
-0.9980				+0.3095			
	-1.5510				-0.3677		

*Computation of corrections.*

1	2	$z_1$	3	4	5	$3a$	$z_2$
+0.700 +0.7	-0.842 -0.857 +0.700 +0.3	-0.842 -0.857 +0.3	+1.039 +1.0	-1.318 -0.098 +1.039 -0.6	+0.301 -1.541 +1.039 -0.9	+1.039 +1.0	+0.301 -1.541 -1.318 -0.098 -1.5
	-0.699 -0.7	-1.399 +0.700		-0.977 -1.0	-1.101 -1.1		-4.156 +1.039
4a	5a	6	7	8	$z_3$	17	15
-0.842 -0.857 +1.592	+0.301 -1.541 +1.592	+0.978 -0.699 +1.592 -0.9	+2.416 -0.699 -2.062 +0.055 +1.592 +1.4	+1.254 +0.149 -5.289 +0.479 +1.502 -2.1	+4.107 -3.649 -2.062 +0.055 -5.289 +0.479 -1.6	+0.857 -0.131 +1.860	+2.416 -0.699 -2.062 +0.055 +1.860 -2.7
-0.107 -0.1	+0.352 +0.4	+0.971 +1.0	+2.702 +2.7	-3.915 -3.9	-7.959 +1.592	+2.586 +2.6	-1.130 -1.1
16	$z_4$	14	9	10	11	12	13
-1.318 -0.098 +1.860 -1.9	+2.416 -0.699 -2.523 -0.174 -4.6	-0.753 -0.8	+2.624 -1.836 -0.753 +0.4	-4.045 +3.792 -0.753 +0.7	-2.295 +3.174 -0.753 +1.8	+0.857 -0.131 -0.753 -0.6	+0.978 -0.699 -0.753 -0.2
-1.456 -1.4	-5.580 +1.860		+0.435 +0.4	-0.306 -0.3	+1.926 +1.9	-0.627 -0.7	-0.674 -0.7
$z_5$	18	19	20	21	22	$z_6$	
+0.978 -0.699 +0.857 -0.131 -2.295 +3.174 +2.624 -1.836 -4.045 +3.792 +2.1	+1.254 +0.149 -5.289 +0.479 +3.728 +0.321 +0.3	+1.440 +0.062 -7.119 +2.545 +3.728 -1.8 -1.144 -1.2	-2.295 +3.174 +3.728 -2.8 +1.807 +1.8	+2.373 +2.275 +1.094 -6.750 +3.728 -2.9 -0.180 -0.2	+6.980 +1.976 -6.342 -5.645 +3.728 -1.5 -0.803 -0.8	+1.254 +0.149 +1.440 +0.062 -5.351 +10.449 +1.094 -6.750 -6.342 -5.645 -9.0	
+4.519 -0.753						-18.640 +3.728	
28	29	30	31	32	$z_7$		
-2.560 -1.066 +1.454 +2.604 -0.633	-1.495 -6.247 -4.579 +13.489 -0.633 +0.3	-0.179 -13.085 -0.352 +12.601 -0.633 +0.9	+2.373 +2.275 +1.094 -6.750 -0.633 +0.6	+2.624 -1.836 -0.632 +1.0 +1.155 +1.2	+2.373 +2.275 -0.515 -28.983 -0.352 +12.601 +1.454 +2.604 -4.579 +13.489 +2.8 +3.167 -0.633		
-0.201 -0.1	+0.835 +0.9	-0.748 -0.7	-1.041 -1.0				

## Computation of corrections—Continued

23	24	25	26	27	28
+6.980 +1.976 -6.342 -5.645 +1.588	-4.045 +3.792 +1.588 -1.4	-0.179 -13.085 -0.352 +12.601 +1.588 -0.6	+3.834 +4.162 +1.108 -10.559 +1.588 -0.3	+6.977 -2.480 -10.844 +5.562 +1.588 +0.9	+6.980 +1.976 -0.179 -13.085 +0.070 +12.429 +1.108 -10.559 -10.844 +5.562 -1.4
-1.448 -1.4	-0.065 -0.0	-0.027 -0.0	-0.167 -0.2	+1.703 +1.7	-7.942 +1.588
38	39	40	41	42	29
-2.180 -8.534 -0.786 +11.712 -1.284	-1.015 -17.358 -3.727 +25.033 -1.284 +3.4	+0.623 -27.410 +3.352 +24.228 -1.284 -2.2	+3.834 +4.162 +1.108 -10.559 -1.284 +1.6	-2.560 -1.066 +1.459 +2.604 -1.284 +0.7	-2.560 -1.066 +3.834 +4.162 -0.004 -61.258 +3.352 +24.228 -0.786 +11.712 -3.727 +25.033 +3.5
-1.072 -1.1	+5.049 +5.0	-2.691 -2.7	-1.139 -1.2	-0.147 -0.1	+6.420 -1.284
33	34	35	36	37	210
+6.977 -2.480 -10.844 +5.562 -2.538	-1.495 -6.247 -4.579 +13.489 -2.538 +3.0	+0.623 -27.410 +3.352 +24.228 -2.538 +3.5	+6.265 +10.228 -0.420 -15.880 -2.538 +2.2	+8.719 +0.384 -5.957 -0.626 -2.538 +0.1	-1.495 -6.247 +6.977 -2.480 +0.623 -27.410 +2.914 +53.890 -0.420 -15.880 -5.957 -0.626 +8.8
-3.323 -3.3	+1.630 +1.7	+1.755 +1.8	-0.145 -0.1	+0.082 +0.1	+12.689 -2.538
43	44	45	46	47	211
+8.719 +0.384 -5.957 -0.626 -2.716	-1.015 -17.358 -3.727 +25.033 -2.716 -1.8	+0.025 -55.481 -0.254 +58.307 -2.716 -0.6	+4.655 +22.009 +0.329 -23.877 -2.716 -2.0	+10.478 -12.412 -6.509 +14.356 -2.716 +0.9	-1.015 -17.358 +8.719 +0.384 +0.025 -55.481 +5.194 +92.311 +0.329 -23.877 -6.509 +14.356 -3.5
-0.196 -0.2	-1.583 -1.6	-0.719 -0.7	-1.600 -1.6	+4.097 +4.1	+13.578 -2.716

## Computation of corrections—Continued

48	49	50	51	52	$z_{12}$		
+0.720 +0.199 +0.519 -0.226 -1.196	+ 0.391 -29.478 - 2.473 +35.829 - 1.196 - 2.3	+ 0.025 -55.481 - 0.254 +58.308 - 1.196 - 0.3	+ 6.265 +10.228 - 0.420 -15.880 - 1.196 + 0.8	- 2.180 - 8.534 - 0.786 +11.712 - 1.196 - 0.7	- 2.180 - 8.534 + 6.265 +10.228 - 0.071 -88.929 - 0.254 +58.308 + 0.519 - 0.226 - 2.473 +35.829 - 2.5  + 5.982 - 1.196		
+0.016 +0.0	+ 0.773 + 0.8	+ 1.102 + 1.1	- 0.203 - 0.2	- 1.684 - 1.7			
53	54	55	56	57	$z_{13}$		
+10.478 -12.412 - 6.509 +14.356 - 7.305	+ 0.391 -29.478 - 2.473 +35.829 - 7.305 + 0.7	+ 0.177 -48.094 + 1.553 +61.270 - 7.305 - 1.8	+ 1.832 +22.318 - 7.305 -11.0  + 5.845 + 5.8	+ 5.607 -29.918 - 7.305 +33.7  + 2.084 + 2.1	+ 0.391 -29.478 +10.478 -12.412 + 0.177 -48.094 + 0.009 +93.855 +21.6  +36.526 - 7.305		
- 1.394 - 1.4	- 2.336 - 2.3	- 4.199 - 4.2					
58	59	60	61	62	$z_{14}$		
- 0.530 +20.030 - 4.333 -15.2	- 0.116 -36.552 - 4.333 +43.2	+ 0.177 -48.094 + 1.553 +51.270 - 4.333 + 1.3	+ 4.655 +22.009 + 0.329 -23.877 - 4.333 - 0.5	+0.720 +0.199 +0.519 -0.226 -4.333 +0.8	+ 0.720 + 0.199 + 4.655 +22.009 + 0.379 -88.720 + 1.553 +51.270 +29.6  +21.665 - 4.333		
- 0.033 - 0.0	+ 2.199 + 2.2	+ 1.873 + 1.9	- 1.717 - 1.7	-2.321 -2.3			
63	64	65	$z_{15}$	66	67	68	$z_{16}$
+ 5.607 -29.918 -10.240 +33.2	- 0.116 -36.552 -10.240 +46.9	-10.240 +11.6 + 1.360 + 1.3	- 0.116 -36.552 + 5.607 -29.918 +91.7 +30.721 -10.240	- 9.016 +13.3  + 4.284 + 4.2	+ 1.832 +22.318 - 9.016 -13.8  + 1.334 + 1.3	- 0.530 +20.030 - 9.016 -16.1  - 5.616 - 5.6	- 0.530 +20.030 - 9.016 -16.1  +27.050 - 9.016

*Final computation of triangles*

Symbol	Station	Observed angle	Correc- tion	Spher- ical angle	Spher- ical excess	Plane an- gle	Loga- rithm
		° ' "	"	"	"	° ' "	
	Fort Morgan-Dauphin Island east base						4.050203
-4a+5a	Cedar	43 54 21.8	+0.5	22.3	0.1	22.2	0.158967
-1+2	Fort Morgan	42 33 37.7	-1.4	36.3	0.1	36.2	9.830179
+3a-5	Dauphin Island east base	93 31 59.6	+2.1	61.7	0.1	32 01.6	9.999173
			+1.2		0.3		
	Cedar-Dauphin Island east base						4.039349
	Cedar-Fort Morgan						4.208343
	Dauphin Island east base-Dauphin Island west base						4.027832
-5a+6	Cedar	37 26 33.0	+0.6	33.6	0.1	33.5	0.216120
-3+5	Dauphin Island east base	103 55 36.4	-2.1	34.3	0.1	34.2	9.987043
-13+14	Dauphin Island west base	38 37 52.5	-0.1	52.4	0.1	52.3	9.795397
			-1.6		0.3		
	Cedar-Dauphin Island west base						4.230995
	Cedar-Dauphin Island east base						4.039349
-15+16	Cedar-Dauphin Island east base	69 30 32.7	-0.3	32.4	0.0	32.4	0.028387
-5a+7	Cedar	60 11 14.2	+2.3	16.5	0.1	16.4	9.938350
-4+5	Dauphin Island east base	50 18 11.4	-0.1	11.3	0.1	11.2	9.886171
			+1.9		0.2		
	Cat-Dauphin Island east base						4.006086
	Cat-Cedar						3.953907
	Cedar-Dauphin Island west base						4.230995
-15+17	Cat	135 31 54.9	+3.7	58.6	0.1	58.5	0.154592
-6+7	Cedar	22 44 41.2	+1.7	42.9	0.0	42.9	9.587301
-12+13	Dauphin Island west base	21 43 18.7	0.0	18.7	0.1	18.6	9.568320
			+5.4		0.2		
	Cat-Dauphin Island west base						3.972888
	Cat-Cedar						3.953907
	Dauphin Island east base-Dauphin Island west base						4.027832
-16+17	Cat	66 01 22.2	+4.0	26.2	0.1	26.1	0.039189
-3+4	Dauphin Island east base	53 37 25.0	-2.0	23.0	0.1	22.9	9.905867
-12+14	Dauphin Island west base	60 21 11.2	-0.1	11.1	0.1	11.0	9.939065
			+1.9		0.3		
	Cat-Dauphin Island west base						3.972888
	Cat-Dauphin Island east base						4.006086
-18+19	Cedar-Cat						3.953907
-7+8	Pins	23 22 40.3	-1.5	38.8	0.1	38.7	0.401443
	Cedar	30 54 61.5	-6.6	54.9	0.1	54.8	9.710768
	Cat	18.4		26.5	0.0	125 42 26.5	9.909561
					0.2		
	Pins-Cat						4.066118
	Pins-Cedar						4.264911
-18+20	Cedar-Dauphin Island west base						4.230995
-6+8	Pins	58 45 26.0	+1.5	27.5	0.2	27.3	0.068044
-11+13	Cedar	53 39 42.7	-4.9	37.8	0.2	37.6	9.906076
	Dauphin Island west base	67 34 57.9	-2.6	55.3	0.2	55.1	9.965872
			-6.0		0.6		
	Pins-Dauphin Island west base						4.205115
	Pins-Cedar						4.264911
-19+20	Cat-Dauphin Island west base						3.972888
	Pins	35 22 45.7	+3.0	48.7	0.1	48.6	0.237322
-11+12	Cat	35.3		34.9	0.0	98 45 34.9	9.994905
	Dauphin Island west base	45 51 39.2	-2.6	36.6	0.1	36.5	9.855908
					0.2		
	Pins-Dauphin Island west base						4.205115
	Pins-Cat						4.066118

## Final computation of triangles—Continued

Symbol	Station	Observed angle	Correc- tion	Spher- ical angle	Spher- ical excess	Plane an- gle	Loga- rithm
		° ' "	"	"	"	° ' "	
-23+24 -20+22 -10+11	Pins-Dauphin Island west base Grand Pins Dauphin Island west base	54 52 01.6 85 13 07.0 39 54 50.9	+1.4 -2.6 +2.2	03.0 04.4 53.1	0.2 0.1 0.2	02.8 04.3 52.9	4.205115 0.087341 9.998486 9.807296
	Grand-Dauphin Island west base Grand-Pins		+1.0		0.5		4.290942 4.099752
-30+31 -23+25 -21+22	Grand-Pins Petit Grand Pins	33 09 08.7 114 03 53.6 32 46 57.5	-0.3 +1.4 -0.6	08.4 55.0 56.9	0.1 0.1 0.1	08.3 54.9 56.8	4.099752 0.262119 9.960510 9.733559
	Petit-Pins Petit-Grand		+0.5		0.3		4.322381 4.095430 <sup>-1</sup>
-30+32 -24+25 - 9+10	Grand-Dauphin Island west base Petit Grand Dauphin Island west base	81 41 28.2 59 11 52.0 39 06 39.1	+1.9 0.0 -0.7	30.1 52.0 38.4	0.1 0.2 0.2	30.0 51.8 38.2	4.290942 0.004582 9.933962 9.799905
	Petit-Dauphin Island west base Petit-Grand		+1.2		0.5		4.229486 4.095429
-31+32 -20+21 - 9+11	Pins-Dauphin Island west base Petit Pins Dauphin Island west base	48 32 19.5 52 26 09.5 79 01 30.0	+2.2 -2.0 +1.5	21.7 07.5 31.5	0.2 0.2 0.3	21.5 07.3 31.2	4.205115 0.125281 9.899090 9.991984
	Petit-Dauphin Island west base Petit-Pins		+1.7		0.7		4.229486 4.322380 <sup>+1</sup>
-33+34 -25+27 -29+30	Grand-Petit Pascagoula Grand Petit	37 39 20.6 104 18 56.1 38 01 38.6	+5.0 +1.7 -1.6	25.6 57.8 37.0	0.1 0.2 0.1	25.5 57.6 36.9	4.095429 0.214006 9.986300 9.789603
	Pascagoula-Petit Pascagoula-Grand		+5.1		0.4		4.295735 4.099038
-40+41 -33+35 -26+27	Pascagoula-Grand Horn Pascagoula Grand	38 49 39.0 97 55 54.0 43 14 18.9	+1.5 +5.1 +1.9	40.5 59.1 20.8	0.1 0.2 0.1	40.4 58.9 20.7	4.099038 0.202744 9.995824 9.835719
	Horn-Grand Horn-Pascagoula		+8.5		0.4		4.297606 4.137501
-40+42 -34+35 -28+29	Pascagoula-Petit Horn Pascagoula Petit	77 06 13.2 60 16 33.4 42 37 10.3	+2.6 +0.1 +1.0	15.8 33.5 11.3	0.2 0.2 0.2	15.6 33.3 11.1	4.295735 0.011094 9.938731 9.830672
	Horn-Petit Horn-Pascagoula		+3.7		0.6		4.245560 4.137501
-41+42 -25+26 -28+30	Grand-Petit Horn Grand Petit	38 16 34.2 61 04 37.2 80 38 48.9	+1.1 -0.2 -0.6	35.3 37.0 48.3	0.2 0.2 0.2	35.1 36.8 48.1	4.095429 0.207989 9.942142 9.994187
	Horn-Petit Horn-Grand		+0.3		0.6		4.245560 4.297605 <sup>+1</sup>

*Final computation of triangles—Continued*

Symbol	Station	Observed angle	Cor- rec- tion	Spher- ical angle	Spher- ical excess	Plane an- gle	Loga <sup>a</sup> rithm
		° ' "	"	"	"	° ' "	
	Pascagoula-Horn						4. 137501
-43+44	Belle	48 58 49.8	- 1.4	48.4	0.1	48.3	0. 122351
-35+37	Pascagoula	69 43 28.0	- 1.7	26.3	0.2	26.1	9. 972219
-39+40	Horn	61 17 53.5	- 7.7	45.8	0.2	45.6	9. 943055
			-10.8		0.5		
	Belle-Horn						4. 232071
	Belle-Pascagoula						4. 202907
	Belle-Pascagoula						4. 202907
-50+51	Club	58 40 43.3	- 1.3	42.0	0.1	41.9	0. 068410
-43+45	Belle	89 28 55.3	- 0.5	54.8	0.2	54.6	9. 999982
-36+37	Pascagoula	31 50 23.4	+ 0.2	23.6	0.1	23.5	9. 722261
			- 1.6		0.4		
	Club-Pascagoula						4. 271299
	Club-Belle						3. 993578
	Belle-Horn						4. 232071
-50+52	Club	105 43 56.9	- 2.8	54.1	0.1	54.0	0. 016580
-44+45	Belle	40 30 05.5	+ 0.9	06.4	0.1	06.3	9. 812560
-38+39	Horn	33 45 53.7	+ 6.1	59.8	0.1	59.7	9. 744927
			+ 4.2		0.3		
	Club-Horn						4. 061211
	Club-Belle						3. 993578
	Pascagoula-Horn						4. 137501
-51+52	Club	47 03 13.6	- 1.5	12.1	0.1	12.0	0. 135496
-35+36	Pascagoula	37 53 04.6	- 1.9	02.7	0.2	02.5	9. 788214
-38+40	Horn	95 03 47.2	- 1.6	45.6	0.1	45.5	9. 998302
			- 5.0		0.4		
	Club-Horn						4. 061211
	Club-Pascagoula						4. 271299
	Belle-Club						3. 993578
-53+54	Deer	41 02 10.7	- 0.9	09.8	0.1	09.7	0. 182743
-45+47	Belle	102 35 18.5	+ 4.8	23.3	0.0	23.3	9. 989430
-49+50	Club	36 22 26.8	+ 0.3	27.1	0.1	27.0	9. 773096
			+ 4.2		0.2		
	Deer-Club						4. 165751
	Deer-Belle						3. 949417
	Deer-Belle						3. 949417
-60+61	Ship	33 10 60.2	- 3.6	56.6	0.1	56.5	0. 261770
-53+55	Deer	97 49 37.9	- 2.8	35.1	0.1	35.0	9. 995936
-46+47	Belle	48 59 22.9	+ 5.7	28.6	0.1	28.5	9. 877722
			- 0.7		0.3		
	Ship-Belle						4. 207123
	Ship-Deer						4. 088909
	Deer-Club						4. 165751
-60+62	Ship	70 52 35.0	- 4.2	30.8	0.2	30.6	0. 024657
-54+55	Deer	56 47 27.2	- 1.9	25.3	0.1	25.2	9. 922555
-48+49	Club	52 20 03.5	+ 0.8	04.3	0.1	04.2	9. 898501
			- 5.3		0.4		
	Ship-Club						4. 112963
	Ship-Deer						4. 088909
	Belle-Club						3. 993578
-61+62	Ship	37 41 34.8	- 0.6	34.2	0.1	34.1	0. 213655
-45+46	Belle	53 35 55.6	- 0.9	54.7	0.1	54.6	9. 905730
-48+50	Club	88 42 30.3	+ 1.1	31.4	0.1	31.3	9. 999890
			- 0.4		0.3		
	Ship-Club						4. 112963
	Ship-Belle						4. 207123

*Final computation of triangles—Continued*

Sym- bol	Station	Observed angle	Cor- rec- tion	Spher- ical angle	Spher- ical excess	Plane an- gle	Loga- rithm
		" ' "	"	"	"	" ' "	
-63+64	Deer-Ship	48 11 17.4	+ 1.3	18.7	0.1	18.6	4.088909
-55+57	Biloxi Lighthouse	96 30 31.2	+ 6.3	37.5	0.1	37.4	0.127644
-59+60	Deer	35 18 04.4	- 0.3	04.1	0.1	04.0	9.997190
	Ship						9.761833
			+ 7.3		0.3		
	Biloxi Lighthouse-Ship						4.213743
	Biloxi Lighthouse-Deer						3.978386
	Ship Island Lighthouse-Biloxi						4.323999
	Lighthouse						
-56+57	Deer	72 34 26.4	- 3.7	22.7	0.1	22.6	0.020407
-66+67	Ship Island Lighthouse	25 29 52.1	- 2.9	59.2	0.2	59.0	9.633980
-63+65	Biloxi Lighthouse	81 55 36.0	+ 2.6	38.6	0.2	38.4	9.995675
			- 4.0		0.5		
	Deer-Biloxi Lighthouse						3.978386
	Deer-Ship Island Lighthouse						4.340081
	Ship Island Lighthouse-Biloxi						4.323999
	Lighthouse						
-58+59	Ship	95 44 07.0	+ 2.2	09.2	0.1	09.1	0.002180
-66+68	Ship Island Lighthouse	50 31 41.2	- 9.8	31.4	0.2	31.2	9.887564
-64+65	Biloxi Lighthouse	33 44 18.6	+ 1.3	19.9	0.2	19.7	9.744612
			- 6.3		0.5		
	Ship-Biloxi Lighthouse						4.213743
	Ship-Ship Island Lighthouse						4.070791
-67+68	Deer-Ship	25 01 39.1	- 6.9	32.2	0.1	32.1	4.088909
-55+56	Ship Island Lighthouse	23 56 04.8	+10.0	14.8	0.1	14.7	0.373636
-58+60	Deer	131 02 11.4	+ 1.9	13.3	0.1	13.2	9.608246
	Ship						9.877536
			+ 5.0		0.3		
	Ship Island Lighthouse-Ship						4.070791
	Ship Island Lighthouse-Deer						4.340081

Final position computation,

## STATION CEDAR

$\alpha$	East base to west base						$^{\circ}$	$'$	$''$
Second angle	West base and Cedar						84	14	41.9
$\alpha$							+103	55	34.3
$\Delta\alpha$	East base to Cedar						188	10	16.2
							+		29.4
$\alpha'$	Cedar to east base						180	00	00.00
							8	10	45.6
							37	26	33.6
	First angle of triangle								
$\phi$	$^{\circ}$	$'$	$''$	Dauphin Island east base		$\lambda$	88	08	14.288
$\Delta\phi$	+	5	51.937			$\Delta\lambda$	—		58.262
$\phi'$	30	20	48.316	Cedar		$\lambda'$	88	07	16.026
			+1						
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$	$'$	$''$	$s$	$4.039349$	$s^2$	$8.0787$	$b^2$	$5.093$
	30	17	52	$\cos \alpha$	$9.995568$	$C$	$8.3054$	$D$	$2.332$
				$B$	$8.511556$		$1.1712$		
1st term	$''$			$h$	$2.546473$		$7.5553$		$7.425$
2d and 3d terms	—351.9435						+0.0036		+0.0027
$-\Delta\phi$	+ 0.0063								
	—351.9372								
	$s$	$4.039349$		$\Delta\lambda$	$1.765385$				
	$\sin \alpha$	$9.152688$		$\sin \frac{1}{2}(\phi+\phi')$	$9.702857$				
	$A'$	$8.509351$							
	$\sec \phi'$	$0.063997$							
		$1.765385$			$1.468242$				
		$''$			$''$				
	$\Delta\lambda$	—58.2620		$-\Delta\alpha$	—29.39				

## STATION CAT

$\alpha$	East base to west base						$^{\circ}$	$'$	$''$
Second angle	West base and Cat						+ 53	14	41.9
$\alpha$								37	23.0
$\Delta\alpha$	East base to Cat						137	52	04.9
							—	2	08.4
$\alpha'$	Cat to east base						180	00	00.00
							317	49	56.5
							66	01	26.2
	First angle of triangle								
$\phi$	$^{\circ}$	$'$	$''$	Dauphin Island east base		$\lambda$	88	08	14.288
$\Delta\phi$	+	4	04.168			$\Delta\lambda$	+	6	14.637
$\phi'$	30	19	00.547	Cat		$\lambda'$	88	14	28.925
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$	$'$	$''$	$s$	$4.006086$	$s^2$	$8.0122$	$b^2$	$4.776$
	30	16	58	$\cos \alpha$	$9.870171$	$C$	$9.6532$	$D$	$2.332$
				$B$	$8.511556$		$1.1712$		
1st term	$''$			$h$	$2.387813$		$8.8366$		$7.108$
2d, 3d, and 4th terms	—244.2379						+0.0686		+0.0013
$-\Delta\phi$	+ 0.0700								
	—244.1679								
	$s$	$4.006086$		$\Delta\lambda$	$2.405922$				
	$\sin \alpha$	$9.826619$		$\sin \frac{1}{2}(\phi+\phi')$	$9.702663$				
	$A'$	$8.509352$							
	$\sec \phi'$	$0.063865$							
		$2.405922$			$2.108585$				
		$''$			$''$				
	$\Delta\lambda$	+254.6373		$-\Delta\alpha$	+128.40				

## secondary triangulation

## STATION CEDAR

$\alpha$ Third angle	West base to east base Cedar and east base						° 264 — 35	' 11 37	'' 22.1 52.4		
$\alpha$ $\Delta\alpha$	West base to Cedar						225 +	33 3	29.7 49.5		
$\alpha'$	Cedar to west base						180 45	00 37	00.00 19.2		
$\phi$	30	14	21.492	Dauphin Island west base	$\lambda$	88	14	51.034			
$\Delta\phi$	+	6	26.825								
$\phi'$	30	20	48.317	Cedar	$\lambda'$	88	07	16.026			
$\frac{1}{2}(\phi+\phi')$  1st term 2d, 3d, and 4th terms }  $-\Delta\phi$	$\alpha$ 30	$\alpha'$ 17	$\alpha''$ 35	$s$ cos $\alpha$	4.230995 9.845212 8.511557	$s^2$ C	8.4620 9.7073 1.1711	$h^2$ D	5.176 2.331	$-\frac{h}{s^2 \sin^2 \alpha}$ E	2.588 8.169 5.917
	$\alpha''$			h	2.587764		9.3404 +0.2190		7.507 +0.0032		6.674 +0.0005
	—387.0473										
	+ 0.2227										
—386.8246											
	$s$ sin $\alpha$	4.230995 9.853676		$\sin \frac{\Delta\lambda}{2}(\phi+\phi')$	2.658019 9.702795						
	$\Lambda'$	8.509351 0.063997									
	sec $\phi'$	2.658019									
	$\Delta\lambda$	—455.0080									
		$\alpha''$	$-\Delta\alpha$	$-\Delta\alpha$			—229.52				

## STATION CAT

$\alpha$ Third angle	West base to east base Cat and east base					$^{\circ}$ 264 — 60	$'$ 11 21	$''$ 22.1 11.1			
$\alpha$ $\Delta\alpha$	West base to Cat					203 +	50 1	11.0 11.7			
$\alpha'$	Cat to west base					180 23	00 51	00.00 22.7			
$\phi$	30	14	21.492	Dauphin Island west base	$\lambda$	88	14	51.034			
$\Delta\phi$	+	4	39.055			$\Delta\lambda$	—	2	22.109		
$\phi'$	30	19	00.547	Cat	$\lambda'$	88	12	28.925			
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30	$'$ 16	$''$ 41	$s$ cos $\alpha$	3.972888 9.961280 8.511557	$s^2$ C	7.9458 9.2130 1.1711	$h^2$ D	4.891 2.331	$-\frac{h}{s^2 \sin^2 \alpha}$ E	2.445 7.159 5.917
1st term 2d, 3d, and 4th terms } $-\Delta\phi$	$''$ —279.0776			h	2.445725		8.3299 +0.0214		7.222 +0.0017		5.521
	+ 0.0231										
	—279.0545										
	$s$ sin $\alpha$	3.972888 9.606517		$\sin \frac{\Delta\lambda}{2}(\phi+\phi')$	2.152622 9.702600						
	$\Lambda'$	8.509352 0.063865									
	sec $\phi'$	2.152622									
	$\Delta\lambda$	—142.1092									
		$''$ —142.1092		$-\Delta\alpha$	$''$ —71.65						

*Final position computation,*

## STATION PINS

$\alpha$ Second angle	Cedar to west base West base and Pins					$^{\circ}$ 45 + 53	$'$ 37 39	$''$ 19.2 37.8			
$\alpha$ $\Delta\alpha$	Cedar to Pins					99 —	16 5	57.0 43.8			
$\alpha'$	Pins to Cedar					First angle of triangle					
$\phi$ $\Delta\phi$	$^{\circ}$ 30 +	$'$ 20 1	$''$ 48.317 35.914	Cedar	$\lambda$ $\Delta\lambda$	88 +	07 11	16.026 20.225			
$\phi'$	30	22	24.231	Pins	$\lambda'$	88	18	36.251 +1			
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30 "	$'$ 21 "	$''$ 36 "	$s$ $\cos \alpha$ B	4.264911 9.207641 8.511550	$s^2$ $\sin^2 \alpha$ C	8.5298 9.9885 1.1729	$h^2$ D	3.968 2.332	$-\frac{h}{E}$ $s^2 \sin^2 \alpha$ E	1.984 8.518 5.919
1st term 2d, 3d, and 4th terms — $\Delta\phi$	—96.4055 + 0.4916			h	1.984102	9.6912 +0.4911	6.300 +0.0002	6.421 +0.0003			
	—95.9139										
	$s$ $\sin \alpha$ A'	4.264911 9.9942754 8.5093503	$\sin \frac{1}{2}(\phi+\phi')$	2.832653 9.703663							
	$\sec \phi'$	0.064116		2.536316							
		2.8326527		2.536316							
		"		"							
	$\Delta\lambda$	+680.225	— $\Delta\alpha$	+343.81							

## secondary triangulation—Continued

## STATION PINS

[illegible]

## STATION GRAND

$\alpha$	West base to Pins						$^{\circ}$	'	"
Third angle	Grand and Pins						157 — 39	58 54	34.4 53.1
$\alpha$	West base to Grand						118 —	03 5	41.3 25.5
$d\alpha$							180 297	00 58	00.00 15.8
$\alpha'$	Grand to west base								
$\phi$	$^{\circ}$ 30	' 14	" 21.492	Dauphin Island west base	$\lambda$	88	14	51.034	
$d\phi$	+	4	58.083						
$\phi'$	30	19	19.575	Grand	$\lambda'$	88	25	36.493	
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30	' 16	" 50	$s$ $\cos \alpha$ B	$s^2$ $\sin^2 \alpha$ C	$h^2$ D	$4.950$ $2.331$	$-\frac{h}{s^2} \sin^2 \alpha$ E	$2.475$ $8.473$ $5.917$
1st term	—298.5266			h	2.474983				6.865
2d, 3d, and 4th terms }	+ 0.4435					9.6443 +0.4409	7.281 +0.0019		+0.0007
— $d\phi$	—298.0831								
	$s$ $\sin \alpha$ A' $\sec \phi'$	4.290942 9.945687 8.509352 0.063888	$d\lambda$ $\frac{1}{2}(\phi+\phi')$	2.809869 9.702633					
		2.809869		2.512502					
	$d\lambda$	+645.4595	— $d\alpha$	+325.46					

*Final position computation,*

## STATION PETIT

$\alpha$ Second angle	Grand to west base West base and Petit					$^{\circ}$ 297 + 59	$'$ 58 11	$''$ 15.8 52.0	
$\alpha$ $\Delta\alpha$	Grand to Petit					357 +	10	07.8 11.6	
$\alpha'$	Petit to Grand			First angle of triangle		180 177 81	00 10 41	00.00 19.4 30.1	
$\phi$ $\Delta\phi$	$^{\circ}$ 30 —	$'$ 19 6	$''$ 19.575 44.068	Grand	$\lambda$ $\Delta\lambda$	88 —	25	36.493 23.006	
$\phi'$	30	12	35.507	Petit	$\lambda'$	88	25	13.487	
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30 15 58	$'$ 15	$''$ 58	$\cos \alpha$ B	$\frac{s}{B}$ 4.095429 9.999470 8.511551	$\sin^2 \alpha$ C	$\frac{s^2}{C}$ 8.1909 7.3873 1.1725	$h^2$ D	$\frac{h^2}{D}$ 5.213 2.332
1st term 2d and 3d terms	+404.0639 + 0.0041			h	2.606450		6.7507 +0.0006		7.545 +0.0035
— $\Delta\phi$	+404.0680								
	$\frac{s}{\sin \alpha}$ $\Delta'$ $\sec \phi'$	4.095429 8.693666 8.509354 0.063392	$\sin \frac{\Delta\lambda}{2(\phi+\phi')}$		1.361841 9.702445	1.064286			
	1.361841								
	$\Delta\lambda$		$-\Delta\alpha$		$-\Delta\lambda$				

## STATION HORN

$\alpha$ Second angle	Grand to Petit Petit and Horn					$^{\circ}$ 357 + 61	$'$ 10 04	$''$ 07.8 37.0		
	Grand to Horn					58 —	14 5	44.8 18.1		
	Horn to Grand			First angle of triangle		180 238 38	00 09 16	00.00 26.7 35.3		
$\phi$ $\Delta\phi$	$^{\circ}$ 30 —	$'$ 19 5	$''$ 19.575 39.559	Grand	$\lambda$ $\Delta\lambda$	88 +	25 10	36.493 30.972		
$\phi'$	30	13	40.016	Horn	$\lambda'$	88	36	07.465		
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30 16 30	$'$ 16	$''$ 30	$\cos \alpha$ B	$\frac{s}{B}$ 4.297606 9.721214 8.5115514	$\sin^2 \alpha$ C	$\frac{s^2}{C}$ 8.5953 9.8592 1.1725	$h^2$ D	$\frac{h^2}{D}$ 5.061 2.332	$-\frac{h}{E}$ $s^2 \sin^2 \alpha$ 2.530 8.454 5.918
1st term 2d, 3d, and 4th terms }  $-\Delta\phi$	+339.1338			h	2.5303714	9.6270 +0.4236	7.393 +0.0025			
	+ 0.4253									
	+339.5591									
	$\frac{s}{\sin \alpha}$ A'	4.297606 9.929579 8.509354 0.063471	$\sin \frac{\Delta\lambda}{2(\phi+\phi')}$	2.800010 9.702560	2.502570					
	$\sec \phi'$	2.800010								
	$\Delta\lambda$	2.800010 +630.9719								$-\Delta\alpha$

## secondary triangulation—Continued

## STATION PETIT

$\alpha$ Third angle	West base to Grand Petit and Grand				$\lambda$	$\Delta\lambda$	$\lambda'$	$^{\circ}$ 118 — 39	$'$ 03 06	$''$ 41.3 38.4					
	West base to Petit							78 —	57 5	02.9 13.3					
	Petit to west base							180 258	00 51	00.00 49.6 — .1					
	$\phi$	$^{\circ}$ 30	$'$ 14	$''$ 21.492				Dauphin Island west base	88	14	51.034				
	$\Delta\phi$	—	1	45.985					+	10	22.452				
$\phi'$	30	12	35.507	Petit	88	25	13.486 +1								
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30	$'$ 13	$''$ 28	$\cos \alpha$ B	$\frac{s}{B}$ 4.229486 9.282513 8.511557	$\sin^2 \alpha$ C	$\frac{s^2}{C}$ 8.4590 9.9837 1.1711	$h^2$ D	$\frac{-h}{s^2 \sin^2 \alpha}$ E	$\frac{2.024}{8.443}$ 5.917					
	$''$ +105.5737			$h$	2.023556	9.6138 +0.4109	6.378 +0.0002	6.384 —0.0002							
	+ 0.4109														
	+105.9846														
	1st term 2d, 3d, and 4th terms }														
$-\Delta\phi$															
	$\frac{s}{\sin \alpha}$ A'	$\frac{s}{\sec \phi'}$		$\sin \frac{\Delta\lambda}{\frac{1}{2}(\phi+\phi')}$	$\frac{s}{\sin \alpha}$ A'	$\frac{s}{\sec \phi'}$	$\frac{s}{\sin \alpha}$ A'	$\frac{s}{\sec \phi'}$	$\frac{s}{\sin \alpha}$ A'	$\frac{s}{\sec \phi'}$					
$\Delta\lambda$	+622.4522		$-\Delta\alpha$	+313.34											

## STATION HORN

$\alpha$ Third angle	Petit to Grand Horn and Grand						$^{\circ}$ 177 — 80	$'$ 38	$''$ 19.4 48.3		
$\alpha$ $\Delta\alpha$	Petit to Horn						96 —	31 5	31.1 29.1		
$\alpha'$	Horn to Petit						180 276	00 26	00.00 02.0		
$\phi$ $\Delta\phi$ $\phi'$	$^{\circ}$ 30 +	$'$ 12 1	$''$ 35.507 04.509	Petit	$\lambda$ $\Delta\lambda$	88 +	25 10	13.487 53.978			
	30	13	40.016	Horn	$\lambda'$	88	36	07.465			
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30	$'$ 13	$''$ 08	$\cos \alpha$ B	$\frac{s}{B}$ 4.245560 9.055539 8.511559	$\sin^2 \alpha$ C	$\frac{s^2}{C}$ 8.4912 9.9943 1.1706	$h^2$ D	$\frac{3.625}{2.331}$ 3.625 2.331	$\frac{-h}{s^2 \sin^2 \alpha}$ E	$\frac{1.813}{8.485}$ 1.813 8.485
1st term 2d, 3d, and 4th terms } $-\Delta\phi$	$''$ —64.9618 + 0.4533 —64.5085			$h$	1.812658		9.6561 +0.4530	5.956 +0.0001		6.214 +0.0002	
	$\frac{s}{\sin \alpha}$ A'	$\frac{s}{\sec \phi'}$	$\frac{s}{\sin \alpha}$ A'	$\sin \frac{\Delta\lambda}{2(\phi+\phi')}$			2.815563 9.701830				
			2.815563				2.517393				
	$\Delta\lambda$	$''$ +653.9778	$-\Delta\alpha$	$''$ +329.15							

*Final position computation,*

## STATION PASCAGOULA

$\alpha$	Grand to Horn						$^{\circ}$	$'$	$''$
Second angle	Horn and Pascagoula						58	14	44.8
$\alpha$							+ 43	14	20.8
$\Delta\alpha$	Grand to Pascagoula						101	29	05.6
							—	3	52.8
$\alpha'$	Pascagoula to Grand						180	00	00.00
							281	25	12.8
							97	55	59.1
	First angle of triangle								
$\phi$	$^{\circ}$	$'$	$''$	Grand		$\lambda$	$^{\circ}$	$'$	$''$
$\Delta\phi$	30	19	19.575			$\Delta\lambda$	88	25	36.493
	+	1	20.998				+	7	40.886
$\phi'$	30	20	40.573	Pascagoula		$\lambda'$	88	33	17.379
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$	$'$	$''$	$s$	$4.099038$	$\sin^2 \alpha$	$8.1981$	$h^2$	$3.820$
	30	20	00	$\cos \alpha$	$9.299092$	$C$	$9.9824$	$D$	$2.332$
				$B$	$8.511551$		$1.1725$		$-h$
									$\sin^2 \alpha$
1st term	—81.2232			$h$	$1.909681$		$9.3530$		$6.152$
2d, 3d, and 4th terms	+ 0.2256						+0.2254		+0.0001
$-\Delta\phi$	—80.9976								6.008
									+0.0001
	$s$	$4.099038$		$\Delta\lambda$	$2.663593$				
	$\sin \alpha$	$9.991216$		$\sin \frac{1}{2}(\phi+\phi')$	$2.663593$				
	$\Delta'$	$8.509351$			$2.66910$				
	$\sec \phi'$	$0.063988$							
		$2.663593$							
		$''$							
	$\Delta\lambda$	+460.8859		$-\Delta\alpha$	+232.76				

## STATION BELLE

$\alpha$	Pascagoula to Horn						$^{\circ}$	$'$	$''$
Second angle	Horn and Belle						19	21	11.9
$\alpha$							+ 69	43	26.3
$\Delta\alpha$	Pascagoula to Belle						89	04	38.2
							—	5	01.7
$\alpha'$	Belle to Pascagoula						180	00	00.00
							268	59	36.5
							48	58	48.4
	First angle of triangle								
$\phi$	$^{\circ}$	$'$	$''$	Pascagoula		$\lambda$	$^{\circ}$	$'$	$''$
$\Delta\phi$	30	20	40.573			$\Delta\lambda$	88	33	17.379
	—		08.724				+	9	57.281
$\phi'$	30	20	31.849	Belle		$\lambda'$	88	43	14.660
									+2
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$	$'$	$''$	$s$	$4.202907$	$\sin^2 \alpha$	$8.4059$	$h^2$	$1.844$
	30	20	36	$\cos \alpha$	$8.206975$	$C$	$9.9999$	$D$	$2.332$
				$B$	$8.511550$		$1.1729$		
1st term	+8.3451			$h$	$0.921432$		$9.5787$		$4.176$
2d and 3d terms	+0.3790						+0.3790		
$-\Delta\phi$	+8.7241								
	$s$	$4.202907$		$\Delta\lambda$	$2.776179$				
	$\sin \alpha$	$9.999944$		$\sin \frac{1}{2}(\phi+\phi')$	$2.776179$				
	$\Delta'$	$8.509351$			$2.479626$				
	$\sec \phi'$	$0.063977$							
		$2.776179$							
		$''$							
	$\Delta\lambda$	+597.2814		$-\Delta\alpha$	+301.73				

## secondary triangulation—Continued

## STATION PASCAGOULA

$\alpha$ Third angle	Horn to Grand Pascagoula and Grand					$^{\circ}$ 238 — 38	$'$ 09 49	$''$ 26.7 40.5			
$\alpha$ $\Delta\alpha$	Horn to Pascagoula					199 +	19 1	46.2 25.8			
$\alpha'$	Pascagoula to Horn					180 19	00 21	00.00 12.0 — .1			
$\phi$ $\Delta\phi$	$^{\circ}$ 30 +	$'$ 13 7	$''$ 40.016 00.557	Horn	$\lambda$ $\Delta\lambda$	88 —	36 2	07.465 50.086			
$\phi'$	30	20	40.573	Pascagoula	$\lambda'$	88	33	17.379			
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30	$'$ 17	$''$ 10	$s$ cos $\alpha$ B	4.137501 9.974802 8.511558	$s^2$ sin <sup>2</sup> $\alpha$ C	8.2750 9.0396 1.1709	$h^2$ D	5.248 2.331	$-\frac{h}{s^2} \sin^2 \alpha$ E	2.624 7.315 5.917
1st term	—420.5020			$h$	2.623861		8.4855	7.579		5.856	
2d, 3d, and 4th terms }	+ 0.0345						+0.0306	+0.0038		+0.0001	
$-\Delta\phi$	—420.5575										
	$s$ sin $\alpha$ $\Delta'$ sec $\phi'$	4.137501 9.519828 8.509351 0.063988	$\Delta\lambda$ sin $\frac{1}{2}(\phi+\phi')$	2.230667 9.702706							
		2.230668		1.933373							
	$\Delta\lambda$	$''$ —170.0858	$-\Delta\alpha$	$''$ —85.78							

## STATION BELLE

$\alpha$ Third angle	Horn to Pascagoula Belle and Pascagoula						° 199 — 61	' 19 17	" 46.2 45.8		
$\alpha$ $\Delta\alpha$	Horn to Belle						138 —	02 3	00.4 35.4		
$\alpha'$	Belle to Horn						180 317	00 58	00.00 25.0 — .1		
$\phi$ $\Delta\phi$	° 30 +	' 13 6	" 40.016 51.834	Horn		$\lambda$ $\Delta\lambda$	88 +	36 7	07.465 07.197		
$\phi'$	30	20	31.850 —1	Belle		$\lambda'$	88	43	14.662		
$\frac{1}{2}(\phi+\phi')$	° 30	' 17	" 06	$s$ cos $\alpha$ B	4.232071 9.871302 8.511558	$s^2$ sin <sup>2</sup> $\alpha$ C	8.4642 9.6505 1.1709	$h^2$ D	5.230 2.331	$-\frac{h}{s^2} \sin^2 \alpha$ E	2.615 8.115 5.917
1st term 2d, 3d, and 4th terms } $-\Delta\phi$	—412.0321 + 0.1976 —411.8345			h	2.614931		9.2856 +0.1936	7.561 +0.0036		6.647 +0.0004	
	$s$ sin $\alpha$ $\Delta'$ sec $\phi'$	4.232071 9.825229 8.509351 0.063977		sin $\frac{1}{2}(\phi+\phi')$		2.630628 9.702690					
		2.630628				2.333318					
	$\Delta\lambda$	" +427.1968		$-\Delta\alpha$		" +215.44					

*Final position computation,*

## STATION CLUB

$\alpha$ Second angle	Belle to Horn Horn and Club						° 317 + 40	' 58 30	" 24.9 06.4
	Belle to Club						358 +	28	31.3 05.0
	Club to Belle			First angle of triangle			180 178 105	00 28 43	00.00 36.3 54.1
$\phi$ $\Delta\phi$	30 —	20 5	31.849 19.873	Belle	$\lambda$ $\Delta\lambda$		88 —	43	14.662 09.806
$\phi'$	30	15	11.976	Club	$\lambda'$		88	43	04.856
$\frac{1}{2}(\phi+\phi')$  1st term 2d and 3d terms — $\Delta\phi$	° 30	' 17	" 52	$\frac{s}{\cos \alpha}$ B	3.993578 9.999846 8.511550	$\frac{s^2}{C}$ $\sin^2 \alpha$ C	7.9872 6.8500 1.1729	$\frac{h^2}{D}$	5.010 2.332
	"			h	2.504974		6.0101 +0.0001		7.342 +0.0022
	+319.8704								
	+ 0.0023								
	+319.8727								
	$\frac{s}{\sin \alpha}$ A'	3.993578 8.424985 8.509353 0.063584	$\sin \frac{\Delta\lambda}{2(\phi+\phi')}$	$\frac{\Delta\lambda}{2(\phi+\phi')}$	0.991500 9.702856				
	$\sec \phi'$								
		0.991500							
		"							
	$\Delta\lambda$	—9.8062	— $\Delta\alpha$		—4.95				

## STATION DEER

$\alpha$ Second angle	Belle to Club Club and Deer						$^{\circ}$ 358 +102	$'$ 28 35	$''$ 31.3 23.3
	Belle to Deer						101 —	03 2	54.6 45.3
	Deer to Belle			First angle of triangle			180 281 41	00 01 02	00.00 09.3 09.8
				Belle	$\lambda$ $\Delta\lambda$		88 +	43 5	14.662 27.087
$\phi$ $\Delta\phi$ $\phi'$	$^{\circ}$ 30 +	$'$ 20	$''$ 31.849 55.361	Deer	$\lambda'$		88	48	41.749
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30	$'$ 21	$''$ 00	$\frac{s}{\cos \alpha}$ B	3.949417 9.283133 8.511550	$\frac{s^2}{C}$ $\sin^2 \alpha$ C	7.8989 9.9837 1.1729	$\frac{h^2}{D}$	3.488 2.332
1st term 2d and 3d terms — $\Delta\phi$	$''$			h	1.744100		9.0555 +0.1136		5.820 +0.0001
	—55.4752								
	+ 0.1137								
	—55.3615								
	$\frac{s}{\sin \alpha}$ A'	3.949417 9.991850 8.509351 0.064045		$\sin \frac{\Delta\lambda}{2(\phi+\phi')}$	2.514663 9.703531				
	$\sec \phi'$								
		2.514663							
	$\Delta\lambda$	+327.0872		— $\Delta\alpha$	+165.27				

## secondary triangulation—Continued

## STATION CLUB

$\alpha$ Third angle	Horn to Belle Club and Belle						$^{\circ}$ 138 —33	$'$ 02 45	$''$ 00.4 59.8		
$\alpha$ $\Delta\alpha$	Horn to Club						104 —	16 3	00.6 30.2		
$\alpha'$	Club to Horn						180 284	00 12	00.00 30.4		
$\phi$ $\Delta\phi$	$^{\circ}$ 30 +	$'$ 13 1	$''$ 40.016 31.961	Horn	$\lambda$ $\Delta\lambda$		88 +	36 6	07.465 57.391		
$\phi'$	30	15	11.977 —1	Club	$\lambda'$		88	43	04.856		
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30	$'$ 14	$''$ 26	$s$ $\cos \alpha$ B	4.061211 9.391708 8.511558	$s^2$ $\sin^2 \alpha$ C	8.1225 9.9728 1.1709	$h^2$ D	3.929 2.331	$-\frac{h}{E}$ $s^2 \sin^2 \alpha$ E	1.964 8.095 5.917
1st term 2d, 3d, and 4th terms — $\Delta\phi$	$''$ —92.1459			h	1.964477	$s^2$ 9.2662 +0.1845	$h^2$ 6.260 +0.0002	$-\frac{h}{E}$ $s^2 \sin^2 \alpha$	5.976 +0.0001		
	+ 0.1848										
	—91.9611										
	$s$ $\sin \alpha$ A'	4.061211 9.986395 8.509353	$\Delta\lambda$ $\sin \frac{1}{2}(\phi+\phi')$		2.620543 9.702113						
	$\sec \phi'$	0.063584									
		2.620543									
		$''$ +417.3909									
	$\Delta\lambda$		— $\Delta\alpha$		+210.21						

## STATION DEER

$\alpha$ Third angle	Club to Belle Deer and Belle					$^{\circ}$ 178 —36	$'$ 28 22	$''$ 36.3 27.1			
$\alpha$ $\Delta\alpha$	Club to Deer					142 —	06 2	09.2 50.0			
$\alpha'$	Deer to Club					180 322	00 03	00.00 19.2 —1			
$\phi$ $\Delta\phi$	$^{\circ}$ 30 +	$'$ 15 6	$''$ 11.976 15.234	Club	$\lambda$ $\Delta\lambda$	88 +	43 5	04.856 36.893			
$\phi'$	30	21	27.210	Deer	$\lambda'$	88	48	41.749			
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30	$'$ 18	$''$ 20	$s$ $\cos \alpha$ B	4.165751 9.897138 8.511556	$s^2$ $\sin^2 \alpha$ C	8.3315 9.5767 1.1713	$h^2$ D	5.149 2.331	$-\frac{h}{E}$ $s^2 \sin^2 \alpha$	2.574 7.908 5.918
1st term 2d, 3d, and 4th terms } $-\Delta\phi$	$''$ —375.3574			h	2.574445	9.0795 +0.1200	7.480 +0.0030	6.400 +0.0003			
	+ 0.1233										
	—375.2341										
	$s$ $\sin \alpha$ A'	4.165751 9.788345 8.509351	$\Delta\lambda$ $\sin \frac{1}{2}(\phi+\phi')$	2.527492 9.702957	2.230449	"	"				
	$\sec \phi'$	0.064045									
		2.527492									
	$\Delta\lambda$	+336.8930	$-\Delta\alpha$	+170.00							

*Final position computation,*

## STATION SHIP

$\alpha$ Second angle	Deer to Club Club and Ship						$^{\circ}$ 322 + 56	$'$ 03 47	$''$ 19.1 25.3
$\alpha$ $\Delta\alpha$	Deer to Ship						18 —	50 1	44.4 14.8
$\alpha'$	Ship to Deer			First angle of triangle			180 198 70	00 49 52	00.00 29.6 30.8
$\phi$ $\Delta\phi$	$^{\circ}$ 30 —	$'$ 21 6	$''$ 27.210 17.189	Deer	$\lambda$ $\Delta\lambda$	88 +	48 2	41.749 28.277	
$\phi'$	30	15	10.021	Ship	$\lambda'$	88	51	10.026	
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30 18 19	$'$ 11	$''$ 19	$s$ $\cos \alpha$ B	4.088909 9.976071 8.511549	$s^2$ $\sin^2 \alpha$ C	8.1778 9.0184 1.1731	$h^2$ D	5.153 2.332
1st term	+377.1629	h		2.576529	8.3693	7.485			
2d and 3d terms	+ 0.0265				+0.0234	+0.0031			
$-\Delta\phi$	+377.1894								
	$s$ $\sin \alpha$ $\Delta\lambda'$ $\sec \phi'$	4.088909 9.509230 8.509353 0.063581	$\Delta\lambda$ $\sin \frac{1}{2}(\phi+\phi')$	2.171073 9.702952					
		2.171073		1.874025					
	$\Delta\lambda$	+148.2767	$-\Delta\alpha$	+74.82					

## STATION BILOXI LIGHTHOUSE

$\alpha$ Second angle	Deer to Ship Ship and Biloxi Lighthouse					$^{\circ}$ 18 + 96	$'$ 50 30	$''$ 44.4 37.5			
$\alpha$ $\Delta\alpha$	Deer to Biloxi Lighthouse					115 —	21 2	21.9 42.9			
$\alpha'$	Biloxi Lighthouse to Deer			First angle of triangle		180 295 48	00 18 11	00.00 39.0 18.7			
$\phi$ $\Delta\phi$	$^{\circ}$ 30 +	$'$ 21 2	$''$ 27.210 12.209		Deer	$\lambda$ $\Delta\lambda$	88 +	48 5	41.749 22.071		
$\phi'$	30	23	39.419	Biloxi Lighthouse	$\lambda'$	88	54	03.820			
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30 22 33	$'$ 22	$''$ 33	$s$ $\cos \alpha$ B	3.978386 9.631690 8.511549	$s^2$ $\sin^2 \alpha$ C	7.9568 9.9120 1.1731	$h^2$ D	4.243 2.332	$-\frac{h}{s^2 \sin^2 \alpha}$ E	2.121 7.869 5.919
1st term 2d, 3d, and 4th terms	-132.3198 + 0.1107			h	2.121625	9.0419 +0.1102	6.575 +0.0004	5.909 +0.0001			
$-\Delta\phi$	-132.2091										
	$s$ $\sin \alpha$ $\Delta'$ $\sec \phi'$	3.978386 9.956007 8.509350 0.064209	$\Delta\lambda$ $\sin \frac{1}{2}(\phi+\phi')$	2.507952 9.703868							
		2.507952		2.211820							
		$''$ $\Delta\lambda$	$-\Delta\alpha$	$''$ +162.86							

## secondary triangulation—Continued

## STATION SHIP

$\alpha$ Third angle	Club to Deer Ship and Deer					$^{\circ}$ 142 — 52	$'$ 06 20	$''$ 09.2 04.3
$\alpha$ $\Delta\alpha$	Club to Ship					89 —	46 4	04.9 04.4
$\alpha'$	Ship to Club					180 269	00 42	00.00 00.5 — .1
$\phi$ $\Delta\phi$	$^{\circ}$ 30 —	$'$ 15	$''$ 11.976 01.955	Club	$\lambda$ $\Delta\lambda$	88 +	43 8	04.856 05.170
$\phi'$	30	15	10.021	Ship	$\lambda'$	88	51	10.026
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30	$'$ 15	$''$ 11	$s$ $\cos \alpha$ B	$\frac{s^2}{C}$ $\sin^2 \alpha$ C	8.2260 0.0000 1.1713		
1st term	+1.7054	h	0.231833			9.3973 +0.2496		
2d term	+0.2496							
$-\Delta\phi$	+1.9550							
	$s$ $\sin \alpha$ $\Delta'$ $\sec \phi'$	4.112963 9.9999964 8.5093531 0.0635812	$\Delta\lambda$ $\sin \frac{1}{2}(\phi+\phi')$	2.685894 9.702275				
		2.6858937		2.388169				
	$\Delta\lambda$	+485.1697	$-\Delta\alpha$	+244.44				

## STATION BILOXI LIGHTHOUSE

$\alpha$ Third angle	Ship to Deer Biloxi Lighthouse and Deer					$^{\circ}$ 198 — 35	$'$ 49 18	$''$ 29.6 04.1
$\alpha$ $\Delta\alpha$	Ship to Biloxi Lighthouse					163 —	31 1	25.5 27.7
$\alpha'$	Biloxi Lighthouse to Ship					180 343	00 29	00.00 57.8 — .1
$\phi$ $\Delta\phi$	$^{\circ}$ 30 +	$'$ 15 8	$''$ 10.021 29.398	Ship	$\lambda$ $\Delta\lambda$	88 +	51 2	10.026 53.794
$\phi'$	30 Fixed value	23	39.419 39.419	Biloxi Lighthouse	$\lambda'$	88	54	03.820 03.820
$\frac{1}{2}(\phi+\phi')$	$^{\circ}$ 30	$'$ 19	$''$ 25	$s$ $\cos \alpha$ B	$\frac{s^2}{C}$ $\sin^2 \alpha$ C	8.4275 8.9056 1.1713	$h^2$ D	$-\frac{h}{E}$ $s^2 \sin^2 \alpha$ E
1st term	—509.4353	h	2.707089			8.5044 +0.0319	5.414 2.331	2.707 7.333
2d, 3d, and 4th terms	+ 0.0376					+0.0056	7.745 +0.0056	5.958 +0.0001
$-\Delta\phi$	—509.3977							
	$s$ $\sin \alpha$ $\Delta'$ $\sec \phi'$	4.213743 9.452733 8.509350 0.064209	$\Delta\lambda$ $\sin \frac{1}{2}(\phi+\phi')$	2.240035 9.703190				
		2.240035		1.943225				
	$\Delta\lambda$	+173.7941	$-\Delta\alpha$	+87.74				



## secondary triangulation—Continued

## STATION SHIP ISLAND LIGHTHOUSE

$\alpha$ Third angle	Ship to Biloxi Lighthouse					°	163	31	25.5								
	Ship Island Lighthouse and Biloxi Lighthouse					—	95	44	09.2								
	Ship to Ship Island Lighthouse						67	47	16.3								
						—		3	25.2								
$\Delta\alpha$							180	00	00.00								
							247	43	51.1								
$\alpha'$	Ship Island Lighthouse to Ship																
$\phi$ $\Delta\phi$	°	30	15	10.021	Ship	$\lambda$ $\Delta\lambda$	88	51	10.026								
	—		2	24.681													
$\phi'$	30	12	45.340	Ship Island Light-house	$\lambda'$	88	57	57.464									
			+1 45.341														
Fixed value																	
$\frac{1}{2}(\phi+\phi')$	°	30	13	58	$s$ $\cos \alpha$ B	4.070791 9.577534 8.511556	$s^2$ $\sin^2 \alpha$ C	8.1416 9.9330 1.1713	$h^2$ D	4.320 2.331	$-h$ $s^2 \sin^2 \alpha$ E	2.160 8.075 5.918					
	1st term	"											h	2.159881	9.2459 +0.1762	6.651 +0.0004	6.153 —0.0001
	2d, 3d, and	+144.5043															
	4th terms	+ 0.1765															
	— $\Delta\phi$			+144.6808													
$\sin \alpha$ A' sec $\phi'$	°	4.070791 9.966513 8.509354 0.063404		$\Delta\lambda$ $\sin \frac{1}{2}(\phi+\phi')$	2.610062 9.702011	2.312073	"	+205.15									
		2.610062															
		"															
		+407.4385															
		— $\Delta\alpha$															

## ADJUSTMENTS BY THE ANGLE METHOD

If the adjustment be made according to the angle method\* the complications due to the presence of the  $z$ 's are avoided. An angle is the difference of two directions and the observation equation for an observed angle is the difference of the observation equations of its two sides, and in taking the difference the  $z$  drops out. To illustrate this suppose that at station Gunner, Figure 6, page 104, the following angles were observed: Duck to Indian Point, Indian Point to Larrabee, Larrabee to Mam, Mam to Lubec Channel Lighthouse, and Lubec Channel Lighthouse to Lubec Church spire. Call the corrections to the observed angles  $u_1, u_2, u_3, u_4$ , and  $u_5$  respectively, and suppose the observed and assumed values to be as given on page 115. Then

$$u_1 = v_2 - v_1 = -6131\delta\phi_1 + 1800\delta\lambda_1 - 2.2$$

$$u_2 = v_3 - v_2 = -1997\delta\phi_1 + 236\delta\lambda_1 + 7.0$$

In a similar way,

$$u_3 = -2732\delta\phi_1 + 85\delta\lambda_1 - 1.2$$

$$u_4 = -5565\delta\phi_1 - 2445\delta\lambda_1 + 3.5$$

$$u_5 = +5886\delta\phi_1 + 5413\delta\lambda_1 + 18.4$$

These contain no  $z$ 's and the normal equations may be formed in the usual way.

Observation equations of this kind would arise when at an unknown point angles are taken on known points, as for example when angles are taken with a sextant from a point off-shore to determine its position, and for such observations the angle method is both easier and more logical than the direction method.

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\* Wright and Hayford, Adjustment of observations, p. 180.

## ADJUSTMENT OF VERTICAL OBSERVATIONS

## GENERAL STATEMENT

When reciprocal vertical observations are made over the lines of a triangulation scheme a computation of the differences of elevations is made by the usual Coast and Geodetic Survey formula. For an account of these observations and of the method of computation, see United States Coast and Geodetic Survey Special Publication No. 19, page 140 et seq. As there are always several lines from each station, rigid conditions are present in the figure. Thus it becomes necessary to make an adjustment of the observed values by the method of least squares. In the following figure the differences of elevations as observed are first computed and then the results are adjusted by the method employed in the United States Coast and Geodetic Survey.

The formula used in the following computations is the one given in Special Publication No. 19, mentioned above. On pages 205 et seq. there is given a new development of the formula that takes into account some of the small terms that are needed in computation over longer and higher lines. The final form of the new formula differs slightly from the one used in this computation.

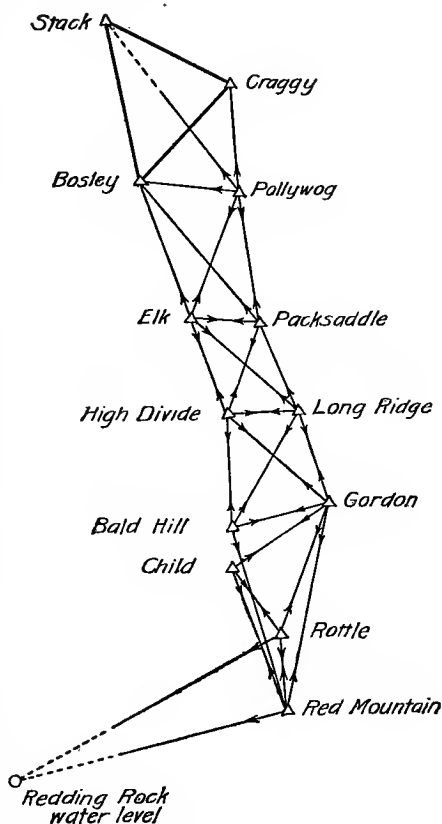


FIG. 8.

## Computation of elevations from reciprocal observations

Station 1	Pollywog	Pollywog	Pack Sad-	Pack Sad-	Elk	Elk
Station 2	Pack Sad-	Elk	dle High Di-	dle Elk	Long Ridge	High Di-
	dle		vide			vide
	° ' "	° ' "	° ' "	° ' "	° ' "	° ' "
$\zeta_1$	90 04 13	90 57 44	90 30 00	91 53 50	88 33 16	89 15 25
$\zeta_2$	90 04 59	89 11 15	89 37 33	88 10 59	91 36 54	90 51 04
$\zeta_2 - \zeta_1$	+ 23	- 46 29	- 52 27	- 3 42 51	+ 3 03 38	+ 1 35 39
$\frac{1}{2}(\zeta_2 - \zeta_1)$	+ 11	- 23 14	- 26 14	- 1 51 26	+ 1 31 49	+ 0 47 50
$\tan \frac{1}{2}(\zeta_2 - \zeta_1)$	6.04732	8.18994	7.88258	8.51090	8.42675	8.14348
$\log s$	4.27444	4.29253	4.15543	3.98141	4.31524	4.17150
$\log s \tan \frac{1}{2}(\zeta_2 - \zeta_1)$	0.32176	2.48247	2.08801	2.49234	3.74199	2.31498
$s \tan \frac{1}{2}(\zeta_2 - \zeta_1)$	+2.10	-303.72	-109.15	-310.70	+552.06	+206.53
Second term	0.00	0.00	0.00	0.00	0.00	0.00
Third term	0.00	0.03	0.01	0.03	0.06	0.02
$h_2 - h_1$	+2.10	-303.75	-109.16	-310.73	+552.12	+206.55
$2 \log s$	8.549	8.585	8.311	7.963	8.630	8.343
$\log p = 9 - 2 \log s$	0.451	0.415	0.689	1.037	0.370	0.657
$p$ of $h_2 - h_1$	2.82	2.60	4.89	10.89	2.34	4.54

Station 1	High Di-	High Di-	High Di-	Long Ridge	Long Ridge	Gordon
Station 2	vide Bald Hill	vide Gordon	vide Long Ridge	Gordon	Bald Hill	Bald Hill
	° ' "	° ' "	° ' "	° ' "	° ' "	° ' "
$\zeta_1$	89 29 43	88 28 23	88 12 34	89 14 01	91 27 22	92 45 09
$\zeta_2$	89 37 50	91 40 22	91 53 03	90 52 54	88 41 46	87 21 10
$\zeta_2 - \zeta_1$	+ 11 53	+ 3 11 59	+ 3 40 31	+ 1 38 53	- 2 45 36	- 5 23 59
$\frac{1}{2}(\zeta_2 - \zeta_1)$	+ 5 56	+ 1 36 00	+ 1 50 16	+ 49 26	- 1 22 48	- 2 42 00
$\tan \frac{1}{2}(\zeta_2 - \zeta_1)$	7.87759	8.44611	8.50632	8.15777	8.38184	8.67357
$\log s$	4.21546	4.29195	4.03443	4.14252	4.29013	4.15348
$\log s \tan \frac{1}{2}(\zeta_2 - \zeta_1)$	2.09305	2.73806	2.54075	2.30029	2.67197	2.82705
$s \tan \frac{1}{2}(\zeta_2 - \zeta_1)$	-123.89	+547.09	+347.34	+199.66	-469.86	-671.51
Second term	0.00	0.00	0.00	0.00	0.00	0.00
Third term	0.01	0.08	0.05	0.03	0.06	0.07
$h_2 - h_1$	-123.90	+547.17	+347.39	+199.69	-469.92	-671.58
$2 \log s$	8.431	8.584	8.069	8.285	8.580	8.307
$\log p = 9 - 2 \log s$	0.569	0.416	0.931	0.715	0.420	0.693
$p$ of $h_2 - h_1$	3.71	2.61	8.53	5.19	2.63	4.93

Station 1	Gordon	Gordon	Gordon	Child	Child	Rattle
Station 2	Red Moun-	Rattle	Child	Rattle	Red Moun-	Red Moun-
	tain				tain	tain
	° ' "	° ' "	° ' "	° ' "	° ' "	° ' "
$\zeta_1$	90 03 28	90 31 05	91 56 00	87 56 00	88 33 47	89 07 26
$\zeta_2$	90 10 44	89 38 48	88 11 27	92 09 03	91 36 13	90 57 56
$\zeta_2 - \zeta_1$	+ 07 16	- 52 17	- 3 44 33	+ 4 13 03	+ 3 02 26	+ 1 50 30
$\frac{1}{2}(\zeta_2 - \zeta_1)$	+ 03 38	- 26 08.5	- 1 52 16.5	+ 2 06 31.5	+ 1 31 13	+ 0 55 15
$\tan \frac{1}{2}(\zeta_2 - \zeta_1)$	7.02404	7.88106	8.51416	8.56610	8.42390	8.20610
$\log s$	4.48786	4.31274	4.23223	4.03839	4.34662	4.06606
$\log s \tan \frac{1}{2}(\zeta_2 - \zeta_1)$	1.51190	2.19380	2.74639	2.60449	2.77052	2.27216
$s \tan \frac{1}{2}(\zeta_2 - \zeta_1)$	+32.50	-156.24	-557.69	+402.24	+589.55	+187.14
Second term	0.00	0.00	0.00	0.00	0.00	0.00
Third term	0.01	0.03	0.09	0.06	0.09	0.04
$h_2 - h_1$	+32.51	-156.27	-557.78	+402.30	+589.64	+187.18
$2 \log s$	8.976	8.625	8.464	8.077	8.693	8.132
$\log p = 9 - 2 \log s$	0.024	0.375	0.536	0.923	0.307	0.868
$p$ of $h_2 - h_1$	1.06	2.37	3.43	8.38	2.03	7.38

*Computation of elevations from nonreciprocal observations*

Station Occ. 1.	Pollywog	Pollywog	Pollywog	Elk	Pack Sad-	Long Ridge
Station Obs. 2.	Bosley	Stack	Craggy	Bosley	dle Bosley	Pack Sad-
Obj. sighted						dle
$\zeta$	° / "	° / "	° / "	° / "	° / "	° / "
$90^\circ - \zeta$	89 11 00	89 40 00	88 04 00	88 40 30	89 37 38	91 01 00
$90^\circ - \zeta$ in secs.	49 00	20 00	1 56 00	1 19 30	22 22	-1 01 00
log ditto	2940	1200	6960	4770	1342	3660
T	3.46835	3.07918	3.84261	3.67852	3.12775	3.56348
log $s$	4.68560	4.68558	4.68574	4.68565	4.68558	4.68562
log $s$ cot $\zeta$	4.17163	4.49539	4.20117	4.33291	4.42745	4.15591
$\alpha$ and mean $\phi$	2.32558	2.26015	2.72952	2.69708	2.24078	2.40501
log (0.5-m)	95 42.2	141 42.3	175.9 42.3	157.0 42.1	137.8 42.1	151.8 42.0
2 log $s$	9.64167	9.64167	9.64167	9.63990	9.66260	9.66001
	8.34326	8.99078	8.40234	8.66582	8.85490	8.31182
log (0.5-m) $s^2$	7.98493	8.63245	8.04401	8.30572	8.51750	7.97183
log $\rho$	6.80535	6.80439	6.80376	6.80399	6.80446	6.80409
log (2d term)	1.17958	1.82806	1.24025	1.50173	1.71304	1.16774
$s$ cot $\zeta$	+211.63	+182.03	+536.44	+497.83	+174.09	-254.10
Second term	15.12	67.31	17.39	31.75	51.65	14.71
Third term	0.01	0.01	0.04	0.04	0.01	0.01
t-o	1.51	1.51	1.51	1.45	1.50	1.40
$h_2 - h_1$	+228.27	+250.86	+555.38	+531.07	+227.25	-237.98
log $p = 9 - 2 \log s$	0.657	0.009	0.598	0.334	0.145	0.688
$p$	4.54	1.02	3.96	2.16	1.40	4.88
Station Occ. 1.	Bald Hill	Rattle	Red Moun-			
Station Obs. 2.	Red Moun-	Redding	tain			
Obj. sighted		Rock	Redding			
		Water level	Rock			
			Water level			
$\zeta$	° / "	° / "	° / "			
$90^\circ - \zeta$	88 41 22.5	91 51 30	92 32 45			
$\tan 90^\circ - \zeta$	+ 1 18 37.5	- 1 51 30	- 2 32 45			
log $s$	8.35936	8.51115	8.64799			
log $s$ cot $\zeta$	4.44942	4.56765	4.48393			
$\alpha$ and mean $\phi$	2.80878	3.07880	3.13192			
log (0.5-m)	21.4 41.6	31.3 41.5	48.0 41.4			
2 log $s$	9.62941	9.64207	9.62747			
	8.98884	9.13530	8.96786			
log (0.5-m) $s^2$	8.52825	8.77737	8.59533			
log $\rho$	6.80387	6.80410	6.80466			
log (2d term)	1.72438	1.97327	1.79067			
$s$ cot $\zeta$	+643.84	-1198.95	-1354.94			
Second term	+ 53.01	+ 94.03	+ 61.75			
Third term	0.06	0.21	0.27			
t-o	1.36	1.45	1.43			
$h_2 - h_1$	+698.27	-1103.26	-1291.49			
Cor. for reduc-		+ 0.15	+ 0.82			
tion to mean						
sea level						
$h_2 - h_1$ (corrected)		-1103.11	-1290.67			
log $p = 9 - 2 \log s$	0.101	9.865	0.032			
$p$	1.26	0.73	1.07			

The adjustment of vertical observations as practiced in the United States Coast and Geodetic Survey is made by means of observation equations and differs somewhat from the method of conditions. Of course condition equations could be employed if it were desired, just as triangulation can be adjusted by observation equations. (See the adjustment by the Variation of Geographic Coordinates, p. 91 et seq.)

Elevations for the various stations are assumed somewhat near what the final values will be. To these are added  $x$ 's to be determined by the adjustment. (See table of assumed elevations on p. 200.) By means of these, observation equations are formed by the comparison of the assumed  $h_2 - h_1$  with that determined by computation.

The method of formation is shown below and a tabulated form of all of the computation on page 201.

*Fixed elevations.*

	Meters
Bosley.....	1037.35
Stack.....	1062.69
Craggy.....	1368.31
Redding Rock.....	} 0.00
Mean sea level.....	

*Assumed and adjusted elevations*

Station	Elevation	
	Assumed + correc- tion	Adjusted
Pollywog	<i>Meters</i> 811 + $x_1$	<i>Meters</i> 811.06
Elk	507 + $x_2$	504.61
Pack Saddle	817 + $x_3$	815.74
High Divide	710 + $x_4$	708.77
Long Ridge	1059 + $x_5$	1055.96
Bald Hill	589 + $x_6$	585.16
Gordon	1259 + $x_7$	1256.12
Child	701 + $x_8$	698.19
Rattle	1103 + $x_9$	1100.46
Red Mountain	1290 + $x_{10}$	1287.70

## FORMATION OF OBSERVATION EQUATIONS

The observation equations are formed as follows:

$$\begin{aligned}
 (1) \text{ Pollywog, assumed elevation}^* &= 811 + x_1 \\
 (2) \text{ Craggy, fixed elevation} &= 1368.31 \\
 h_2 - h_1 \text{ (assumed)}^* &= +557.31 - x_1 \\
 h_2 - h_1 \text{ (observed)}^* &= +555.38 + v_1 \\
 \text{Observed} - \text{assumed}^* &= -1.93 + x_1 + v_1 = 0 \\
 -v_1 &= -1.93 + x_1 \\
 p &= \frac{1}{3} \text{ of } 3.96 = 1.32 \\
 (1) \text{ Elk, assumed elevation}^* &= 507 + x_2 \\
 (2) \text{ Pollywog, assumed elevation}^* &= 811 + x_1 \\
 h_2 - h_1 \text{ (assumed)}^* &= +304 + x_1 - x_2 \\
 h_2 - h_1 \text{ (observed)}^* &= +303.75 + v_5 \\
 \text{Observed} - \text{assumed}^* &= -0.25 - x_1 + x_2 + v_5 = 0 \\
 -v_5 &= -0.25 - x_1 + x_2 \\
 p &= 2.60
 \end{aligned}$$

In a similar manner the remaining equations are formed. These are usually formed as in the following table. The constant term is found in the column "Observed minus assumed," and the remainder of the equation in the column "Symbol."

\*Including symbolic correction.

Table of formation of observation equations

Station 1	Station 2	Weight $p$	Assumed difference of elevation $h_2-h_1$	Observed difference of elevation $h_2-h_1$	Observed minus assumed	Symbol	Adjusted difference of elevation $h_2-h_1$	Adjusted minus observed $v$	$pv$	Minus $\dagger$ $pv$	$pv^2$
*Pollywog	Craggy	1.32	557.31	555.38	-1.93	$+z_1$	557.25	+1.87	2.47		4.6159
*Pollywog	Stack	0.34	251.69	250.86	-0.83	$+z_1$	251.63	+0.77	+0.26		0.2016
*Pollywog	Bosley	1.51	226.35	226.27	-0.08	$+z_1$	226.29	-1.98	2.99		5.9198
*Elk	Bosley	0.72	530.35	531.07	+0.72	$+z_2$	532.74	+1.67	1.20		2.0080
Elk	Pollywog	2.60	304	303.75	-0.25	$-z_1+z_2$	306.45	+2.70	+7.02	-7.02	18.9540
*Elk	Elk	10.89	310	310.73	-0.73	$-z_2+z_3$	311.13	-0.40	4.36	+4.36	1.9424
*Pack Saddle	Pollywog	0.47	220.35	227.25	+6.90	$+z_3$	221.61	-5.64	-7.65		14.9505
*Pack Saddle	Elk	2.82	6	2.10	+3.90	$-z_1+z_3$	4.68	-2.58	-7.28	+7.28	18.7710
*High Divide	Elk	4.54	203	206.55	+3.55	$-z_2+z_4$	204.18	-2.19	-10.85	+10.85	25.9329
*High Divide	Pack Saddle	4.89	107	109.16	+2.16	$-z_3+z_4$	106.97	-2.39	-10.71	+10.71	23.4329
*Long Ridge	Elk	8.53	349	347.30	+1.61	$-z_4+z_5$	347.19	+0.20	1.71	-1.71	0.3412
*Long Ridge	Elk	552	552	552.12	-0.12	$-z_5+z_6$	551.35	+0.77	1.80	-1.80	1.3874
*Long Ridge	Pack Saddle	1.63	242	237.98	+4.02	$-z_3+z_6$	240.22	-2.24	3.65	+3.65	8.1787
*Long Ridge	High Divide	3.71	121	123.90	+2.90	$-z_4+z_6$	123.61	-0.29	1.08	+1.08	0.3120
*Long Ridge	Long Ridge	2.63	470	469.92	-0.08	$-z_5+z_6$	470.80	+0.88	2.31	-2.31	2.0367
*Long Ridge	Bald Hill	4.93	670	671.58	+1.58	$-z_6+z_7$	670.96	+0.62	3.06	-3.06	1.8951
*Long Ridge	High Divide	2.61	549	547.17	+1.83	$-z_4+z_7$	547.35	-0.18	0.47	+0.47	0.0846
*Long Ridge	Gordon	5.19	200	199.69	+0.31	$-z_5+z_7$	200.16	-0.47	2.44	+2.44	1.1465
*Long Ridge	Long Ridge	0.36	1290	1290.67	-0.67	$+z_{10}$	1287.70	+2.97	1.07	-1.07	3.1755
*Long Ridge	Redding Rock	0.42	701	698.27	+2.73	$-z_6+z_{10}$	702.54	-4.27	1.79	+1.79	7.6578
*Long Ridge	Bald Hill	1.06	31	32.51	+1.51	$-z_7+z_{10}$	31.58	+0.93	0.99	-0.99	0.9168
*Long Ridge	Gordon	3.43	558	557.78	-0.22	$-z_7+z_{10}$	557.93	+0.15	0.51	-0.51	0.0772
*Long Ridge	Red Mountain	2.03	589	589.64	+0.64	$+z_8-z_{10}$	589.51	-0.13	0.26	+0.26	0.0343
*Long Ridge	Red Mountain	0.24	1103	1103.11	-0.11	$+z_9$	1100.46	+2.65	0.25	-0.25	1.0354
*Long Ridge	Child	8.38	402	402.30	-0.30	$-z_8+z_9$	402.27	+0.03	1.45	+1.45	0.0975
*Long Ridge	Battle	2.37	156	156.27	+0.27	$-z_7+z_9$	155.60	-0.61	1.45	-1.45	0.8819
*Long Ridge	Gordon	7.38	157	157.18	+0.18	$+z_9-z_{10}$	157.24	+0.06	0.44	-0.44	0.0266
*Long Ridge	Red Mountain										146.3942

\* Computed from nonreciprocal observations. Weight used =  $\frac{1}{3} p$  from the computation.

† This column added for convenience in computing.

## COMPUTATION OF PROBABLE ERROR

$$\begin{aligned}\text{Probable error} &= \pm 0.6745 \sqrt{\frac{\Sigma pv^2}{\text{No. observations} - \text{No. unknowns}}} \\ &= \pm \sqrt{\frac{0.455 \Sigma pv^2}{n_o - n_u}}\end{aligned}$$

$$\Sigma pv^2 = 146.394, \log = 2.16552$$

$$\text{No. observations} - \text{No. unknowns} = 27 - 10 = 17, \quad \text{colog} = 8.76955$$

$$\text{Constant} = 0.455, \quad \log = 9.65801$$

$$\log (\text{probable error})^2 = 0.59308$$

$$\log \text{probable error} = 0.29654$$

$$\text{Probable error of unit weight} = \pm 1.98 * \text{ m.}$$

$$\log (\text{probable error, unit weight})^2 = 0.59308$$

$$\text{Weight coefficient for Long Ridge} = 2.843, \log = 0.45378$$

$$\log (\text{probable error})^2 = 0.13930$$

$$\log \text{probable error} = 0.06965$$

$$\text{Probable error for Long Ridge} = \pm 1.17 \text{ m.}$$

## FORMATION OF NORMAL EQUATIONS BY DIFFERENTIATION

The following equations are formed from the table just given:

$$-v_1 = -1.93 + x_1$$

$$-v_2 = -0.83 + x_1$$

$$-v_3 = +1.92 + x_1$$

$$-v_4 = +0.72 + x_2$$

$$-v_5 = -0.25 - x_1 + x_2$$

and so on for the rest of the 27  $v$ 's.

The function  $u$  to be made a minimum is  $\Sigma p_n v_n^2$ , or

$$\begin{aligned}u &= +1.32(-1.93 + x_1)^2 + 0.34(-0.83 + x_1)^2 + 1.51(+1.92 + x_1)^2 + 0.72(+0.72 + x_2)^2 \\ &+ 2.60(-0.25 - x_1 + x_2)^2 + 10.89(-0.73 - x_2 + x_3)^2 + 0.47(+6.90 + x_3)^2 + 2.82(+3.90 \\ &- x_1 + x_3)^2 + 4.54(-3.55 - x_2 + x_4)^2 + 4.89(+2.16 - x_3 + x_4)^2 + 8.53(+1.61 - x_4 + x_5)^2 \\ &+ 2.34(-0.12 - x_2 + x_5)^2 + 1.63(+4.02 - x_3 + x_5)^2 + 3.71(+2.90 - x_4 + x_5)^2 + 2.63(-0.08 \\ &- x_5 + x_6)^2 + 4.93(-1.58 - x_6 + x_7)^2 + 2.61(+1.83 - x_4 + x_7)^2 + 5.19(+0.31 - x_5 + x_7)^2 \\ &+ 0.36(-0.67 + x_{10})^2 + 0.42(+2.73 - x_6 + x_{10})^2 + 1.06(-1.51 - x_7 + x_{10})^2 + 3.43 \\ &(-0.22 - x_7 + x_8)^2 + 2.03(+0.64 + x_8 - x_{10})^2 + 0.24(-0.11 + x_9)^2 + 8.38(-0.30 - x_8 + x_9)^2 \\ &+ 2.37(+0.27 - x_7 + x_9)^2 + 7.38(+0.18 + x_9 - x_{10})^2.\end{aligned}$$

The function will be rendered a minimum by equating to zero the partial differential coefficients with respect to  $x_1, x_2$ , etc. By this means the following equations are derived:

$$\begin{aligned}&+1.32(-1.93 + x_1) + 0.34(-0.83 + x_1) + 1.51(+1.92 + x_1) - 2.60(-0.25 - x_1 + x_2) - 2.82 \\ &(+3.90 - x_1 + x_3) = 0 \\ &+0.72(+0.72 + x_2) + 2.60(-0.25 - x_1 + x_2) - 10.89(-0.73 - x_2 + x_3) - 4.54(-3.55 - x_2 + x_4) \\ &- 2.34(-0.12 - x_2 + x_5) = 0\end{aligned}$$

---

\* This vertical net is not of a high degree of accuracy, it being a small spur of secondary triangulation that was executed in some haste with slight attention to vertical observations. It was selected on account of its small size. The more accurate work is usually in larger nets. See list of probable errors ranging from  $\pm 0.23$  m. to  $\pm 1.83$  m. in United States Coast and Geodetic Survey Special Publication No. 13.

$$\begin{aligned}
&+10.89(-0.73-x_2+x_3)+0.47(+6.90+x_3)+2.82(+3.90-x_1+x_3)-4.89(+2.16-x_3+x_4) \\
&\quad -1.63(+4.02-x_3+x_5)=0 \\
&+4.54(-3.55-x_2+x_4)+4.89(+2.16-x_3+x_4)-8.53(+1.61-x_4+x_5)-3.71(+2.90 \\
&\quad -x_4+x_6)-2.61(+1.83-x_4+x_7)=0 \\
&+8.53(+1.61-x_4+x_5)+2.34(-0.12-x_2+x_5)+1.63(+4.02-x_3+x_5)-2.63(-0.08 \\
&\quad -x_5+x_6)-5.19(+0.31-x_5+x_7)=0 \\
&+3.71(+2.90-x_4+x_6)+2.63(-0.08-x_5+x_6)-4.93(-1.58-x_6+x_7)-0.42(+2.73 \\
&\quad -x_6+x_{10})=0 \\
&+4.93(-1.58-x_6+x_7)+2.61(+1.83-x_4+x_7)+5.19(+0.31-x_5+x_7)-1.06(-1.51 \\
&\quad -x_7+x_{10})-3.43(-0.22-x_7+x_8)-2.37(+0.27-x_7+x_9)=0 \\
&+3.43(-0.22-x_7+x_8)+2.03(+0.64+x_8-x_{10})-8.38(-0.30-x_8+x_9)=0 \\
&+0.24(-0.11+x_9)+8.38(-0.30-x_8+x_9)+2.37(+0.27-x_7+x_9)+7.38(+0.18+x_9 \\
&\quad -x_{10})=0 \\
&+0.36(-0.67+x_{10})+0.42(+2.37-x_6+x_{10})+1.06(-1.51-x_7+x_{10})-2.03(+0.64 \\
&\quad +x_8-x_{10})-7.38(+0.18+x_9-x_{10})=0
\end{aligned}$$

By multiplying and collecting, we obtain the following normals:

$$\begin{aligned}
&+8.59x_1-2.60x_2-2.82x_3\ldots\ldots\ldots-10.2786=0 \\
&-2.60x_1+21.09x_2-10.89x_3-4.54x_4-2.34x_5\ldots\ldots\ldots+24.2159=0 \\
&-2.82x_1-10.89x_2+20.70x_3-4.89x_4-1.63x_5\ldots\ldots\ldots-10.8237=0 \\
&\quad -4.54x_2-4.89x_3+24.28x_4-8.53x_5-3.71x_6-2.61x_7\ldots\ldots\ldots-34.8232=0 \\
&\quad -2.34x_2-1.63x_5-8.53x_4+20.32x_5-2.63x_6-5.19x_7\ldots\ldots\ldots+18.6066=0 \\
&\quad -3.71x_4-2.63x_5+11.69x_6-4.93x_7\ldots\ldots\ldots+0.42x_{10}\ldots\ldots\ldots+17.1914=0 \\
&\quad -2.61x_4-5.19x_5-4.93x_6+19.59x_7-3.43x_8-2.37x_9-1.06x_{10}\ldots\ldots\ldots+0.3111=0 \\
&\quad -3.43x_7+13.84x_8-8.38x_9-2.03x_{10}\ldots\ldots\ldots+3.0586=0 \\
&\quad -2.37x_7-8.38x_8+18.37x_9-7.38x_{10}\ldots\ldots\ldots-0.5721=0 \\
&\quad -0.42x_6-1.06x_7-2.03x_8-7.38x_9+11.25x_{10}\ldots\ldots\ldots-3.3228=0
\end{aligned}$$

(See the table of normals on p. 204.)

The normals are most conveniently formed from the table given on page 204. The various observation equations are written along the horizontal lines in the columns of their respective  $x$ 's. The normals are then formed as in condition equations, except that the constant terms must also be multiplied by each column and the sums taken for the constant terms in the normals, as may be seen from the direct computation of the normals above.

After the  $x$ 's are determined from the solution of the normals, they are added to the assumed elevations, giving the adjusted final elevations. The  $v$ 's are most easily determined by computing  $h_2-h_1$  from the adjusted values; if the observed  $h_2-h_1$  is subtracted from the adjusted value the respective  $v$  results. They could, of course, be computed by substituting the  $x$ 's in the observation equations, but this would require more work.

For a check the  $\sum pv$  at any station should equal zero, with the possible exception of a small amount due to dropping the decimals on the  $x$ 's. In the table on page 201, use  $pv$  from the first column if the  $x$  is positive and from the second column if the  $x$  is negative.

Table for formation of normal equations

	$p$	$N$	1	2	3	4	5	6	7	8	9	10	$pN$		$x$ 's
1	1.32	-1.93	+1										-2.5476	1	+0.0595
2	0.34	-0.83	+1										-0.2822	2	-2.3925
3	1.51	+1.92	+1										+2.8992	3	-1.2577
4	0.72	+0.72		+1									+0.5184	4	-1.2304
5	2.60	-0.25	-1	+1									-0.6500	5	-3.0402
6	10.89	-0.73		-1	+1								-7.9497	6	-3.8403
7	0.47	+6.90			+1								+3.2430	7	-2.8757
8	2.82	+3.90	-1		+1								+10.9980	8	-2.8107
9	4.54	-3.55		-1		+1							-16.1170	9	-2.5441
10	4.89	+2.16			-1	+1							+10.5624	10	-2.2951
11	8.53	+1.61				-1	+1						+13.7333		
12	2.34	-0.12		-1			+1						-0.2808		
13	1.63	+4.02			-1		+1						+6.5526		
14	3.71	+2.90				-1		+1					+10.7590		
15	2.63	-0.08					-1	+1					-0.2104		
16	4.93	-1.58						-1	+1				-7.7894		
17	2.61	+1.83				-1			+1				+4.7763		
18	5.19	+0.31					-1		+1				+1.6089		
19	0.36	-0.67						-1				+1	-0.2412		
20	0.42	+2.73							-1			+1	+1.1466		
21	1.06	-1.51								-1		+1	-1.6006		
22	3.43	-0.22									+1		-0.7546		
23	2.03	+0.64										-1	+1.2992		
24	0.24	-0.11									+1		-0.0264		
25	8.38	-0.30									+1		-2.5140		
26	2.37	+0.27									+1		+0.6399		
27	7.38	+0.18									+1	-1	+1.3284		

## Normal equations

	1	2	3	4	5	6	7	8	9	10	$\eta$	$\Sigma$
1	+8.59	-2.60	-2.82								-10.2786	-7.1086
2		+21.09	-10.89	-4.54	-2.34						+24.2159	+24.9359
3			+20.70	-4.89	-1.63						-10.8237	-10.3537
4				+24.28	-8.53	-3.71	-2.61				-34.8232	-34.8232
5					+20.32	-2.63	-5.19				+18.6066	+18.6066
6						+11.69	-4.93				+17.1914	+17.1914
7							+19.59				-1.06	+0.3111
8								-3.43	-2.37	-0.42	+2.03	+3.0586
9								+13.84	-8.38	-2.03	+0.5721	-0.3321
10									+18.37	+11.25	-3.3228	-2.9628

## Solution of normal equations

1	2	3	4	5	6	7	$\eta$	$\Sigma$
+8.59	-2.60	-2.82					-10.2786	-7.1086
$x_1$	+0.30268	+0.32829					+1.19658	+0.82754
	+21.09	-10.89	-4.54	-2.34			+24.2159	+24.9359
1	-0.7870	-0.8536					-3.1111	-2.1516
	+20.3030	-11.7436	-4.54	-2.34			+21.1048	+22.7842
		+0.57842	+0.22361	+0.11525			-1.03949	-1.12221
	+20.70	-4.89	-1.63				-10.8237	-10.3537
1	-0.9258						-3.3744	-2.3337
2	-6.7927	-2.6280	-1.3535				+12.2074	+13.1788
	+12.9815	-7.5160	-2.9835				-1.9907	+0.4913
	$x_2$	+0.57898	+0.22983				+0.15335	-0.03785
		+24.28	-8.53	-3.71	-2.61		-34.8232	-34.8232
		-1.0152	-0.5232				+4.7192	+5.0948
		-4.3516	-1.7274				-1.1526	+0.2845
		+18.9132	-10.7806	-3.71	-2.61		-31.2566	-29.4440
		$x_4$	+0.570004	+0.196159	+0.137999		+1.652634	+1.556796

*Solution of normal equations—Continued*

9	8	10	7	6	5	$\eta$	$\Sigma$
+18.37 $z_9$	- 8.38 + 0.45618	- 7.38 + 0.40174	- 2.37 + 0.12901 <sup>9</sup>			- 0.5721 + 0.03114	- 0.3321 + 0.01808
9	+13.84 - 3.8228	- 2.03 - 3.3666	- 3.43 - 1.0811			+ 3.0586 - 0.2610	+ 3.0586 - 0.1515
	+10.0172 $z_8$	- 5.3966 + 0.53873	- 4.5111 + 0.45034			+ 2.716 - 0.27928	+ 2.9071 - 0.29021
	9 8	+11.25 - 2.9648 - 2.9073	- 1.86 - 0.9521 - 2.4303	- 0.42		- 3.3228 - 0.2298 + 1.5072	- 2.9628 - 0.1334 + 1.5661
		+ 5.3779 $z_{10}$	- 4.4424 + 0.82605	- 0.42 + 0.07810		- 2.0454 + 0.38033	- 1.5299 + 0.28448
		4 9 8 10	+19.59 - 0.3602 - 0.3058 - 2.0315 - 3.6696	- 4.93 - 0.5120  - 0.3469	- 5.19 - 1.4877	+ 0.3111 - 4.3134 - 0.0738 + 1.2599 - 1.6896	+ 0.3111 - 4.0632 - 0.0428 + 1.3092 - 1.2638
			+13.2229 $z_7$	- 5.7889 + 0.43779	- 6.6777 + 0.50501	- 4.5058 + 0.34076	- 3.7495 + 0.28356
			4 10 7	+11.69 - 0.7277 - 0.0328 - 2.5343	- 2.63 - 2.1148  - 2.9234	+17.1914 - 6.1313 - 0.1597 - 1.9726	+17.1914 - 5.7757 - 0.1195 - 1.6415
				+ 8.3952 $z_6$	- 7.6681 + 0.91339	+ 8.9279 - 1.06345	+ 9.6550 - 1.15006
				2 3 4 7 6	+20.32 - 0.2697 - 0.6857 - 6.1450 - 3.3723 - 7.0040	+18.6066 + 2.4323 + 0.4575 -17.8164 - 0.2755 + 8.1547	+18.6066 + 2.6259 + 0.1129 -16.7832 - 1.8935 + 8.8188
					+ 2.8433 $z_5$	+ 8.6442 - 3.04020	+11.4875 - 4.04020

*Back solution*

5	6	7	10	8	9	4	3	2	1
-3.0402	-1.0634 -2.7769	+0.3408 -1.5353 -1.6812	+0.3803 -0.2999 -2.3755	-0.2793 -1.2950 -1.2364	+0.0311 -0.3710 -0.9220 -1.2822	+1.6526 -1.7329 -0.7533 -0.3968	+0.1534 -0.6987 -0.7124	-1.0395 -0.3504 -0.2751 -0.7275	+1.1966 -0.4129 -0.7242
-3.0402	-3.8403	-2.8757	-2.2951	-2.8107	-2.5441	-1.2304	-1.2577	-2.3925	+0.0595

## DEVELOPMENT OF FORMULAS FOR TRIGONOMETRIC LEVELING

## GENERAL STATEMENT

The formulas used on pages 198 and 199 in the computation of vertical observations were found to be lacking in some of the quantities that were appreciable when the lines were very long and high. Accordingly, a new derivation is now given that takes into account some of these quantities. As a result, the formulas derived in this development differ slightly from those used in the computation cited above, but they ought to give practically the same result in computing over lines of such length as occur therein.

The following derivation of the formulas for trigonometric leveling is based on certain approximate assumptions which fall under four general heads:

1. *Geometric approximations.*—The verticals at the two points ( $P_1$  the point occupied and  $P_2$  the point sighted on) are treated as if they lay in one plane and the intersection of this plane with the ellipsoid that represents the surface of the earth is treated as the arc of a circle whose radius is the mean radius of curvature of a vertical section through  $P_1$  and  $P_2$ . Helmert (in his *Höhere Geodäsie*, Vol. I, p. 520, and Vol. II, p. 563) investigates the error arising from these assumptions and finds it to be about 1/40 meter at a maximum when the distance  $P_1 P_2$  is about 100 kilometers.

2. *Geodetic approximations.*—The difference between the geodetic zenith and the astronomic zenith, i. e., the deflection of the plumb line, is ignored. If these deflections are known, corrections may be applied to the measured zenith distances (which, of course, are referred to the astronomical zenith) to reduce them to the geodetic zenith. Furthermore, the elevations obtained by trigonometric leveling between two points are referred to an assumed ellipsoid, while spirit leveling gives elevations referred to the geoid, so that the distances between geoid and ellipsoid must be known to make the two kinds of leveling comparable. If trigonometric leveling could be carried out with great precision, its use in connection with spirit leveling would give just this information as to the distance of the ellipsoid from the geoid. The change in the distance from geoid to ellipsoid occurring between  $P_1$  and  $P_2$  may be found from the deflections of the vertical at those points, *provided* it is assumed that the deflections vary uniformly between  $P_1$  and  $P_2$ , an assumption which may be considerably in error.

3. *Optical approximations.*—The path of the ray of light between  $P_1$  and  $P_2$  is assumed to be the arc of a circle in a vertical plane through  $P_1$  and  $P_2$ . The angle between the chord  $P_1 P_2$  and the tangent to the circle at either point is the refraction in zenith distance and it is evidently implied that this refraction is equal at  $P_1$  and  $P_2$ . If we call  $O$  (see figure 9) the center of the circle referred to in approximation 1, and call the angle  $P_1 O P_2 = \theta$ , the refraction in zenith distance of the angle  $TP_1 P_2$  ( $= \angle TP_2 P_1$ ) is written as  $m\theta$  and  $m$  is termed the coefficient of refraction. The course of a ray of light through the atmosphere depends on the variations in pressure, temperature and humidity of the medium through which it passes and may be far from circular. Our lack of knowledge of the conditions which govern the refraction is the greatest obstacle to precision in trigonometric leveling.

4. *Algebraic approximations.*—After the approximations mentioned above have been made, there is the further approximation arising from the dropping of small terms after an expansion in series. In the following developments it will be seen that only extremely small terms are dropped, and that in cases arising in practice their effect even on the sixth place of logarithms is unimportant, while in fact logarithms of only five places are commonly used for this sort of computation. The accuracy of the developments is confirmed by the numerical agreement between the approximate and the exact formulas in the examples given. (*Exact* is used in the sense of dispensing with the use of series. The formula is *inexact*, owing to the first three sets of approximations.) The examples represent rather extreme cases of those arising in practice, and other numerical examples of extreme cases give a similar agreement.

#### DEVELOPMENT OF THE FORMULAS

Figure 9 represents the vertical plane of approximation 1 common to  $P_1$  and  $P_2$ , being in fact the plane parallel to both verticals (see Helmert, *Höhere Geodäsie*, Vol. I, p. 519) on which the several points are projected.

The measured zenith distances are assumed equal to

$$\angle V_1 P_1 T = \zeta_1$$

$$\text{and } \angle V_2 P_2 T = \zeta_2.$$

The measurements are not made exactly in this plane, but the error, which is part of that involved in approximation 1, is negligible.

The refraction in zenith distance is, according to approximation 3,

$$\Delta\zeta = \angle TP_1 P_2 = \angle TP_2 P_1 = m\theta.$$

$S_1$  and  $S_2$  are points on the earth's surface in the verticals of  $P_1$  and  $P_2$ , so that the respective elevations of  $P_1$  and  $P_2$  above the surface are

$$h_1 = S_1 P_1$$

$$\text{and } h_2 = S_2 P_2.$$

The mean radius of curvature  $\rho$  of approximation 1 is given by

$$\rho = OS_1 = OS_2.$$

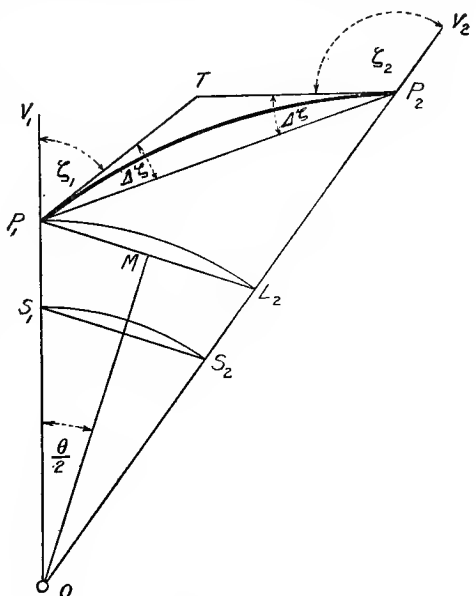


FIG. 9.

If  $s$  denotes the distance  $P_1P_2$  measured along the arc and if  $\theta$  be expressed in radians,

$$s = \rho\theta$$

or if  $\theta$  be in seconds,

$$\theta'' = \frac{s}{\rho \sin 1''}.$$

There are two cases to be considered according as to whether both or only one of the zenith distances have been measured.

### CASE I. Reciprocal zenith distances

In the triangle  $P_1OP_2$

$$\angle P_2P_1O = 180^\circ - \zeta_1 - \Delta\zeta = 180^\circ - \zeta_1 - m\theta$$

$$\angle P_1P_2O = 180^\circ - \zeta_2 - \Delta\zeta = 180^\circ - \zeta_2 - m\theta$$

$$\begin{array}{l} \text{also} \quad OP_1 = \rho + h_1 \\ \text{and} \quad OP_2 = \rho + h_2. \end{array}$$

Therefore by the law of sines

$$\frac{\rho + h_1}{\rho + h_2} = \frac{\sin(\zeta_2 + m\theta)}{\sin(\zeta_1 + m\theta)}.$$

Treating this as a proportion and taking by division,

$$\frac{(\rho + h_2) - (\rho + h_1)}{\rho + h_1} = \frac{\sin(\zeta_1 + m\theta) - \sin(\zeta_2 + m\theta)}{\sin(\zeta_2 + m\theta)}$$

or

$$h_2 - h_1 = \frac{2(\rho + h_1) \sin\left(\frac{\zeta_1 - \zeta_2}{2}\right) \cos\left(\frac{\zeta_1 + \zeta_2}{2} + m\theta\right)}{\sin(\zeta_2 + m\theta)}. \quad (A)$$

Since the sum of the angles of a triangle is  $180^\circ$ ,

$$180^\circ - \zeta_1 - m\theta + 180^\circ - \zeta_2 - m\theta + \theta = 180^\circ$$

which gives

$$\frac{\zeta_1 + \zeta_2}{2} + m\theta = 90^\circ + \frac{\theta}{2}$$

also

$$\zeta_2 + m\theta = \frac{\zeta_2 + \zeta_1}{2} + m\theta + \frac{\zeta_2 - \zeta_1}{2} = 90^\circ + \frac{\theta}{2} + \frac{\zeta_2 - \zeta_1}{2}$$

whence (A) becomes

$$h_2 - h_1 = \frac{2(\rho + h_1) \sin\left(\frac{\zeta_2 - \zeta_1}{2}\right) \sin\frac{\theta}{2}}{\cos\left(\frac{\zeta_2 - \zeta_1}{2} + \frac{\theta}{2}\right)}. \quad (1)$$

The quantity  $2(\rho + h_1) \sin\frac{\theta}{2}$  has a simple geometrical interpretation. In the figure make  $OL_2 = OP_1$  and draw  $OM \perp P_1L_2$ . Then

$$P_1M = L_2M = OP_1 \sin P_1OM = (\rho + h_1) \sin\frac{\theta}{2}.$$

Then  $2(\rho + h_1) \sin \frac{\theta}{2}$  is the chord  $P_1L_2$  or the chord  $S_1S_2$  increased to allow for the elevation of  $P_1$  above the earth's surface. In fact, the relation (1) might have been obtained by applying the law of sines directly to the triangle  $P_1P_2L_2$ , which makes it evident why  $P_1L_2$  appears.

For convenient computation\* (1) may be transformed as follows: By the sine series

$$\begin{aligned} 2 \sin \frac{\theta}{2} &= 2 \left[ \frac{\theta}{2} - \frac{1}{6} \left( \frac{\theta}{2} \right)^3 + \dots \right] \\ 2(\rho + h_1) \sin \frac{\theta}{2} &= \rho \left( 1 + \frac{h_1}{\rho} \right) \left( \theta - \frac{\theta^3}{24} + \dots \right) \\ &= \rho \theta \left( 1 + \frac{h_1}{\rho} \right) \left( 1 - \frac{\theta^2}{24} \right) = s \left( 1 + \frac{h_1}{\rho} \right) \left( 1 - \frac{s^2}{24\rho^2} \right). \quad (2) \end{aligned}$$

The remaining factors of the right-hand side of (1) may be written,

$$\begin{aligned} \frac{\sin \left( \frac{\zeta_2 - \zeta_1}{2} \right)}{\cos \left( \frac{\zeta_2 - \zeta_1}{2} + \frac{\theta}{2} \right)} &= \frac{\sin \left( \frac{\zeta_2 - \zeta_1}{2} \right)}{\cos \left( \frac{\zeta_2 - \zeta_1}{2} \right) \cos \frac{\theta}{2} - \sin \left( \frac{\zeta_2 - \zeta_1}{2} \right) \sin \frac{\theta}{2}} \\ &= \frac{\tan \left( \frac{\zeta_2 - \zeta_1}{2} \right) \sec \frac{\theta}{2}}{1 - \tan \frac{\theta}{2} \tan \left( \frac{\zeta_2 - \zeta_1}{2} \right)} \\ &= \tan \left( \frac{\zeta_2 - \zeta_1}{2} \right) \left( 1 + \frac{\theta^2}{8} \right) \left[ 1 + \frac{\theta}{2} \tan \left( \frac{\zeta_2 - \zeta_1}{2} \right) \right]. \quad (3) \end{aligned}$$

The last transformation comes by expanding  $\sec \frac{\theta}{2}$  in powers of  $\theta$  and noting that  $\tan \frac{\theta}{2} = \frac{\theta}{2}$  nearly, and that the product  $\frac{\theta}{2} \tan \frac{\zeta_2 - \zeta_1}{2}$  is small, so that,

$$\frac{1}{1 - \frac{\theta}{2} \tan \left( \frac{\zeta_2 - \zeta_1}{2} \right)} = 1 + \frac{\theta}{2} \tan \left( \frac{\zeta_2 - \zeta_1}{2} \right) \text{ very nearly.}$$

By combining (2) and (3) and using  $\theta = \frac{s}{\rho}$ , equation (1) becomes

$$\left. \begin{aligned} h_2 - h_1 &= s \left( 1 + \frac{h_1}{\rho} \right) \tan \left( \frac{\zeta_2 - \zeta_1}{2} \right) \left[ 1 + \frac{s}{2\rho} \tan \left( \frac{\zeta_2 - \zeta_1}{2} \right) \right] \left[ 1 + \frac{s^2}{12\rho^2} \right] \\ \text{or } h_2 - h_1 &= s \tan \left( \frac{\zeta_2 - \zeta_1}{2} \right) ABC \end{aligned} \right\} \quad (4)$$

---

\* See also note 1, p. 219.

where  $A = 1 + \frac{h_1}{\rho}$  = correction for elevation of station whose elevation is known,

$B = 1 + \frac{s}{2\rho} \tan\left(\frac{\zeta_2 - \zeta_1}{2}\right)$  = correction for approximate difference of elevation,

$C = 1 + \frac{s^2}{12\rho^2}$  = correction for distance.

The logarithms of  $A$ ,  $B$ , and  $C$  are given in the tables on pages 218 and 219 with the respective arguments  $h_1$ ,  $\log\left[s \tan\left(\frac{\zeta_2 - \zeta_1}{2}\right)\right]$ , and  $\log s$ . The tables show the limiting values of the respective arguments for which logarithms of  $A$ ,  $B$ , and  $C$  become 1, 2, 3, etc., units of the fifth place of decimals.

Equation (4) may be compared with the expression more commonly given for  $h_2 - h_1$ ,

$$h_2 - h_1 = s \tan\left(\frac{\zeta_2 - \zeta_1}{2}\right) \left[1 + \frac{h_1 + h_2}{2\rho} + \frac{s^2}{12\rho^2}\right] \quad (5)$$

With the tables here given ~~(5)~~ will probably be found slightly more convenient for logarithmic computation than ~~(4)~~.<sup>5</sup> The two forms are equally accurate.

#### Case II. Only one zenith distance ( $\zeta_1$ ) observed

Where two zenith distances are known, the formula, either (4) or (5), does not involve the coefficient of refraction ( $m$ ) explicitly. Where only one zenith distance is known, a value of  $m$  must be assumed from the best sources of information available.

In the triangle  $P_1 L_2 P_2$

$$\angle P_1 L_2 P_2 = 90^\circ + \frac{\theta}{2} = \angle V_1 P_1 L_2$$

$$\angle P_2 P_1 L = \angle V_1 P_1 L_2 - \angle V_1 P_1 P_2$$

$$= 90^\circ + \frac{\theta}{2} - (\zeta_1 + A\zeta) = 90^\circ - \zeta + \left(\frac{1}{2} - m\right)\theta$$

For the third angle we find, by subtracting the sum of the other angles from  $180^\circ$

$$\angle P_1 P_2 L = \zeta_1 - (1 - m)\theta.$$

By the law of sines

$$\frac{L_2 P_2}{P_1 L_2} = \frac{\sin P_2 P_1 L_2}{\sin P_1 P_2 L_2}$$

or

$$h_2 - h_1 = P_1 L_2 \frac{\cos [\zeta_1 - (\frac{1}{2} - m)\theta]}{\sin [\zeta_1 - (1 - m)\theta]}. \quad (6)$$

The chord  $P_1 L_2 = \text{chord } S_1 S_2 \times \frac{\rho + h_1}{\rho} = \text{chord } S_1 S_2 \times A$ ,  $A$  having the meaning previously given; chord  $S_1 S_2 = \text{arc } s$  very nearly; or, if greater precision is desired,  $P_1 L_2 = sAR$ , where  $R$  is the reduction factor from arc to chord.

The logarithm of the reduction factor from arc to sine is given in the Coast and Geodetic Survey Special Publication No. 8 (Formulæ and Tables for the Computation of Geodetic Positions), page 17. The logarithm of the reduction to chord is very nearly one-fourth of the reduction from arc to sine. Granting approximations 1, 2, and 3, equation (6) may be rewritten as the so-called exact formula in the following form:

$$h_2 - h_1 = sAR \frac{\cos \left( \zeta_1 - \left( \frac{1}{2} - m \right) \frac{s}{\rho \sin 1''} \right)}{\sin \left[ \zeta_1 - \left( \frac{1}{2} - m \right) \frac{s}{\rho \sin 1''} - \frac{1}{2} \frac{s}{\rho \sin 1''} \right]}. \quad (7)$$

$\sin 1''$  is introduced to convert the angle from radians to seconds of arc.  $A$  and  $R$  have the meanings previously indicated. The quantity  $(\frac{1}{2} - m)$  appears in the computation of the refraction from reciprocal zenith distances on the Coast and Geodetic Survey forms. A mean of the determinations of  $(\frac{1}{2} - m)$  from the reciprocal zenith distances should be used in computing the nonreciprocal observations.

Having found the angle  $\zeta_1 - (\frac{1}{2} - m) \frac{s}{\rho \sin 1''}$  for the numerator, the computer should subtract  $\frac{1}{2} \frac{s}{\rho \sin 1''}$  from it to get the angle for the denominator. The angle in the denominator need not be carried out very accurately, as it is always near  $90^\circ$  where the sine varies slowly.

The former Coast and Geodetic Survey formula was

$$h_2 - h_1 = s \cot \zeta_1 + \frac{(\frac{1}{2} - m) s^2}{\rho} + \frac{(1 - m) s^2 \cot^2 \zeta_1}{\rho}. \quad (8)$$

It is obtained from (6) or (7) by expanding in series and dropping certain small quantities. On some of the longer lines the quantities dropped are appreciable in computations with five-place logarithms. The development hereafter given will show that the general form of (8) may be retained by the introduction of correction-factors  $D_1$  and  $D_2$ , which are nearly unity, and by the further factor  $A$ , the correction-factor for elevation of the occupied station. The full formula will then be,

$$h_2 - h_1 = A D_1 s \cot \zeta_1 + \frac{(\frac{1}{2} - m) A D_2 s^2}{\rho} + \frac{(1 - m) s^2 \cot^2 \zeta_1}{\rho}.$$

This form may be obtained from (6) as follows:

As before

$$P_1 L_2 = 2(\rho + h_1) \sin \frac{\theta}{2}$$

or expanding by the sine series

$$P_1 L_2 = \rho \left( 1 + \frac{h_1}{\rho} \right) \left[ \theta - \frac{\theta^3}{24} + \dots \right] = A \rho \left( \theta - \frac{\theta^3}{24} + \dots \right). \quad (9)$$

The factor  $\cos [\zeta_1 - (\frac{1}{2} - m) \theta]$  in (6) may be written

$$\cos [\zeta_1 - (\frac{1}{2} - m) \theta] = \cos \zeta_1 \cos [(\frac{1}{2} - m) \theta] + \sin \zeta_1 \sin [(\frac{1}{2} - m) \theta]$$

Since  $(\frac{1}{2} - m) \theta$  is a small quantity, the series forms for its sine and cosine may be used, giving

$$\begin{aligned} \cos [\zeta_1 - (\frac{1}{2} - m) \theta] = \cos \zeta_1 \left[ 1 - (\frac{1}{2} - m)^2 \frac{\theta^2}{2} + \dots \right] \\ + \sin \zeta_1 \left[ (\frac{1}{2} - m) \theta - (\frac{1}{2} - m)^3 \frac{\theta^3}{6} + \dots \right] \end{aligned} \quad (10)$$

The third factor on the right-hand side of (6), namely,

$$\frac{1}{\sin [\zeta_1 - (1 - m) \theta]} = \operatorname{cosec} [\zeta_1 - (1 - m) \theta]$$

may be expanded in powers of  $(1 - m)\theta$  by Taylor's theorem.

$$\begin{aligned} f(\zeta_1) &= \operatorname{cosec} \zeta_1 \\ f'(\zeta_1) &= -\cot \zeta_1 \operatorname{cosec} \zeta_1 \\ f''(\zeta_1) &= \operatorname{cosec} \zeta_1 (1 + 2 \cot^2 \zeta_1) \\ f'''(\zeta_1) &= -6 \operatorname{cosec}^3 \zeta_1 \cot \zeta_1 + \cot \zeta_1 \operatorname{cosec} \zeta_1. \end{aligned}$$

This gives,

$$\begin{aligned} \frac{1}{\sin [\zeta_1 - (1 - m) \theta]} &= \operatorname{cosec} \zeta_1 + \operatorname{cosec} \zeta_1 \cot \zeta_1 (1 - m) \theta \\ &+ \operatorname{cosec} \zeta_1 (1 + 2 \cot^2 \zeta_1) (1 - m)^2 \frac{\theta^2}{2} \\ &+ \operatorname{cosec} \zeta_1 \cot \zeta_1 (6 \operatorname{cosec}^2 \zeta_1 - 1) (1 - m)^3 \frac{\theta^3}{6} + \dots \end{aligned} \quad (11)$$

The expressions (9), (10), and (11) for the factors on the right-hand side of (6) are now to be multiplied together.

In cases that actually occur,  $\theta$  and  $\cot \zeta_1$  are small quantities of about the same order of magnitude. If we call  $\cot \zeta_1$  a quantity of the first order, it is evident that  $\operatorname{cosec} \zeta_1$  differs from unity by a quantity of the second order. In forming the product from (9), (10), and (11) it is seen that the product is of the second order, and will moreover contain only terms of even order, so that if terms of the fourth order are retained the error will be of the sixth order, or the proportional error (the error as compared with the quantity itself) will be of the fourth order or of the order of  $\frac{1}{30^4}$  part of the difference of elevation,

if we suppose a quantity of the first order may be as large as  $\frac{1}{30}$ , a liberal allowance. The error, then, of the omitted terms should not affect the fifth place of logarithms and probably not the sixth. It will be seen that the expansions (9), (10), and (11) have been carried out sufficiently far for the purpose in hand, and if these expressions

be multiplied together, retaining in the product no terms of higher order than the fourth, the result may be written:

$$h_2 - h_1 = (\rho + h_1) \left\{ \theta \cot \zeta_1 \left[ 1 + \frac{6(1-m)^2 - 1}{6} \cdot \theta^2 \right] + \left( \frac{1}{2} - m \right) \theta^2 \left[ 1 + \frac{5 - 10m + 4m^2}{12} \cdot \theta^2 \right] + (1-m) \theta^2 \cot^2 \zeta_1 \right\} \quad (12)$$

Since  $\theta = \frac{s}{\rho}$ , we may write

$$h_2 - h_1 = AD_1 s \cot \zeta_1 + \frac{AD_2 \left( \frac{1}{2} - m \right) s^2}{\rho} + \frac{(1-m)s^2 \cot^2 \zeta_1}{\rho} \quad (13)$$

$$\text{where } D_1 = 1 + \frac{6(1-m)^2 - 1}{6} \cdot \frac{s^2}{\rho^2} \quad \left. \begin{array}{l} D_2 = 1 + \frac{5 - 10m + 4m^2}{12} \cdot \frac{s^2}{\rho^2} \end{array} \right\} \quad (13a)$$

The factor  $A$  has been omitted from the last term as being unnecessary, the latter being small and  $A$  near unity.  $D_1$  and  $D_2$  are also near unity. Their logarithms are tabulated in the same manner as the other quantities, the tables showing the limiting values of the argument between which  $\log D_1$  or  $\log D_2$  may be taken as 1, 2, 3, etc., units of the fifth decimal.

It may be noted that in some European surveys the term  $(1-m) \frac{s^2 \cot^2 \zeta_1}{\rho}$  is dropped and the formula for difference of elevation written as

$$h_2 - h_1 = s \cot \zeta_1 + \left( \frac{1}{2} - m \right) \frac{s^2}{\rho} \quad (14)$$

The dropped terms or factors all represent quantities of the fourth order in our expansion. The term  $\frac{(1-m)s^2 \cot^2 \zeta_1}{\rho}$  is, however, the largest of such quantities as a rule, and might be noticeable where  $D_1$  and  $D_2$  would not be.

Probably for short lines and small differences of elevation the most convenient formula would be

$$h_2 - h_1 = As \cot \zeta_1 + A \left( \frac{1}{2} - m \right) \frac{s^2}{\rho} \quad (15)$$

and for other lines formula (7).

## EXAMPLES

The data for the following examples, which illustrate the use of the formulas, come from The Transcontinental Triangulation, Special Publication No. 4, page 273, et. seq.

	At Snow Mountain West			At Ross Mountain		
	°	'	"	°	'	"
$\zeta$	91	13	39.1	89	34	04.8
$\phi$	39	22	38	38	30	20
$\alpha$	18	56	18	197	49	29
Approx. elev.	2146 meters			672 meters		

$\log s = 5.007341$ .

For mean  $\alpha$  and  $\phi$  on the Clarke spheroid of 1866,  $\log \rho = 6.80369$ .

Example 1. Difference of elevation for reciprocal zenith distances, assuming Snow Mountain West as the known elevation from formula (5).

Example 2. Same data as Example 1 worked by formula (4).

Example 3. Assuming Ross Mountain as known elevation, solve by (4).

Example 4. With refraction from reciprocal zenith distances, but with only zenith distance at Snow Mountain West appearing explicitly, find difference of elevation by (7).

Example 5. Same data as Example 4, worked by (13).

Example 6. Like example 4, except zenith distance at Ross Mountain is used.

Example 7. Like example 5, except zenith distance at Ross Mountain is used.

The agreement of the differences of elevation as computed by the various combinations of data and formulas will give an idea of the accuracy of the latter.

Example 1			Example 2	Example 3
Station 1	{ Snow Mountain			
Station 2	{ West			
	Ross Mountain			
$\zeta_1$	° ' "			
$\zeta_2$	91 13 39.1			
$\zeta_2 - \zeta_1$	89 34 04.8			
$\frac{1}{2}(\zeta_2 - \zeta_1)$	- 1 39 34.3			
$\frac{1}{2}(\zeta_2 - \zeta_1)$ in secs.	- 49 47.15			
log ditto	- 2987.15			
$\log s$	3.475257n			
$\log s \tan \frac{1}{2}(\zeta_2 - \zeta_1)$	4.685605			
Second term*	5.007341			
Third term*	3.168203n			
$h_2 - h_1$	-1473.36			
$h_1$	2145.66			
$h_2$	672.30			
$\zeta_1 + \zeta_2 - 180^\circ$	47 43.9			
$\zeta_1 + \zeta_2 - 180$ in secs.	2863.9			
log ditto	3.456958			
log $\rho$	6.803690			
log $\frac{1}{s}$	4.992659			
log $\frac{\sin 1''}{2} = 4.38454$	4.384545			
log (0.5-m)	9.637852			
(0.5-m)	0.43436			
		log $s \tan \frac{1}{2}(\zeta_2 - \zeta_1)$	3.168203n	3.168203
		log A	+146	+46
		log B	- 50	+50
		log C	+ 11	+11
		log $\langle h_2 - h_1 \rangle$	3.168310n	3.168310
		$h_2 - h_1$	-1473.36	+1473.36

Example 4		Example 6
log s.	5.007341	(Same as example 4)
colog $\rho$	3.196310	
colog sin 1''	5.314425	
log $\theta \dagger$	3.518076	
log (0.5-m)	9.637852	
log (0.5-m) $\theta$	3.155928	
(0.5-m) $\theta$	1431.''95	
$\zeta_1$	91° 13' 39.''1	89° 34' 04.''8
(0.5-m) $\theta$	23 51. 95	23 51. 95
$\zeta_1 - (0.5-m)\theta$	90 49 47. 15	89 10 12. 85
$\frac{\theta}{2}$	27 28	27 28
$\zeta_1 - (1-m)\theta$	90 22 19	88 42 45
log s	5.007341	5.007341
log A	+ 146	+ 46
log B	- 5	- 5
log cos [ $\zeta_1 - (0.5-m)\theta$ ]	8.160817n	8.160817
colog sin [ $\zeta_1 - (1-m)\theta$ ]	+ 9	+ 110
log $\langle h_2 - h_1 \rangle$	3.168308n	3.168309
$h_2 - h_1$	-1473.36	+1473.36

\*Second and third terms in example 1 computed by aid of table in General Instructions for Field Work, Coast and Geodetic Survey, pp. 36-37 (edition of 1903).

$\dagger \theta$  is used for  $\frac{s}{\rho \sin 1''}$ , as in the text.

Example 5					
log $s$	5.007341	log (0.5- $m$ )	9.637852	log $s^2$	10.015
log $A$	+146	log $s^2$	10.014682	colog $\rho$	3.196
log $D_1$	+79	colog $\rho$	3.196310	cot <sup>2</sup> $\zeta_1$	6.662
cot $\zeta_1$	8.330975 $n$	log $A$	146	log (1- $m$ )	9.971
log first	3.338541 $n$	log $D_2$	39	log third	9.844
First	-2180.424	log second	2.849029		
Second	+706.365				
Third	+ .698				
$h_2-h_1$	-1473.361				

Example 7					
log $s$	5.007341	log (0.5- $m$ )	9.637852	log $s^2$	10.015
log $A$	+46	log $s^2$	10.014682	colog $\rho$	3.196
log $D_1$	+79	colog $\rho$	3.196310	cot <sup>2</sup> $\zeta_1$	5.755
cot $\zeta_1$	7.877369	log $A$	+46	log (1- $m$ )	9.971
log first	2.884835	log $D_2$	+39	log third	8.937
First	+767.070	log second	2.848929		
Second	+706.202				
Third	+ .087				
$h_2-h_1$	+1473.359				

## RECAPITULATION OF FORMULAS

(Numbered as in foregoing discussion)

*Case I. Reciprocal observations*

Former Coast and Geodetic Survey form,

$$h_2-h_1 = s \tan \left( \frac{\zeta_2-\zeta_1}{2} \right) \left[ 1 + \frac{h_1+h_2}{2\rho} + \frac{s^2}{12\rho^2} \right]. \quad (5)$$

Reference: Page 210 and General Instructions for Field Work Coast and Geodetic Survey, pages 34-37 (edition of 1908).

Logarithmic form,

$$h_2-h_1 = s \tan \left( \frac{\zeta_2-\zeta_1}{2} \right) ABC. \quad (4)$$

Reference: Page 209 and tables.

*Case II. Nonreciprocal observations*

Former Coast and Geodetic Survey form,

$$h_2-h_1 = s \cot \zeta_1 + \frac{0.5-m}{\rho} s^2 + \frac{(1-m) s^2 \cot^2 \zeta_1}{\rho}. \quad (8)$$

Reference: Page 211 and General Instructions for Field Work Coast and Geodetic Survey, pages 34-37 (edition of 1908).

Corrected form,

$$h_2-h_1 = AD_1 s \cot \zeta_1 + \frac{0.5-m}{\rho} AD_2 s^2 + \frac{(1-m) s^2 \cot^2 \zeta_1}{\rho}. \quad (13)$$

Reference: Page 213 and tables.

“Exact” form,

$$h_2 - h_1 = sAR \frac{\cos \left[ \zeta_1 - \left( \frac{1}{2} - m \right) \frac{s}{\rho \sin 1''} \right]}{\sin \left[ \zeta_1 - \left( \frac{1}{2} - m \right) \frac{s}{\rho \sin 1''} - 2 \frac{s}{\rho \sin 1''} \right]} \quad (7)$$

Reference: Page 211 and tables; also Formule and Tables for Position Computation, Coast and Geodetic Survey Special Publication No. 8, for  $R$ .

See also additional note, page 220.

#### NOTES ON CONSTRUCTION AND USE OF TABLES

The tables are constructed with mean values of  $\rho$  and  $m$ .

$\log \rho = 6.80444$  corresponding to mean radius of curvature in latitude  $40^\circ$  for Clarke's spheroid of 1866.

$m = 0.06$ .  $m$  varies between 0.05 and 0.10 in the great majority of cases. This value near the smaller limit was taken as probably nearer the truth for the high lines, in which the correction terms tabulated are most likely to appear, than an intermediate value of 0.07 or 0.08.

$A$ ,  $B$ ,  $C$ ,  $D_1$  and  $D_2$  are all very near unity. To compute their logarithms the approximate expression  $\log (1+x) = Mx$  was used,  $M$  being the modulus of common logarithms = 0.43429.

Formulas for constructing tables:

$$A = 1 + \frac{h}{\rho} \quad \log A = \frac{Mh}{\rho} \quad h = \frac{\rho}{M} \log A = 146.78 \log A$$

$\log A$  being in units of fifth place.

$$B = 1 + \frac{s}{2\rho} \tan \left( \frac{\zeta_2 - \zeta_1}{2} \right) \quad \log B = \frac{Ms}{2\rho} \tan \left( \frac{\zeta_2 - \zeta_1}{2} \right)$$

$$\log \left[ s \tan \left( \frac{\zeta_2 - \zeta_1}{2} \right) \right] = \log \frac{2\rho}{M} + \log (\log B)$$

$$= 7.4677 + \log (\log B)$$

$$C = 1 + \frac{s^2}{12\rho^2} \quad \log C = \frac{Ms^2}{12\rho^2} \quad \log s = \log \left( \rho \sqrt{\frac{12}{M}} \right) + \frac{1}{2} \log (\log C)$$

$$= 7.5251 + \frac{1}{2} \log (\log C)$$

$$D_1 = 1 + \frac{6(1-m)^2 - 1}{6} \frac{s^2}{\rho^2} \quad \log D_1 = M \frac{6(1-m)^2 - 1}{6} \frac{s^2}{\rho^2}$$

$$\log s = 7.0578 + \frac{1}{2} \log (\log D_1)$$

$$D_2 = 1 + \frac{10 - 20m + 8m^2}{24} \frac{s^2}{\rho^2} \quad \log D_2 = M \left( \frac{5 - 10m + 4m^2}{12} \right) \frac{s^2}{\rho^2}$$

$$\log s = 7.2027 + \frac{1}{2} \log (\log D_2)$$

The values of  $\log A$ ,  $\log B$ , etc., were taken successively at 0.5, 1.5, 2.5, etc., units of the fifth place, namely, at the point where the value

of  $\log A$ ,  $\log B$ , as rounded off to 5 decimals would change by one in the fifth place. The corresponding values of  $h, \log \left[ s \tan \left( \frac{\zeta_2 - \zeta_1}{2} \right) \right]$  and  $\log s$  were then computed by the formulas above. These values are carried out far enough so that the values of  $\log A$ ,  $\log B$ , etc., may be obtained by interpolation to *six* decimals. In the numerical examples here given the values of  $\log A$ ,  $\log B$ , etc., were computed independently for the actual values of  $\rho$  and  $m$ . These results as used in the example all agree within a unit of the sixth decimal place with those found by interpolating in the tables.

The unit of length throughout the tables and formulas is the meter.

Tables

Elevation of occupied station $h_1$	$\log A$ units of fifth place	Elevation of occupied station $h_1$	$\log A$ units of fifth place
<i>Meters</i>		<i>Meters</i>	
0	0.0	3009	20.5
73	0.5	3156	21.5
220	1.5	3303	22.5
367	2.5	3449	23.5
514	3.5	3596	24.5
661	4.5	3743	25.5
807	5.5	3890	26.5
954	6.5	4036	27.5
1101	7.5	4183	28.5
1248	8.5	4330	29.5
1394	9.5	4477	30.5
1541	10.5	4624	31.5
1688	11.5	4770	32.5
1835	12.5	4917	33.5
1982	13.5	5064	34.5
2128	14.5	5211	35.5
2275	15.5	5357	36.5
2422	16.5	5504	37.5
2569	17.5	5651	38.5
2715	18.5	5798	39.5
2862	19.5	5945	40.5

$\log A$  is *positive* except in the rare case when  $h_1$  indicates a depression below mean sea level.

$A$  is used for both reciprocal and nonreciprocal observations.

For reciprocal observations only (unless formula, p. —, is used)				For nonreciprocal observations			
log approximate difference elevation = $\log s \tan \left( \frac{\zeta_2 - \zeta_1}{2} \right)^*$	log $B$ units of 5th place	log $s$	log $C$	log $s$	log $D_1$ units of 5th place	log $s$	log $D_2$ units of 5th place
2.167	0.0	4.875	0.0	4.407	0.0	4.552	0.0
2.644	0.5	5.113	0.5	4.646	0.5	4.791	0.5
2.866	1.5	5.224	1.5	4.757	1.5	4.902	1.5
3.011	2.5	5.297	2.5	4.830	2.5	4.975	2.5
3.121	3.5	5.352	3.5	4.884	3.5	5.029	3.5
3.208	4.5	5.395	4.5	4.928	4.5	5.073	4.5
3.281	5.5	5.432	5.5	4.964	5.5	5.109	5.5
3.343	6.5	5.463	6.5	4.995	6.5	5.140	6.5
3.397	7.5		7.5	5.023	7.5	5.167	7.5
3.445	8.5			5.047	8.5	5.192	8.5
3.489	9.5			5.068	9.5	5.213	9.5
3.528	10.5			5.088	10.5	5.233	10.5
3.565	11.5			5.106	11.5	5.251	11.5
3.598	12.5			5.123	12.5		12.5
3.629	13.5			5.138	13.5		
3.658	14.5			5.153	14.5		
3.685	15.5			5.167	15.5		
3.711	16.5			5.179	16.5		
3.735	17.5			5.191	17.5		
3.758	18.5			5.203	18.5		
3.779	19.5			5.214	19.5		
3.800	20.5			5.224	20.5		
3.820	21.5			5.234	21.5		
3.839	22.5			5.243	22.5		
3.857	23.5			5.252	23.5		
3.874	24.5			5.261	24.5		
	25.5				25.5		

\* Or  $\log s \cot \left[ \zeta_1 - (0.5 - m) \frac{s}{\rho \sin 1''} \right]$  for nonreciprocal observations. (See note 2, p. 220.)

$\log B$  has the same sign as the approximate difference of elevation.

$\log C$  is always positive.

$\log D_1$  and  $\log D_2$  are always positive.

#### NOTES ON THE DEVELOPMENTS

NOTE 1.—The transformation of (1), page 208, may be conducted rather more simply than is there given.

$$h_2 - h_1 = \frac{2(\rho + h_1) \sin \left( \frac{\zeta_2 - \zeta_1}{2} \right) \sin \frac{\theta}{2}}{\cos \left[ \left( \frac{\zeta_2 - \zeta_1}{2} \right) + \frac{\theta}{2} \right]} \quad (1)$$

or

$$h_2 - h_1 = \frac{2(\rho + h_1) \sin \left( \frac{\zeta_2 - \zeta_1}{2} \right) \sin \frac{\theta}{2}}{\cos \left( \frac{\zeta_2 - \zeta_1}{2} \right) \cos \frac{\theta}{2} - \sin \left( \frac{\zeta_2 - \zeta_1}{2} \right) \sin \frac{\theta}{2}}$$

Divide numerator and denominator by  $\cos \left( \frac{\zeta_2 - \zeta_1}{2} \right) \cos \frac{\theta}{2}$ ,

$$h_2 - h_1 = \frac{2 (\rho + h_1) \tan \frac{\theta}{2} \tan \left( \frac{\zeta_2 - \zeta_1}{2} \right)}{1 - \tan \frac{\theta}{2} \tan \left( \frac{\zeta_2 - \zeta_1}{2} \right)}$$

or expanding  $\tan \frac{\theta}{2}$  in series and using  $\theta = \frac{s}{\rho}$ ,

$$\begin{aligned} h_2 - h_1 &= \left( 1 + \frac{h}{\rho} \right) s \left( 1 + \frac{s^2}{12\rho^2} \right) \tan \left( \frac{\zeta_2 - \zeta_1}{2} \right) \left[ 1 + \frac{s \tan \left( \frac{\zeta_2 - \zeta_1}{2} \right)}{2\rho} \right] \\ &= s \tan \left( \frac{\zeta_2 - \zeta_1}{2} \right) ABC, \end{aligned}$$

which is equation (4).

NOTE 2.—The formula for nonreciprocal observations may be put in the same form as that for reciprocal observations.

From the equation on page 208

$$\begin{aligned} \zeta_2 &= 180^\circ - \zeta_1 - 2m \theta + \theta \\ \frac{\zeta_2 - \zeta_1}{2} &= 90 - \zeta_1 + (0.5 - m) \theta \\ \tan \left( \frac{\zeta_2 - \zeta_1}{2} \right) &= \cot [\zeta_1 - (0.5 - m) \theta] = \cot \left[ \zeta_1 - (0.5 - m) \frac{s}{\rho \sin 1''} \right] \end{aligned}$$

Substitute in (4)

$$h_2 - h_1 = s \cot \left[ \zeta_1 - (0.5 - m) \frac{s}{\rho \sin 1''} \right] ABC$$

for nonreciprocal observations analogous to

$$h_2 - h_1 = s \tan \left( \frac{\zeta_2 - \zeta_1}{2} \right) ABC$$

for reciprocal observations.  $B$  should be taken from table with argument

$$\log s \cot \left[ \zeta_1 - (0.5 - m) \frac{s}{\rho \sin 1''} \right].$$

This is the present Coast and Geodetic Survey formula for non-reciprocal observations.

















